

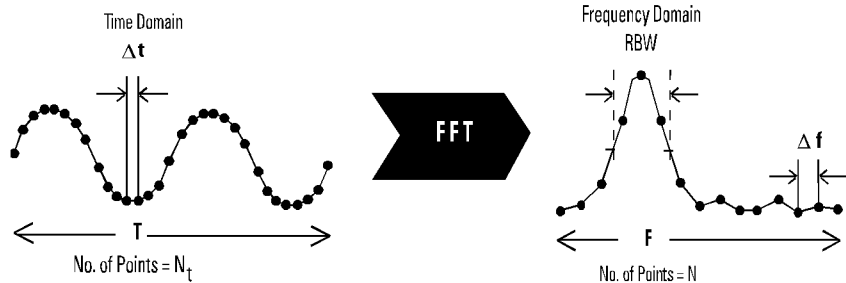
# Understanding Time and Frequency Domain Interactions in the HP 89400 Series Vector Signal Analyzers

## Product Note 89400-12

*A key strength of the HP 89400 Series vector signal analyzers (VSAs) is their ability to move easily between time and frequency domain measurements. This flexibility is ideal for analyzing signals that change with time, whether they are modulated, burst, pulsed or transient.*

*However, when using both domains simultaneously, one quickly finds that measurement parameters in one domain interact with those in the other. Changing resolution bandwidth affects the length of the time display, which may or may not be affected by the frequency span selected, and so forth.*

*Some of these interactions reflect unchanging physical law, while others are the result of algorithms or operating modes built into the analyzer's firmware for operating convenience. Although they can all be described mathematically, the graphical approach developed here will perhaps give the user a more intuitive feel for the analyzer's operation.*



**Figure 1. Description of FFT terms**

### Fundamental concepts

HP 89400 Series VSAs operate by capturing a waveform in the time domain and using it to calculate a spectrum (frequency domain) display. A fast Fourier transform (FFT) algorithm performs the calculation, beginning with a block of time domain samples where:

$T$  = time record length (sec)  
 $N_t$  = number of time samples  
 $\Delta t$  = sampling interval

Each block of  $N_t$  samples yields a block of  $N$  frequency points (i.e. a spectrum) where:

$F$  = spectrum width, or span (Hz)  
 $N$  = number of frequency points  
 $\Delta f$  = frequency point spacing

The frequency domain data is tied to the time domain data by two fundamental relationships that are the foundation for much of the discussion to follow. These are:

$$\begin{aligned} F &\propto 1/\Delta t & (1) \\ \text{RBW} &\propto 1/T & (2) \end{aligned}$$

Note that resolution bandwidth (RBW) is not the same as  $\Delta f$ . Rather, as shown in figure 1,  $\Delta f$  is the spacing of the displayed frequency points, while RBW is the analyzer's 3 dB measurement bandwidth, usually spanning more than one frequency point.

The proportionality constant in equation (1) is simply the ratio of span to sampling frequency, and is equal to 1/2.56 for baseband spans and 1/1.28 for zoom.

Equation (2) may also be written as:

$$\text{RBW} = \text{ws}/T$$

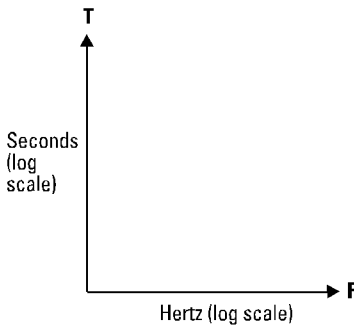
where  $\text{ws}$  is the *window shape* factor. The value for  $\text{ws}$  is dependent on window type, as follows:

| Window Type | ws  |
|-------------|-----|
| Flat Top    | 3.8 |
| Gaussian    | 2.2 |
| Hanning     | 1.5 |
| Uniform     | 1.0 |

Finally, note that this discussion applies only to the analyzer's Vector and Analog Demodulation modes, and not to the Scalar or Digital Demodulation modes.

### Time vs. Frequency — A Graphical Approach

Any HP 89400 Series VSA measurement setup involves a choice of measurement span (F) and time record length (T). The values chosen for these parameters strongly influence measurement performance in areas such as resolution, speed and accuracy—in short, they determine the analyzer's ability to provide the desired insights into the signal under test. Because of this, these two variables are the fundamental axes upon which the analyzer's operating modes will be portrayed (figure 2).

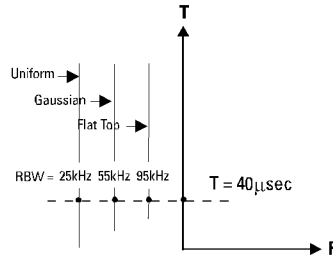


**Figure 2. Time record length (T) vs. frequency span (F) axes**

Beginning with these axes, recall from equation (2) the mathematical relationship that exists between T and resolution bandwidth:

$$RBW = ws/T$$

This allows multiple scales to be drawn for the vertical axis—a time scale for T, and one RBW scale for each available window shape (figure 3). Thus, given any value of T, one may find the corresponding RBW simply by reading directly across to the axis labeled for the type of window in use. The example shows that a 40  $\mu$ sec time record length always corresponds to a 25 kHz RBW when using the uniform window, or a 55 kHz RBW with the Gaussian window, etc.



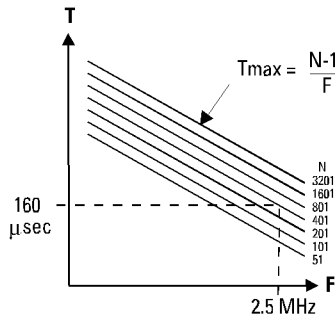
**Figure 3. Addition of RBW scales to the T vs. F axes**

### Tmax Lines

The first lines plotted on the T vs. F axes show Tmax, the maximum time record length that the analyzer can display for a given frequency span and number of frequency points, according to the relationship

$$T_{max} = (N-1)/F$$

As will be shown later, T may be adjusted to be less than Tmax, but can never exceed it. Because N is limited to certain discrete values (51, 101, 201, 401, 801, 1601, or 3201), a family of lines shows all possible values for Tmax (figure 4). As shown, a frequency span of 2.5 MHz with 401 points of resolution yields a maximum displayable time length of 160  $\mu$ sec.



**Figure 4. Tmax as a function of F and N**

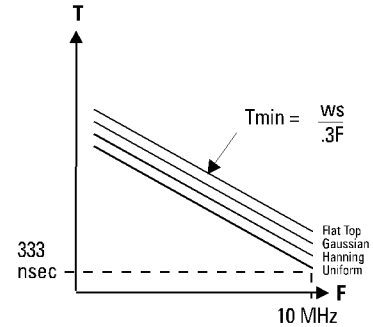
### Tmin Lines

Just as there is a maximum time record length for a given frequency span, there is also a minimum length. This length is calculated from

$$T_{min} = ws/ (.3 \times F)$$

where ws is again the window shape factor for the RBW in use. In practice, Tmin is always rounded up to the next integral multiple of  $\Delta t$ . The four available window shapes result in a family of four Tmin lines, as shown in

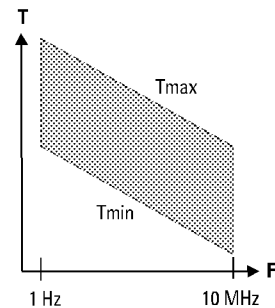
figure 5. (Notice that N, the number of frequency points, has no effect on Tmin). With 10 MHz of span and the uniform window, the analyzer's shortest possible time record is thus 352 nsec, or 333 nsec rounded up to the next multiple of  $\Delta t$  where, from equation (1),  $\Delta t = 1/(2.56 \times 10 \text{ MHz})$ , or 39 nsec.



**Figure 5. Tmin as a function of F and window type**

### Operating Region

Using these concepts, an operating region can now be defined for the HP 89400 analyzers (figure 6), bounded by 1 Hz and 10 MHz on the frequency span axis, and by Tmin and Tmax on the time axis. This region is significant in that it contains all valid combinations of T, F, N and RBW. In other words, given values for any one or more of these parameters, it shows the allowable range of values for all others. Likewise, when a boundary limit is reached for one parameter, it indicates which others can be changed in order to achieve the desired setup condition.



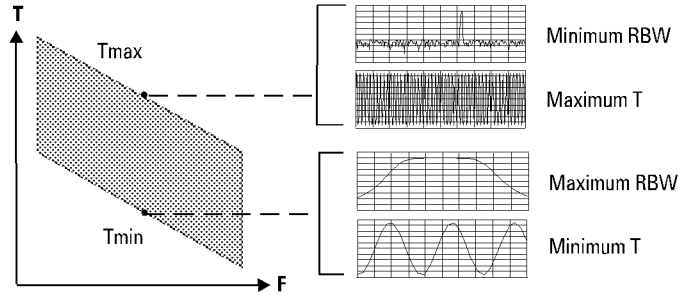
**Figure 6. HP 89400 time and frequency operating region for a given N and window type**

The following examples refer to figure 9 on the last page of this note, and show how the graphic relates to real-world measurement setups.

**Example:** For a frequency span of 100 kHz and  $N = 401$  points, available time record lengths range from 33.3  $\mu\text{sec}$  (plus rounding) to 4 msec. To obtain a longer time record length, it is necessary to either increase  $N$  or decrease the frequency span.

**Example:** For the same setup, the available RBWs range from 30 kHz to 250 Hz (uniform window). As shown on the figure, a narrower RBW would require more frequency points (higher  $N$ ) or a smaller frequency span.

**Example:** A time record length of 1 msec is available over frequency spans from 3.3 kHz to 3.2 MHz. The RBW in all cases is 1 kHz (uniform window), causing the signal resolution to range from .03% of the total span to about 30%.



**Figure 7. Time and frequency resolution tradeoffs within the HP 89400 operating region**

### RBW vs. Time Tradeoffs

Because any given frequency span allows a range of time record lengths (and vice versa), an obvious question is how to select the best combination of span and time length (i.e. the optimal operating point) for a particular measurement. The answer to this lies in the tradeoffs of RBW and time length that occur between  $T_{\text{max}}$  and  $T_{\text{min}}$ .

As shown in figure 7, an operating point on the  $T_{\text{max}}$  line provides the narrowest possible RBW, while employing the longest time record. Conversely, operation along the  $T_{\text{min}}$  line provides a much wider RBW and shorter time record length.

This ability to vary time length and RBW without having to change span is a powerful feature of the HP 89400 Series VSAs. As shown in the next section of this note, taking advantage of this capability can be quite simple.

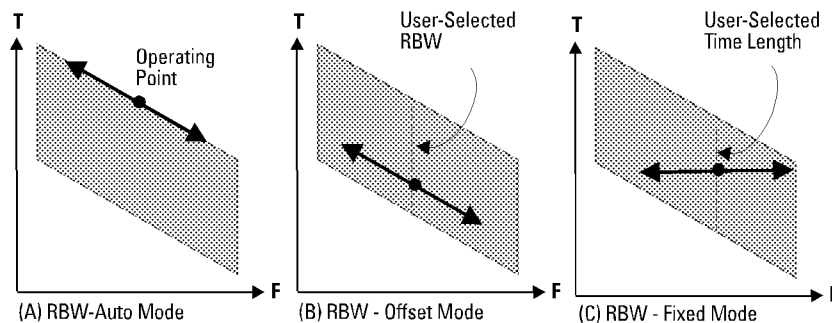
### RBW Coupling Modes

For more convenient operation, the HP 89400 analyzers can automati-

cally adjust the values of  $T$  and RBW. This is accomplished with three different RBW coupling modes.

In RBW-Auto mode, the analyzer automatically selects the narrowest frequency domain resolution (RBW) available for the chosen values of span and  $N$ . Every change in  $F$  (span) produces a proportional change in RBW and an inversely proportional change in  $T$ , with a proportionality constant of  $N-1$ . Notice that this rule precisely describes the  $F$  vs.  $T$  relationship previously derived for  $T_{\text{max}}$ . In other words, *in the RBW-Auto mode, operating points will always be along the upper boundary of the  $T$  vs.  $F$  region (figure 8A).*

(This discussion assumes that the RBW mode is set to allow arbitrary values of bandwidth. Selecting the "1-3-10" RBW mode would restrict the available RBW choices such that actual operation would be near, but rarely equal to, the  $T_{\text{max}}$  line). But what if operation away from the  $T_{\text{max}}$  line is required? This could occur if the user wished a wider



**Figure 8. Time and frequency interactions as a function of RBW coupling mode**

bandwidth than that chosen by RBW-Auto, or it might result from the need for a shorter  $T$  (perhaps to exclude portions of the input waveform). Both of these scenarios are supported by RBW coupling modes of their own.

Figure 8B shows the result of manually changing RBW. First, the analyzer's operating point moves downward to the new RBW selected (with a corresponding new time record length). In addition, the analyzer automatically changes RBW

coupling from "Auto" to "Offset." *In RBW-Offset mode, RBW and  $T$  change with  $F$  to maintain a constant ratio of RBW to span.*

Figure 8C shows the result of manually changing  $T$ . Notice that the analyzer's operating point changes as before, shifting downward to the newly selected value for  $T$  (with its corresponding RBW). However, instead of changing to RBW-Offset mode, the analyzer now selects RBW-Fixed mode. *The RBW-Fixed mode*

*holds  $T$  constant at the value chosen, even as frequency span is varied.*

The only exceptions occur when  $T$  is outside the range of  $T_{\max}$  -  $T_{\min}$  for the newly selected span. In these cases,  $T$  automatically tracks  $T_{\max}$  or  $T_{\min}$ , as appropriate.

By paying careful attention to both the RBW coupling mode and the overall boundaries of the analyzer's operating region, most HP 89400 measurements can be set up quickly and without confusion.

**Figure 9. HP 89400 Series Vector Signal Analyzers  
Time and Frequency Operating Region**

(selected values of  $N$ , window shape)

