

Cryptography, briefly

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802.11b slides from Dan Boneh

Cryptography

- ◆ Is
 - Tremendous tool
 - Basis for many security mechanisms
- ◆ Is not
 - The solution to all security problems
 - Secure unless implemented properly
 - Secure if used improperly

Basic Concepts in Cryptography

- ◆ Encryption scheme:
 - functions to encrypt, decrypt data
 - key generation algorithm
- ◆ Secret vs. public key
 - Public key: publishing key does not reveal key^{-1}
 - Secret key: more efficient; can have $key = key^{-1}$
- ◆ Hash function
 - map text to short hash; ideally, no collisions
- ◆ Signature scheme
 - functions to sign data and confirm signature

Cryptosystem

- ◆ A cryptosystem consists of five parts
 - A set P of plaintexts
 - A set C of ciphertexts
 - A set K of keys
 - A pair of functions
 - encrypt: $K \times P \rightarrow C$
 - decrypt: $K \times C \rightarrow P$
- such that for every key $k \in K$ and plaintext $p \in P$
 $decrypt(k, encrypt(k, p)) = p$

OK def'n for now, but doesn't include key generation or prob encryption.

Primitive example: shift cipher



◆ Shift letters using mod 26 arithmetic

- Set P of plaintexts {a, b, c, ..., x, y, z}
- Set C of ciphertexts {a, b, c, ..., x, y, z}
- Set K of keys {1, 2, 3, ..., 25}
- Encryption and decryption functions

$$\text{encrypt}(\text{key}, \text{letter}) = \text{letter} + \text{key} \pmod{26}$$

$$\text{decrypt}(\text{key}, \text{letter}) = \text{letter} - \text{key} \pmod{26}$$

□ Example

$$\text{encrypt}(3, \text{stanford}) = \text{vwdqirug}$$

Evaluation of shift cipher

◆ Advantages

- Easy to encrypt, decrypt
- Ciphertext does look garbled

◆ Disadvantages

- Not very good for long sequences of English words
 - Few keys -- only 26 possibilities
 - Regular pattern
 - $\text{encrypt}(\text{key}, x)$ is same for all occurrences of letter x
 - can use letter-frequency tables, etc



Letter frequency in English

◆ Five frequency groups [Beker and Piper]

E has probability	0.12
TAOINSHR have probability	0.06 - 0.09
DL have probability	~ 0.04
CUMWFGYPB have probability	0.015 - 0.028
VKJXQZ have probability	< 0.01

Possible to break letter-to-letter substitution ciphers.

- 1400: Arabs did careful analysis of words in Koran
- 1500: realized that letter-frequency could break substitution ciphers

One-time pad

◆ Secret-key encryption scheme (symmetric)

- Encrypt plaintext by xor with sequence of bits
- Decrypt ciphertext by xor with same bit sequence

◆ Scheme for pad of length n

- Set P of plaintexts: all n-bit sequences
- Set C of ciphertexts: all n-bit sequences
- Set K of keys: all n-bit sequences
- Encryption and decryption functions

$$\text{encrypt}(\text{key}, \text{text}) = \text{key} \oplus \text{text} \quad (\text{bit-by-bit})$$

$$\text{decrypt}(\text{key}, \text{text}) = \text{key} \oplus \text{text} \quad (\text{bit-by-bit})$$

Evaluation of one-time pad

◆ Advantages

- Easy to compute encrypt, decrypt from key, text
- As hard to break as possible
 - This is an information-theoretically secure cipher
 - Given ciphertext, all possible plaintexts are equally likely, assuming that key is chosen randomly

◆ Disadvantage

- Key is as long as the plaintext
 - How does sender get key to receiver securely?

Idea can be combined with pseudo-random generators ...

What is a "secure" cryptosystem?

◆ Idea

- If enemy intercepts ciphertext, cannot recover plaintext

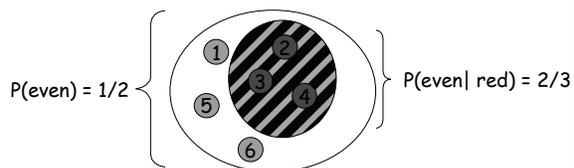
◆ Issues in making this precise

- What else might your enemy know?
 - The kind of encryption function you are using
 - Some plaintext-ciphertext pairs from last year
 - Some information about how you choose keys
- What do we mean by "cannot recover plaintext" ?
 - Ciphertext contains no information about plaintext
 - No efficient computation could make a reasonable guess

Information-theoretic security

◆ Remember conditional probability...

- Random variables X, Y, ...
- Conditional probability $P(X=x|Y=y)$
 - Probability that X takes value x, given that Y=y



Information-theoretic security

◆ Cryptosystem is *info-theoretically secure* if

$$P(\text{Plaintext}=p | \text{Ciphertext}=c) = P(\text{Plaintext}=p)$$

H	Land	1
H	Sea	2
T	Land	2
T	Sea	1

◆ Ciphertext gives no info about plaintext

Prob(1 is for Land) = Prob(1 is for Sea)
assuming that all keys are equally likely

In practice ...

- ◆ Information-theoretic security is possible
 - Shift cipher, one-time pad are info-secure
- ◆ But not practical
 - Keys would be long
 - No public-key system
- ◆ Therefore
 - Cryptosystems in use are either
 - Just found to be hard to crack, or
 - Based on computational notion of security

Example cryptosystems

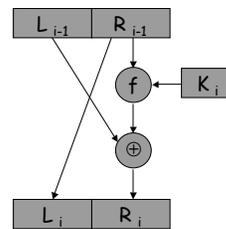
- ◆ Feistel constructions
 - Iterate a "scrambling function"
 - Example: DES, ..., AES (Rijndael)
- ◆ Complexity-based cryptography
 - Multiplication, exponentiation are "one-way" functions
 - Examples: RSA, El Gamal, elliptic curve systems, ...

Feistel networks

- ◆ Many block algorithms are *Feistel networks*
 - Examples
 - DES, Lucifer, FREAL, Khufu, Khafre, LOKI, GOST, CAST, Blowfish, ..., AES
 - Standard form for
 - Iterating a function f on parts of a message
 - Producing invertible transformation

Feistel network

Divide n -bit input in half and repeat



- ◆ Scheme requires
 - Function $f(R_{i-1}, K_i)$
 - Computation for K_i
 - e.g., permutation of key K
- ◆ Advantage
 - Systematic calculation
 - Easy if f is table, etc.
 - Invertible if K_i known
 - Get R_{i-1} from L_i
 - Compute $f(R_{i-1}, K_i)$
 - Compute L_{i-1} by \oplus

Data Encryption Standard

- ◆ Developed at IBM, widely used
- ◆ Feistel structure
 - Permute input bits
 - Repeat application of a *S-box* function
 - Apply inverse permutation to produce output
- ◆ Appears to work well in practice
 - Efficient to encrypt, decrypt
 - Not provably secure
- ◆ Improvements
 - Triple DES, AES (Rijndael)

Review: Complexity Classes



Answer in polynomial space
may need exhaustive search

If yes, can guess and check in
polynomial time



Answer in polynomial time, with
high probability

Answer in polynomial time
compute answer directly

One-way functions

- ◆ A function f is one-way if it is
 - Easy to compute $f(x)$, given x
 - Hard to compute x , given $f(x)$, for most x
- ◆ Examples (we believe)
 - $f(x) = \text{divide bits } x = y@z \text{ and multiply } f(x)=y*z$
 - $f(x) = 3^x \bmod p$, where p is prime
 - $f(x) = x^3 \bmod pq$, where p, q are primes with $|p|=|q|$

One-way trapdoor

- ◆ A function f is *one-way trapdoor* if
 - Easy to compute $f(x)$, given x
 - Hard to compute x , given $f(x)$, for most x
 - Extra "trapdoor" information makes it easy to compute x from $f(x)$
- ◆ Example (we believe)
 - $f(x) = x^3 \bmod pq$, where p, q are primes with $|p|=|q|$
 - Compute cube root using $(p-1)*(q-1)$

Public-key Cryptosystem

◆ Trapdoor function to encrypt and decrypt

- $\text{encrypt}(\text{key}, \text{message})$

↕
key pair
↕

- $\text{decrypt}(\text{key}^{-1}, \text{encrypt}(\text{key}, \text{message})) = \text{message}$

◆ Resists attack

- Cannot compute m from $\text{encrypt}(\text{key}, m)$ and key (without key^{-1})

Example: RSA

◆ Arithmetic modulo pq

- Generate secret primes p, q
- Generate secret numbers a, b with $x^{ab} \equiv x \pmod{pq}$

□ Public encryption key (n, a)

- $\text{Encrypt}(n, a, x) = x^a \pmod{n}$

□ Private decryption key (n, b)

- $\text{Decrypt}(n, b, y) = y^b \pmod{n}$

□ Main properties

- This works
- Cannot compute b from n, a
 - Apparently, need to factor $n = pq$

How RSA works (quick sketch)

◆ Let p, q be two distinct primes and let $n = p \cdot q$

- Encryption, decryption based on group Z_n^*
- For $n = p \cdot q$, order $\phi(n) = (p-1)(q-1)$
 - Proof: $(p-1)(q-1) = p \cdot q - p - q + 1$

□ Key pair: (a, b) with $ab \equiv 1 \pmod{\phi(n)}$

- $\text{Encrypt}(x) = x^a \pmod{n}$
- $\text{Decrypt}(y) = y^b \pmod{n}$
- Since $ab \equiv 1 \pmod{\phi(n)}$, have $x^{ab} \equiv x \pmod{n}$
 - Proof: if $\gcd(x, n) = 1$, then by general group theory, otherwise use "Chinese remainder theorem".

How well does RSA work?

◆ Can generate modulus, keys fairly efficiently

- Efficient rand algorithms for generating primes p, q
 - May fail, but with low probability
- Given primes p, q easy to compute $n = p \cdot q$ and $\phi(n)$
- Choose a randomly with $\gcd(a, \phi(n)) = 1$
- Compute $b = a^{-1} \pmod{\phi(n)}$ by Euclidean algorithm

□ Public key n, a does not reveal b

- This is not proven, but believed

□ But if n can be factored, all is lost ...

Message integrity

- ◆ For RSA as stated, a weak point
 - $\text{encrypt}(k*m) = (k*m)^e = k^e * m^e$
 $= \text{encrypt}(k)*\text{encrypt}(m)$
 - This leads to "chosen ciphertext" form of attack
 - If someone will decrypt *new* messages, then can trick them into decrypting *m* by asking for $\text{decrypt}(k^e * m)$
- ◆ Implementations reflect this problem
 - "The PKCS#1 ... RSA encryption is intended primarily to provide confidentiality. ... It is not intended to provide integrity." RSA Lab. Bulletin
- ◆ Additional mechanisms provide integrity

Digital Signatures

- ◆ Public-key encryption
 - Alice publishes encryption key
 - Anyone can send encrypted message
 - Only Alice can decrypt messages with this key
- ◆ Digital signature scheme
 - Alice publishes key for verifying signatures
 - Anyone can check a message signed by Alice
 - Only Alice can send signed messages

Properties of signatures

- ◆ Functions to sign and verify
 - $\text{Sign}(\text{Key}^{-1}, \text{message})$
 - $\text{Verify}(\text{Key}, x, m) = \begin{cases} \text{true} & \text{if } x = \text{Sign}(\text{Key}^{-1}, m) \\ \text{false} & \text{otherwise} \end{cases}$
- ◆ Resists forgery
 - Cannot compute $\text{Sign}(\text{Key}^{-1}, m)$ from *m* and *Key*
 - Resists existential forgery:
given *Key*, cannot produce $\text{Sign}(\text{Key}^{-1}, m)$ for any random or otherwise arbitrary *m*

Look for where this is used!

RSA Signature Scheme

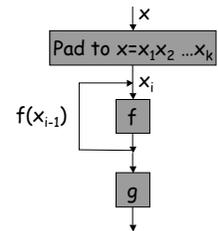
- ◆ Publish decryption instead of encryption key
 - Alice publishes decryption key
 - Anyone can decrypt a message encrypted by Alice
 - Only Alice can send encrypt messages
- ◆ In more detail,
 - Alice generates primes *p*, *q* and key pair (a, b)
 - $\text{Sign}(x) = x^a \text{ mod } n$
 - $\text{Verify}(y) = y^b \text{ mod } n$
 - Since $ab \equiv 1 \text{ mod } \phi(n)$, have $x^{ab} \equiv x \text{ mod } n$

One-way hash functions

- ◆ Length-reducing function h
 - Map arbitrary strings to strings of fixed length
- ◆ One way
 - Given y , hard to find x with $h(x)=y$
 - Given m , hard to find m' with $h(m) = h(m')$
- ◆ Collision resistant
 - Hard to find any distinct m, m' with $h(m)=h(m')$

Iterated hash functions

- ◆ Repeat use of block cipher (like DES, ...)
- Pad input to some multiple of block length
- Iterate a length-reducing function f
 - $f : 2^{2k} \rightarrow 2^k$ reduces bits by 2
 - Repeat $h_0 = \text{some seed}$
 - $h_{i+1} = f(h_i, x_i)$
- Some final function g completes calculation



Applications of one-way hash

- ◆ Password files (one way)
- ◆ Digital signatures (collision resistant)
 - Sign hash of message instead of entire message
- ◆ Data integrity
 - Compute and store hash of some data
 - Check later by recomputing hash and comparing
- ◆ Keyed hash fctns for message authentication

Crypto Summary

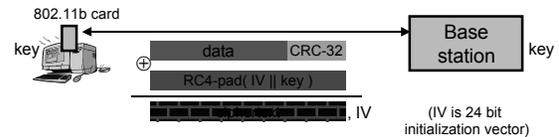
- ◆ Encryption scheme:
 - $\text{encrypt}(\text{key}, \text{plaintext})$ $\text{decrypt}(\text{key}^{-1}, \text{ciphertext})$
- ◆ Secret vs. public key
 - Public key: publishing key does not reveal key^{-1}
 - Secret key: more efficient; can have $\text{key} = \text{key}^{-1}$
- ◆ Hash function
 - map long text to short hash key; ideally, no collisions
- ◆ Signature scheme
 - private key^{-1} and public key provide "authentication"

Limitations of cryptography

- ◆ Most security problems are not crypto problems
 - This is good
 - Cryptography works!
 - This is bad
 - People make other mistakes; crypto doesn't solve them
- ◆ Examples
 - Ineffective use of cryptography
 - Example 802.11b WEP protocol
 - Security properties do not compose
 - Deployment and management problems [Anderson]

Last mile security: wireless Ethernet

- ◆ Many corporate wireless hubs installed without any privacy or authentication.
 - POP/IMAP passwords easily sniffed off the air.
 - Laptops in parking lot can access internal network.
- ◆ Intended "solution": use the WEP protocol (802.11b).
 - Provides 40-bit or 128-bit encryption using RC4 ...



Some mistakes in the design of WEP

- ◆ CRC-32 \square no packet integrity!!
 - CRC-32 is linear
 - Attacker can easily modify packets in transit, e.g. inject "rm -rf *"
 - Should use MAC for integrity
- ◆ Prepending IV is insufficient.
 - Fluhrer-Mantin-Shamir: RC4 is insecure in prepending IV mode
 - Given 106 packets can get key.
 - Implemented by Stubblefield, AirSnort, WEPcrack, ...
 - Correct construction:
 - packet-key = SHA-1(IV || key)
 - use longer IV, random.

What to do?

- ◆ Regard 802.11b networks as public channels.
 - Use SSH, SSL, IPsec, ...
- ◆ Lesson:
 - Insist on open security reviews for upcoming standards
 - Closed standards don't work: e.g. GSM, CMEA, ...
 - Open review worked well for SSL and IPsec

General Problem in Security

- ◆ Divide-and-conquer is fundamental
 - Decompose system requirements into parts
 - Develop independent software modules
 - Combine modules to produce required system
- ◆ Common belief:
 - Security properties do not compose

Difficult system development problem

Failure of Compositionality

- ◆ Example
 - Transmit messages between Alice and Bob.
 - Do not allow any eavesdropper to learn messages.
- ◆ Assumptions
 - Alice and Bob both have public encryption keys, private decryption keys.
 - Anyone can generate $\{\text{message}\}_{K_A}$, but decryption key K_A^{-1} is needed to read this message.

Example protocol

Protocol P1

A \square B : $\{\text{message}\}_{K_B}$

A \square B : K_A^{-1}

- ◆ This satisfies basic requirements
 - Message is transmitted under encryption
 - Revealing secret key K_A^{-1} does not reveal message

Similar protocol

Protocol P2

B \square A : $\{\text{message}'\}_{K_A}$

B \square A : K_B^{-1}

- ◆ Transmits message securely from B to A
 - Message is transmitted under encryption
 - Revealing secret key K_B^{-1} does not reveal message

Composition P1; P2

◆ Sequential composition of two protocols

A \square B : {message} $\}_{KB}$

A \square B : KA^{-1}

B \square A : {message} $\}_{KA}$

B \square A : KB^{-1}

◆ Definitely not secure

- Eavesdropper learns both keys, decrypts messages

Why cryptosystems fail [Anderson]

◆ Security failures not publicized

- Government: top secret
- Military: top secret
- Private companies
 - Embarrassment
 - Stock price
 - Liability

Anderson study of bank ATMs

◆ US Federal Reserve regulations

- Customer not liable unless bank proves fraud

◆ UK regulations significantly weaker

- Banker denial and negligence
- Teenage girl in Ashton under Lyme
 - Convicted of stealing from her father, forced to plead guilty, later determined to be bank error
- Sheffield police sergeant
 - Charged with theft and suspended from job; bank error

◆ 1992 class action suit

Sources of ATM Fraud

◆ Internal Fraud

- PINs issued through branches, not post
 - Bank employees know customer's PIN numbers
- One maintenance engineer modified an ATM
 - Recorded bank account numbers and PINs
- One bank issues "master" cards to employees
 - Can debit cash from customer accounts
- Bank with good security removed control to cut cost
 - No prior study of cost/benefit; no actual cost reduction
 - Increase in internal fraud at significant cost
 - Employees did not report losses to management out of fear

Sources of ATM Fraud

◆ External Fraud

- Full account numbers on ATM receipts
 - Create counterfeit cards
 - Attackers observe customers, record PIN
 - Get account number from discarded receipt
 - One sys: Telephone card treated as previous bank card
 - Apparently programming bug
 - Attackers observe customer, use telephone card
- Attackers produce fake ATMs that record PIN
- Postal interception accounts for 30% of UK fraud
 - Nonetheless, banks have poor postal control procedures
- Many other problems
 - Test sequence causes ATM to output 10 banknotes

Sources of ATM Fraud

◆ PIN number attacks on lost, stolen cards

- Bank suggestion of how to write down PIN
 - Use weak code; easy to break
- Programmer error - all customers issued same PIN
- Banks store encrypted PIN on file
 - Programmer can find own encrypted PIN, look for other accounts with same encrypted PIN
- One large bank stores encrypted PIN on mag strip
 - Possible to change account number on strip, leave encrypted PIN, withdraw money from other account

Additional problems

- ### ◆ Some problems with encryption products
- Special hardware expensive; software insecure
 - Banks buy bad solutions when good ones exist
 - Not knowledgeable enough to tell the difference
 - Poor installation and operating procedures
 - Cryptanalysis possible for homegrown crypto

More sophisticated attacks described in paper

Wider Implications

- ### ◆ Equipment designers and evaluators focus on technical weaknesses
- Banking systems have some loopholes, but these do not contribute significantly to fraud
- ### ◆ Attacks were made possible because
- Banks did not use products properly
 - Basis errors in
 - System design
 - Application programming
 - Administration

-
- ◆ Most security failures occur at the level of implementation
 - ◆ Few operational systems employ features effectively
 - ◆ Military sector experiencing same problems as civilian sector

Suggestions

- ◆ Specification should list all possible failure modes of system
- ◆ Specification should make clear what strategy has been adopted to prevent each failure
- ◆ Specification should detail implementation of each strategy
- ◆ Program must be able to be operated by personnel available

Summary

- ◆ Cryptographic systems suffer from lack of failure information
- ◆ Most security failures due to implementation and management error