



Transfer Functions

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Purpose:

Identify and verify circuit transfer functions and practice transfer function design.

Equipment Required:

- 1 - HP 54600B Oscilloscope
- 1 - HP34401A Digital Multimeter
- 1 - HP 33120A Function Generator
- 1 - Protoboard
- 1 - 1-k Ω Resistor
- 1 - 0.01- μ F Capacitor
- Various Passive Components

Prelab:

Review the sections in Ch. 9* covering pole-zero diagrams and the sections in Chs. 10 and 11 covering transfer functions and transfer function design.

1. Transfer Function Analysis

The circuit of Fig. 1 is a first-order RC circuit with the output $v_{out}(t)$ taken across the capacitor. The input is the unit step function $v_{in}(t) = u(t)$.

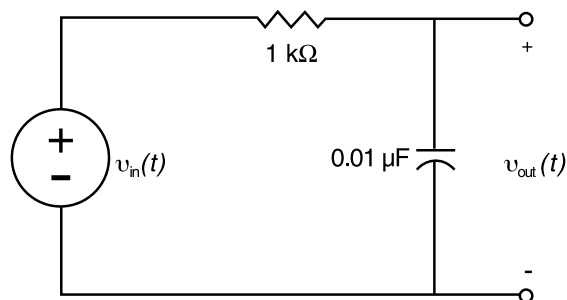


Figure 1

- Determine the circuit voltage transfer function $T_v(s)$, the step response transform $G(s)$ and the output waveform $g(t)$ for $v_{out}(t)$ as indicated in Fig. 1.
- Enter the circuit of Fig. 1 into a CCA program and determine the step response by performing a transient analysis. Plot the results and tape the output into your lab journal.

2. Transfer Function Design

- Use Table 1 to determine your assigned design project by adding your student number to that of your partner. The last digit of this sum determines the project.



- b. The input to the circuit will be a unit step function $v_{in}(t) = u(t)$. Find the step response transform $G(s)$ and the step response waveform $g(t)$ for your assigned transfer function $T_v(s)$.
- c. Design a circuit that will realize the voltage transfer function $T_v(s)$ associated with your assigned project. Use the standard component values available in your laboratory. Fully document your design.

Last Digit	0,9	1,8	2,7	3,6	4,5
Voltage Transfer Function $T_v(s)$	$\frac{10000}{s+10000}$	$\frac{s}{s+10000}$	$\frac{s}{s+20000}$	$\frac{20000}{s+20000}$	$\frac{30000}{s+30000}$

Table 1

- d. Use transient analysis in a CCA program to verify that your circuit design produces the step response waveform $g(t)$ found in step 2b.

Procedure

1. Transfer Function Analysis

- a. Connect the circuit of Fig. 1. Set the output of the function generator to produce a square wave, $1 V_{pp}$ and a DC offset of $+0.5 V_{avg}$. You will need to determine an appropriate frequency so that the circuit reaches its final value before the source changes state back to 0 V. The scope should be triggered on the rising edge of the square wave.
- b. On a half page in your journal, sketch the waveform displayed on the scope. Label the horizontal and vertical axes with the units and scales.

2. Transfer Function Design

- a. Construct your circuit design that realizes the voltage transfer function $T_v(s)$ assigned in the Prelab. The source $v_{in}(t)$ is a square wave with $1 V_{pp}$ and a DC offset V_{avg} of 0.5 V. You will need to determine an appropriate frequency to ensure the circuit will have time to reach its final value.
- b. On a half page in your journal, sketch the waveform displayed on the scope. Label the horizontal and vertical axes with the units and scales.

Conclusion

For the step function $u(t)$, all of the activity occurs at $t = 0$ when the function changes from a value of 0 for all $t < 0$, to a value of 1 for all $t > 0$. In practice this singular event is difficult to capture and record. Our approach uses a square wave that, in effect, repeats the step function over and over again. We ignore the square wave transition from a value of 1 back to 0, and concentrate our efforts on recording the positive-going transition.



The step response for the circuit of Fig. 1 is of the form:

$$g(t) = [A + Be^{-t/T}] u(t) \quad (1)$$

The initial value of $g(t)$, at $t = 0+$ is given by:

$$g(0) = A + B \quad (2)$$

The final value of $g(t)$ is found by letting time t approach infinity. When t reaches infinity, Eq. 1 reduces to:

$$g(\infty) = A \quad (3)$$

Equation 3 can be applied to the data collected in this lab exercise only if the half-period of the input waveform is greater than five time constants of the circuit, $5T_c$. This requirement on the period of the input waveform can be written as:

$$\frac{T_O}{2} \geq 5 T_c \quad (4)$$

If the condition specified in Eq. 4 is satisfied, the final value given in Eq. 3 can be determined from the sketch of the response waveform after an interval of time $t > 5T_c$.

1. Transfer Function Analysis

From the sketch of the response waveform drawn in Procedure 1b, determine the values of A , B , and T_c . (the "Exponential Waveforms" experiment provides details on determining the time constant from a sketch of the circuit response.) Use these experimental values for A , B and T_c and Eq. 5 to determine the realized transfer function $T_v(s)$ for the circuit of Fig. 1.

$$\begin{aligned} T_v(s) &= sG(s) = s \mathcal{L} g(t) \\ &= s \mathcal{L} [A + Be^{-t/T_c}] \\ &= s \left[\frac{A}{s} + \frac{B}{s + 1/T_c} \right] \\ &= \frac{(A+B)s + A/T_c}{s + 1/T_c} \end{aligned} \quad (5)$$

Does the transfer function derived from the empirical data agree with the transfer function computed in Prelab 1a? Plot a pole-zero diagram for this circuit.

2. Transfer Function Design

From the sketch of the response waveform drawn in Procedure 2b, determine the values of A , B , and T_c . Use these values to determine $T_v(s)$, and compare this result with your assigned transfer function listed in Table 1. Does this comparison support the claim that your circuit design realizes your assigned transfer function? Plot a pole-zero diagram for this circuit.

* Roland E. Thomas and Albert J. Rosa, The Analysis and Design of Linear Circuits, Prentice Hall, (New Jersey, 1994)

