PDL Measurements Using the 8169A Polarization Controller Product Note

Introduction

This paper discusses methods of polarization dependent loss (PDL) measurements, particularly PDL measurements using a waveplate-type polarization controller and Mueller-Stokes analysis.

The discussion includes the mathematical derivation and the achievable accuracy. An extension of this technique for the characterization of integrated optics components is also mentioned.

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PDL Measurements Using The Agilent 8169A Polarization Controller

Contents

1. Polarization Dependent Loss (PDL) Measurements

The long fiber optic transmission distances made possible by erbium doped fiber amplifiers make the transmission line optically transparent from the beginning to the end.

This increases the importance of parameters which, until now, were considered less relevant. One typical example is *polarization dependent loss (PDL).*

PDL is the maximum change in transmission of an optical component versus all possible input polarization states.

Many optical components, such as fiber optic couplers, filters, switches, attenuators and isolators, exhibit PDL.

In the communication system, polarization changes occur due to mechanical stress or temperature induced birefringence in the optical fiber. When these changes combine with PDL in fiber optic components, then unwanted signal fluctuations occur.

Therefore, it is important to know the PDL value of all components in the fiber optic transmission system.

Two PDL measurement principles exist. Both use a *polarization controller* as the key instrument for this measurement.

A first PDL measurement principle is to generate all possible polarization states with the polarization controller and to observe the changes of loss at the output power of the test device, so that the minimum and the maximum loss can be determined.

Using this method requires a calibration of the polarization controller for all used states of polarization, because the polarization controller usually produces polarization dependent power changes (internal PDL).

During PDL measurement, the same calibrated polarization states have to be set again. This is very time consuming.

The time for measuring one PDL value is in the order of 30 seconds or more, except if selected components are used for the measurement setup.

If the PDL of the polarization controller is small in comparison with the desired measurement accuracy, then no calibration is necessary and the polarization can be varied randomly.

A measurement time of 2 seconds can be achieved in combination with a fast optical power meter (average time = 20 ms) and taking 100 measurement points. This is the principle of the Agilent E5574A optical loss analyzer.

The second PDL measurement principle is to apply well known polarization states to the device under test (DUT). In this case, the polarization dependent loss is measured with an optical power meter at four different, well known polarization states.

The Mueller / Stokes analysis is then used to calculate the PDL. An advantage of this method is that arbitrary polarization states can be synthesized this way.

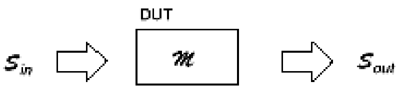
In the following section, the Mueller / Stokes method for determining PDL and its practical realization using an Agilent 8169A polarization controller are described.

2. Theory of the PDL Measurement using Mueller / Stokes Analysis

The Stokes vector $\mathcal{S} = (S0, S1, S2, S3)$ completely describes the power and polarization state of an optical wave. Each element of the vector is based on measured power levels. *S0* is the total intensity. *S1* describes the amount of linear horizontal ($S1 > 0$) or vertical polarization ($S1 < 0$). *S2* describes the amount of linear +45° ($S2 > 0$) or –45° ($S2 < 0$) polarization, and *S3* describes the amount of right-hand ($S3 > 0$) or left-hand circular ($S3 < 0$) polarization. For completely polarized light, the following relationship between the Stokes vector elements applies:

$$S0^2 = S1^2 + S2^2 + S3^2 \quad (1)$$

An incident polarized wave, characterized by the Stokes vector, $\mathcal{S}_{in}$, interacts with the optical component (DUT). The emerging lightwave can be characterized again as a Stokes vector. The interaction of the polarized light with the optical component can be described with the Mueller matrix, $\mathcal{M}$, a 4 x 4 real matrix. So the following matrix equation can be written:



$$\mathcal{S}_{out} = \mathcal{M} \bullet \mathcal{S}_{in} \quad (2)$$

This equation represents four linear equations, but only the first is interesting for PDL calculation, because SO_{out} represents the total output power. The equation derived from the first row of the Mueller matrix is as follows (the m_{1k} elements with k = 1, 2, 3, 4 represent the first row of the Mueller matrix):

$$S0_{out} = m_{11}\, S0_{in} + m_{12} S1_{in} + m_{13} S2_{in} + m_{14} S3_{in} \quad (3)$$

The first task is to determine the m_{1k} elements. To accomplish this, four different, well defined states of polarization, $\mathcal{S}_{in} = (S0, S1, S2, S3)$, are applied to the device under test. These states contain essentially the same optical power: a slight power variation is only introduced by the internal PDL of the polarization controller.

The four states are characterized by their Stokes vectors:

Input state	Input Stokes vector = ($S0_{in}, S1_{in}, S2_{in}, S3_{in}$)	Output power = $S0_{out}$
Linear horizontal (0°)	$S_{in},1 = (P_a, P_a, 0, 0)$	$P_1 = m_{11}P_a + m_{12}P_a$
Linear vertical (90°)	$S_{in},2 = (P_b, P_b, 0, 0)$	$P_2 = m_{11}P_b + m_{12}P_b$
Linear diagonal (+45°)	$S_{in},3 = (P_c, 0, P_c, 0)$	$P_3 = m_{11}P_c + m_{13}P_c$
Circular (right hand)	$S_{in},4 = (P_d, 0, 0, P_d)$	$P_4 = m_{11}P_d + m_{14}P_d$

Table 1: Input and outputs states used in PDL measurement

$P_{a,b,c,d}$ are the optical powers at the input of the device under test (DUT).
$P_{1,2,3,4}$ are the corresponding optical powers measured at the output of the DUT.
All of these powers can simply be measured with a power meter. The setup and measurement principle are illustrated in figure 1.

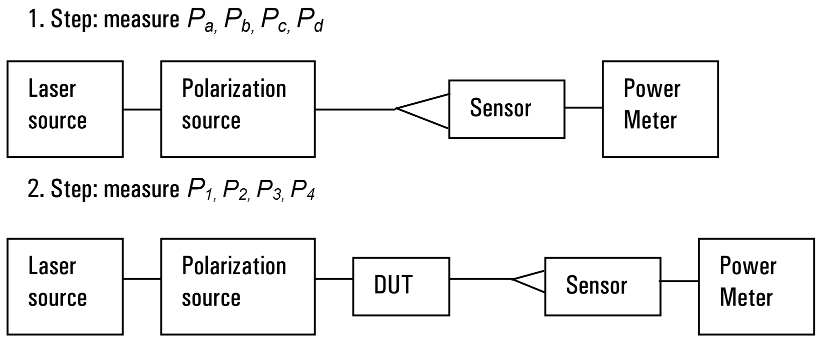


Figure 1: PDL measurement principle using Mueller/Stokes analysis

Solving the equation system of the last column in table 1 yields the first row of the ***Mueller matrix***:

$$\begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\frac{P_1}{P_a} + \frac{P_2}{P_b}\right) \\ \frac{1}{2}\left(\frac{P_1}{P_a} - \frac{P_2}{P_b}\right) \\ \frac{P_3}{P_c} - m_{11} \\ \frac{P_4}{P_d} - m_{11} \end{bmatrix}$$

A vertical representation of the first row was chosen to enhance the clarity. With these matrix elements it is possible to calculate the PDL of the test component. To accomplish this, we re-write equation 3 in the form of a power transmission:

$$T = \frac{S0_{out}}{S0_{in}} = \frac{m_{11}S0_{in} + m_{12}S1_{in} + m_{13}S2_{in} + m_{14}S3_{in}}{S0_{in}} \qquad (4)$$

The goal is now to find those Stokes vectors S_{in} which give minimum and maximum transmission *T*. The search for the extrema of *T*, under the constraint of equation 1, yields the following results (see the Appendix for the mathematical derivation):

Maximum transmission

$$T_{max} = m_{11} + \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \qquad (5)$$

Minimum transmission

$$T_{min} = m_{11} - \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \qquad (6)$$

Once the transmission extrema are known, the PDL can easily be determined as:

$$PDL_{dB} = 10 \log (\mathbf{T_{max}} / \mathbf{T_{min}}) \qquad (7)$$

3. PDL Measurement with the Agilent 8169A Polarization Controller

To obtain highly accurate PDL values, e.g. to the order of ±0.001 dB, it is necessary to do the following:

a) Calibrate the test setup for the four polarization states as described above to determine the insertion loss changes of the polarization controller versus polarization;
b) Use a very stable laser source or monitor the optical power fluctuations of the source by using a fiber optic coupler;
c) Use a detector with lowest PDL.

A PDL test setup using a stable laser source is shown in Fig. 2.
The Agilent 8169A polarization controller is used to generate the necessary polarization states, with the help of the instrument's quarter-wave (Q) and half-wave (H) retardation plates. An optical isolator ensures stable output power. Only one optical head is necessary for this type of measurement, in this case an InGaAs type optical head with depolarizing filter. The accuracy of this setup may be further improved by adding a coupler and a monitor power meter at the output of the polarization controller. By measuring ***power ratios*** instead of just powers, any changes in optical power can be canceled out. The second power meter may have higher polarization dependence without affecting the measurement result. The polarization dependence of the coupler has no influence either. However, the monitoring option is not further discussed here.

4. PDL Measurement Process

Before the calibration, the polarizer (P) at the input of the polarization controller should be adjusted to maximize the output power of the polarization controller, because otherwise a large loss may occur at the polarizer. From here on, this offset angle is called α_p . The angles of the quarter-wave plate (Q) and the half-wave plate (H) must be set with respect to α_p.
During calibration four different polarization states are generated by the polarization controller, as described above. Table 2 shows the settings of the polarizer, of the quarter-wave plate and of the half-wave plate (values in degree). These settings are most precise for the wavelength 1540 nm. At this wavelength, the retarders exhibit their nominal quarter-wave respectively half-wave retardation.

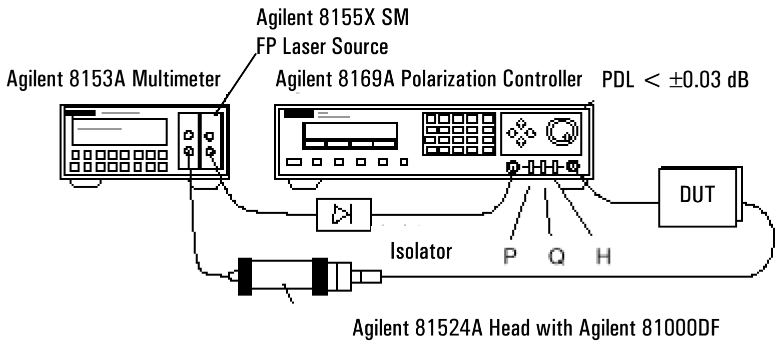


Figure 2: PDL test setup based on stable laser sourece

Polarization controller Output state	Polarizer	Q-Plate	H-Plate	Measured power
Linear horizontal, 0°:	α_p	α_p	α_p	P_a
Linear vertical, 90°:	α_p	α_p	$\alpha_{p + 45°}$	P_b
Linear diagonal, + 45°:	α_p	α_p	$\alpha_{p + 22.5°}$	P_c
Right hand circular:	α_p	$\alpha_{p + 45°}$	α_p	P_d

Table 2: Polarization controller settings and power results calibration

Pol. controller state (same as in table 3)	Measured power	Transmission T
Linear horizontal, 0°:	P_1	$T_1 = P_1 / P_a$
Linear vertical, 90°:	P_2	$T_2 = P_2 / P_b$
Linear diagonal, + 45°:	P_3	$T_3 = P_3 / P_c$
Right hand circular:	P_4	$T_4 = P_4 / P_d$

Table 3: Polarization controller settings and results during DUT measurement

When the calibration is completed, then the device under test is inserted between the output of the polarization controller and the optical head. The same states of polarization as during the calibration are set again. The corresponding optical powers are measured and the transmissions are calculated; see table 3. The Mueller matrix elements can be calculated using the equations derived above (a vertical representation was chosen again to enhance the clarity):

$$\begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \end{bmatrix} = \begin{bmatrix} \frac{T_1 + T_2}{2} \\ \frac{T_1 - T_2}{2} \\ T_3 - m_{11} \\ T_4 - m_{11} \end{bmatrix}$$

Finally, the PDL value is calculated using equations 5 to 7.

It is not necessary to re-calibrate the test set up before each DUT measurement, as long as the relationship between the four output powers of the polarization controller remains the same.

If the laser source has a wavelength which differs from 1540 nm, then the retardation of the quarter-wave plate is no longer quarter-wave, and the retardation of the half-wave plate is no longer half-wave.

The wave plates of the Agilent 8169A polarization controller are retarders of the "zero order type". These retarders have true quarter-wave and half-wave retardations at the nominal wavelength. Retarders of the "low order type" have a retardation of a multiple of waves plus a quarter-wave respectively a half-wave. Such retarders exhibit a stronger wavelength dependence than zero order retarders.

However, even zero order retarders cannot be used without correction when the wavelength is more than ±20 nm away from 1540 nm. At 1300 nm, for example, the retardation has increased by 20 %. This cannot be ignored.

	Linear vertical		Linear diagonal		RH circular	
λ (nm)	Q-Plate	H-Plate	Q-Plate	H-Plate	Q-Plate	H-Plate
1580	2.5°	46.2°	1.7°	22.3°	42.9°	-17.1°
1560	1.2°	45.6°	0.8°	22.9°	44°	-16.5°
1540	0°	45°	0°	22.5°	45°	-15.1°
1520	-1.4°	44.3°	-1°	22°	46.2°	-13.8°
1500	-2.7°	43.6°	-2°	21.4°	47.4°	-12.4°
1340	-14.7°	36.2°	-13.9°	12.8°	58.1°	-0.7°
1320	-16.3°	35.1°	-16°	11°	59.6°	1°
1300	-17.9°	34°	-18.5°	8.9°	61.2°	3°
1280	-19.6°	32.9°	-21.2°	6.5°	62.9°	5.1°
1260	-21.2°	31.7°	-24.2°	3.9°	64.7°	7.4°

Table 4: Numerical values of the waveplate positions for selected wavelengths. For wavelength values between listed values a linear approximation is suggested.

If the desired accuracy of the PDL measurement approaches ±0.001 dB, and the wavelength is outside the 1520 - 1560 nm band, then the wavelength dependent retardation of the wave plates has to be corrected, i.e. wavelength dependent settings for the wave plates have to be used.

Table 4 shows the wavelength dependent positions for retarders of the zero order type for linear vertical (90°), linear diagonal (+45°) and circular (RH) polarized light.

For horizontally polarized light the same positions as before can be used, because the fast axis of the retarders is parallel to the polarization axis of the polarizer. This is wavelength independent.

Figures 3 to 5 show the wavelength dependences of the waveplate positions in graphical form.

Notice that angular position of the half-waveplate at 1540 nm (the design wavelength) is -15.1° instead of 0°. This is necessary to obtain circular polarization state for a broad wavelength range. At 1540 nm, the position of the half-wave plate has no influence on the polarization state because a half-wave plate causes rotation only, and rotating a circular state has no effect.

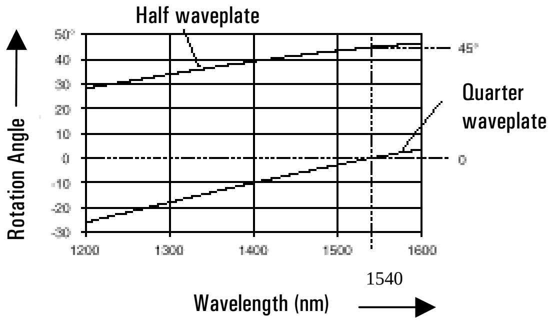


Figure 3: Waveplate positions to obtain linear vertical polarization state

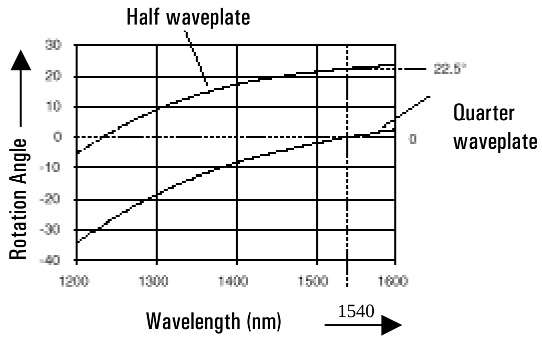


Figure 4: Waveplate positions to obtain linear diagonal polarization state

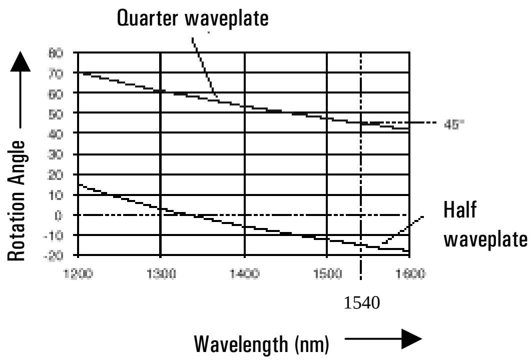


Figure 5: Waveplate positions to obtain right hand circular polarization statel

5. PDL Accuracy Considerations and Measurement Results

The accuracy of the PDL measurement is determined by the accuracy of the four polarization states and the accuracy of the optical power measurements.
The accuracy of the four polarization states depends on the settling accuracy of the polarization controller. The mechanical angular positions of the retarders with respect to the polarizer and the retardation values of half-wave and quarter-wave retarder create the polarization state. The Agilent 8169A's mechanical angular uncertainty of ±0.7° (accuracy home position = ±0.2°, accuracy = ±0.5°) causes a PDL uncertainty of approximately ±1 % of the measured PDL value.

Detector noise (Agilent 81524A with Agilent 8153A multi- meter, at relatively high power levels) causes a PDL uncertainty of typically ±1 % of the PDL value, because the PDL value is calculated from the measurement data. Laser source fluctuations will introduce additional uncertainties. If a fiber coupler is used to monitor source fluctuations, these uncertainties can usually be disregarded.

The most important part of the uncertainty of the optical power measurement is the PDL of the detector itself. This effect causes incorrect optical power measurements when the four different polarization states are set up. The PDL values of typical detectors range from 0 up to approximately 0.060 dB p-p.

For accurate PDL measurements a detector with lowest intrinsic PDL has to be used. One possible solution is to use selected detectors with low PDL. Another solution is to depolarize the laser light in front of the detector: a PDL of typically less than 0.002 dB p-p is achieved with the Agilent 81000DF depolarizer when using a Fabry-Perot laser as source.

If the DUT is connected with fiber optic connectors, then this connection can also cause PDL. The best accuracy is obtained when the DUT is connected by a fusion splice, which has no intrinsic PDL. Moving a fiber will change the polarization states at the output of this fiber. Moving the fiber before the polarization controller changes the absolute power level, which is not relevant here. Changing measurement results may occur due to the change of alignment between the PDL axis of the test component and the PDL axis of the photodetector (notice that the combined PDL of two components depends on the relative alignment between their PDL axes).

Summarizing the above, the total PDL uncertainty is (PDL_{det} = detector PDL expressed as peak-to-peak):
$U_{PDL\text{-}dut} = \pm (PDL_{det} + 2\ \% \text{ of } PDL_{dut})$.

To verify the performance of the PDL measurement setup, a DUT with known PDL was measured. The DUT was a glass plate which was anti-reflection coated on one side and tilted with respect to the optical beam axis. The theoretical PDL value was calculated by applying Fresnel's formulae to the tilted air-to-glass transition.

A fiber coupler was used to monitor the power fluctuations of the laser source. In this measurement the four polarization states reaching the detector were the same during calibration and measurement because this DUT does not alter the polarization state. Therefore, the detector's PDL has no influence on the measured PDL value.

Experimental and theoretical PDL values coincide within 0.001 dB, except in the region between -2° and +2°. Multiple reflections between the DUT and the fiber connector occur in this angle range. This causes non-predictable uncertainties in the measurement result (see figure 6).
The almost perfect agreement between experimental and theoretical data demonstrates how well this measurement method works. The repeatability for the described measurement set was around 0.001 dB.

The time for this PDL measurement depends on the settling time of the polarization controller and the average time of the optical power meter. Both can be very fast. The typical settling time for the Agilent 8169A polarization controller is less than 200 ms and the average time of the Agilent 8153A optical power meter be set to 50 ms. So it will take less than 4 x (200ms + 50 ms) = 1 s to obtain one PDL measurement value.
This method was described by Nyman [1] at the Optical Fiber Conference 1994. Nyman reported the same accuracy and repeatability of 0.001 dB for the PDL measurement.

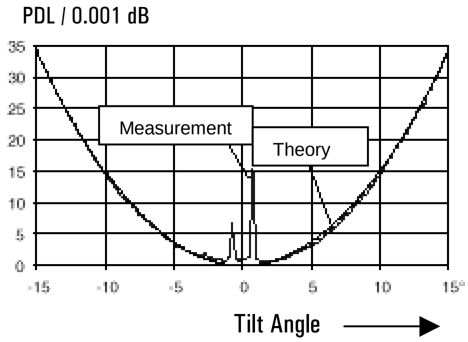


Figure 6: PDL measurement of tilted glass plate

6. Testing integrated optics devices

A frequent question in context with testing integrated optics devices is: how can the desired horizontal and vertical polarization states (at the device input) be generated? The problem is that the state of polarization at the output of a standard single-mode fiber is completely unpredictable. One possibility is using polarization maintaining fiber and mechanical rotators.

However, the necessary rotation precision is very difficult to achieve; quite often, a lateral offset is obtained during the rotation. The other possibility, using a polarization controller and conventional single mode fiber, is usually not applicable because the polarization state at the end of the fiber is unknown. Does an intelligent solution, with electronic control, exist?

Such a possibility is based on the assumption that the test device has sufficient PDL, and that the max / min transmissions occur at the horizontal / vertical (linear) polarization states. In this case, the Mueller method (originally a PDL test method) can be used to set the desired polarization states.

In many cases, a better criterion for the control of the polarization state is the modulation transfer function, e.g. at a fixed frequency of 10 MHz. In lithium-niobate modulators, for example, the insertion loss is only a weak function of the input polarization state. In contrast, the ***modulation depth*** is strongly dependent on the polarization. For such devices, the power meter should be replaced by a combination of (polarization-insensitive) photodetector and selective level meter or electrical spectrum analyzer.

The benefit of using the Mueller method is that the polarization states (Stokes vectors) leading to min /max transmission can be calculated, in addition to the calculation of the PDL.
Figure 8 shows this concept. From the calculated polarization states, the correspondent waveplate positions can be calculated (see Appendix). When this is accomplished, the modulator can be tested for its modulation transfer function, its dependence on the input polarization state, and other characteristics. Of course, the DUT's input fiber must be well secured so that the polarization state remains constant. It is advisable to write a computer program so that taking the measurements and controlling the polarization states can be done automatically. A possible measurement setup is shown in figure 9.

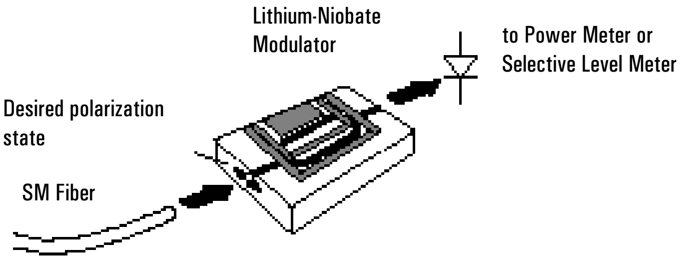


Figure7: Generating horizontal or vertical polarization states test integrated optical devices

1. Calculate Stokes vectors, S $_{2}$ S $_{3}$ S which correspond to min / max modulation index.

2. Set the waveplates of the Agilent 8169A polarization controller to generate these Stokes vectors

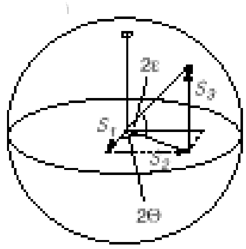


Figure 8: Calculating the polarization controller settings for IO device test.

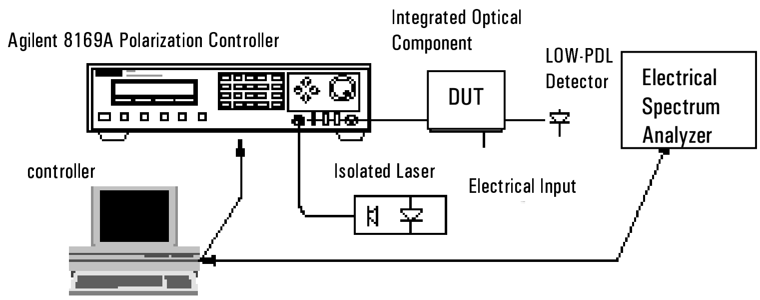


Figure 9: Possible test setup for integrated optical component

7. Summary

This paper describes how to measure PDL of optical components. It was shown that the Mueller matrix method allows fast and accurate PDL measurements. In addition, the Mueller method can be used to generate well defined polarization states for the test of integrated optical devices.
To perform a PDL measurement with this method a highly precise and repeatable polarization controller such as the Agilent 8169A is mandatory.
It was shown that this instrument can not only be used at its design wavelength around 1540 nm, but also around 1300 nm using wavelength dependent retarder settings.

Literature

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[3] E. Collet, "Polarized light, fundamentals and
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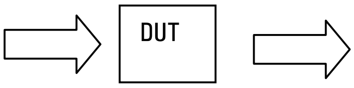
Appendix: Mathematical derivation

When totally polarized light can be assumed, then the transfer of polarized light through a test component (DUT) can be described as shown in figure 10:

<u>Power description:</u>

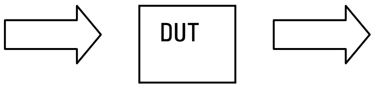
$P_{in} = SO_{in}$



$P_{out} = SO_{out}$

<u>Stokes vector description:</u>

S_{in}



SO_{out}

The input power is SO_{in}. The output power, SO_{out}, can be calculated from the first row of the Mueller matrix of the DUT:

$$SO_{out} = m_{11}S0_{in} + m_{12}S1_{in} + m_{13}S2_{in} + m_{14}S3_{in}$$

For totally polarized light, the Stokes vector components are related by:

$$SO_{in}^2 = S1_{in}^2 + S2_{in}^2 + S3_{in}^2 \quad \text{(A2)}$$

or: $$\left(\frac{S1_{in}}{SO_{in}}\right)\left(\frac{S2_{in}}{SO_{in}}\right)\left(\frac{S3_{in}}{SO_{in}}\right) = 1 \quad \text{(A3)}$$

The power transmission is defined as:

$$T = \frac{SO_{out}}{SO_{in}} \quad \text{(A4)}$$

$$T = \frac{m_{11}SO_{in} + m_{12}S1_{in} + m_{13}S2_{in} + m_{14}S3_{in}}{SO_{in}} \quad \text{(A5))}$$

$$T = m_{11} + m_{12}\frac{S1_{in}}{SO_{in}} + m_{13}\frac{S2_{in}}{SO_{in}} + m_{14}\frac{S2_{in}}{SO_{in}} \quad \text{(A6)}$$

A1. PDL Calculation

The questions are: for a given set of m_{1x} -values, which Stokes vectors lead to a maximum transmission (or modulation index)? And how large is the maximum transmission? After re-writing equation A6, we search the extrema of the following equation, in which $x_{1,2,3}$ are the power ratios leading to minimum and maximum:

$$T = m_{11} + m_{12}x_1 + m_{13}x_2 + m_{14}x_3 \quad \text{(A7)}$$

under the constraint (after re-writing equation A3):

$$\varphi = x_1^2 + x_2^2 + x_3^2 - 1 = 0 \quad \text{(A8)}$$

Searching for the extrema leads to the Lagrange functions (see [2], λ = scaling factor)

$$L_{x1} = \frac{dT}{dx_1} + \frac{\lambda\, d\varphi}{dx_1} = m_{12} + 2\square x_1 = 0 \quad \text{(A9)}$$

$$L_{x2} = \frac{dT}{dx_2} + \frac{\lambda\, d\varphi}{dx_2} = m_{13} + 2\square x_2 = 0 \quad \text{(A10)}$$

$$L_{x3} = \frac{dT}{dx_3} + \frac{\lambda\, d}{dx_3} = m_{14} + 2\square x_3 = 0 \quad \text{(A11)}$$

and $\varphi = x_1^2 + x_2^2 + x_3^2 - 1 = 0$ (A12)

The above represents four equations with four unknowns, x_1 , x_2 , x_3, λ

$$X_1 = \frac{-m_{12}}{2\lambda}, \quad x_2 = \frac{-m_{13}}{2\lambda}, \quad x3 = \frac{-m_{14}}{2\lambda} \quad \text{(A13)}$$

$$\left(\frac{m_{12}}{2\lambda}\right)^2 + \left(\frac{m_{13}}{2\lambda}\right)^2 + \left(\frac{m_{14}}{2\lambda}\right)^2 = 1 \quad \text{(A14)}$$

$$m_{12}^2 + m_{13}^2 + m_{14}^2 = 4\lambda^2 \quad \text{(A15)}$$

$$2\lambda = \pm\sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \quad \text{(A16)}$$

Inserting this result into equations A9 to A11 yields:

$$X_1 = \frac{S1_{in}}{SO_{in}} = \frac{m_{12}}{\pm\sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}} \quad \text{(A17)}$$

$$X_2 = \frac{S2_{in}}{SO_{in}} = \frac{m_{13}}{\pm\sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}} \quad \text{(A18)}$$

$$X_3 = \frac{S3_{in}}{SO_{in}} = \frac{m_{14}}{\pm\sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}} \quad \text{(A19)}$$

Calculating the transmission:

$$T = m_{11} + m_{12}x_1 + m_{13}x_2 + m_{14}x_3 \quad \text{(A20)}$$

$$T = m_{11} \pm \frac{1}{\sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}}\left(m_{12}^2 + m_{13}^2 + m_{14}^2\right) \quad \text{(A21)}$$

The result, equation A22, shows that the power transmission extrema can be calcuated from the elements of the first row of the Mueller matrix:

$$T = m_{11} \pm \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \quad \text{(A22)}$$

A2. Calculation and Setting of Min / Max Polarization States on the Agilent 8169A

The mathematical derivation is based on the book from Collet [3]. Because of its complexity, only the important steps are mentioned here.
First, determine the first row of the Mueller matrix, m_{11} to m_{14}, as described in section 4. Then calculate the input Stokes vectors for min, max transmission as in equations A17 to A19. For simplicity, we use the symbols S_1, S_2 and S_2 instead of x_1, x_2 and x_2 (this is equivalent with setting the input power $SO_{in} = 1$). We also use the acronyms *J*-state and *K*-state instead of min/ max state, because it is not *a priori* clear which waveplate position will generate which state.

In the calculation of the waveplate settings for the *J*-state, we start by assuming linear, horizontal input state, $S_1 = 1$, at the input of the quarter-wave plate (Q). Generally, rotating the Q-plate is responsible for S_3. More specifically, $-S_3$ must be generated at the output of the Q-plate because the half-waveplate (H) will then convert $-S_3$ to $+S_3$. The Q-plate also generates an undesired angle θ_{offset} in the horizontal plane of the Poincaré sphere. Rotating the H-plate is used to set the angle $2\theta_j$ (the factor of 2 was chosen because a 180° rotation of the polarization ellipse corresponds to 360° rotation on the Poincaré sphere).

The result is the following: to obtain the *J*-state, the waveplates must be rotated by the following mechanical angles (starting at 0°), to be calculated from the known Stokes elements S_1 to S_3. The last part of equation A24 represents the correction for the undesired θ_{offset} caused by the Q-plate:

$$\alpha_Q = \frac{\arcsin(S_3)}{2} \qquad (A23)$$

$$\alpha_H = \frac{\theta j}{2} + \frac{\arcsin(S_3)}{4} \qquad (A24)$$

where $2\theta_j$ is the angle obtained from rectangular-to-polar conversion of *S1* and *S2* (the additional angle of 180° is applicable when the Stokes elements *S1* and *S2* form an angle $2\theta_j$ outside of the –90° to +90° range):

$$2\theta j = \arctan\left(\frac{S_2}{S_1}\right) + \begin{matrix} 0° \\ 180° \end{matrix} \qquad (A25)$$

To obtain the *K*-state, all *S*-values must be replaced by minus *S*. The angle $2\theta_k$ can be calculated by adding 180° to $2\theta_k$ because the *J*- and *K*-states are on opposite sides of the Poincaré sphere.

Finally, we must take into account that the input polarizer was rotated to maximize the power. This is a simple correction: we assume that the polarizer was mechanically rotated by an angle α_p. Then α_p must be added to both α_Q and α_H in the equations above.

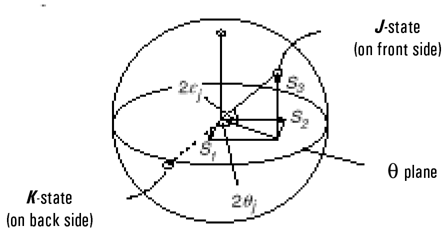


Figure 11: Constructing the desired polarization states

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