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Laser vibrometer

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Laser Sicherheitshinweise

Hiermit erklärt die Firma LD Didactic GmbH,
dass es sich bei dem angebotenen Lasersystem um einen Aufbau handelt, der sowohl in Komponenten als auch im fertigen Aufbau einem Laser der Klasse 3A, 3B oder 4 nach DIN EN 60 825-1 entspricht. Typischerweise ist die Pumpdiode eine Nd:YAG Laser Klasse 4, ein HeNe Laser mit Auskoppler Klasse 3A, aber ein HeNe mit zwei hochreflektierenden Spiegeln nur Klasse 1. Bitte die Anleitung oder Aufkleber beachten.

Aus Haftungsgründen dürfen diese Geräte oder Gerätesammlungen nicht an Privatleute verkauft werden. Der Einsatz von Lasern oberhalb Klasse 2 an allgemeinbildendenden Schulen ist in Deutschland nicht gestattet.

Gewerbliche Abnehmer, Schulen und Universitäten werden hiermit darauf hingewiesen, dass aus dem missbräuchlichen Betrieb der Geräte ein Verletzungsrisiko, speziell für die Augen, resultiert.

Dem Benutzer obliegt insbesondere:
- Die relevanten Unfallverhütungsvorschriften zu beachten, zur Zeit beispielsweise BGV B2 und BGI 832
- die OstrV zu beachten „Verordnung zum Schutz der Beschäftigten vor Gefährdungen durch künstliche optische Strahlung“
- Der Betrieb der Geräte muss rechtzeitig beim Gewerbeaufsichtsamt und der Berufsgenossenschaft angezeigt werden.
- Der Betreiber muss schriftlich einen Laserschutzbeauftragten benennen, der für die Einhaltung der Schutzmaßnahmen verantwortlich ist.
- Die Geräte sind nur für den Betreib in umschlossenen Räumen vorgesehen, deren Wände die Ausbreitung des Laserstrahls begrenzen.
- Der Laserbereich ist deutlich und dauerhaft zu kennzeichnen.
- Ab Laserklasse 4 ist eine Laser-Warnleuchte am Raumzugang notwendig.

- Die Geräte sind zur Lehre und Ausbildung in Berufsschulen, Universitäten oder ähnlichen Einrichtungen gedacht.
- Die Geräte nur innerhalb der in den Anleitungen vorgegebenen Betriebsbedingungen betreiben.
- Die Geräte nur von entsprechend unterwiesenen Mitarbeitern und Studierenden benutzen lassen.

Bei Handhabung des Gerätes durch Studenten müssen diese von entsprechend geschultem Personal überwacht werden.

Als praktische Ratschläge:

- Vor dem Einschalten auf Beschädigungen prüfen
- Nicht in den Strahl blicken
- Den Laserstrahl so führen, dass sich keine Personen, Kinder oder Tiere ungewollt im Strahlbereich befinden können
- Den Laserstrahl nicht auf reflektierende Flächen oder in den freien Raum richten
- Nicht mit reflektierenden Gegenständen im Laserstrahl arbeiten
- Armbanduhr, Schmuck und andere reflektierende Gegenstände ablegen.
- Beim Einsetzen optischer Bauteile den Laserstrahl an der Quelle abschalten oder geeignet abdecken, bis die Bauteile positioniert sind
- Teilweise wird mit unsichtbaren Laserstrahlen gearbeitet, deren Verlauf nicht sichtbar ist.
- Falls nötig, Laserschutzbrillen oder Laserjustierbrillen benutzen.

Die Firma LD Didactic GmbH haftet nicht für eine missbräuchliche Verwendung der Geräte durch den Kunden.
Der Kunde verpflichtet sich hiermit die Geräte nur entsprechend der rechtlichen Grenzen einzusetzen und insbesondere den Laserstrahl nicht im Straßenverkehr oder Luftraum zu verwenden oder in anderer Form auf Personen und Tiere zu richten.

Der Kunde bestätigt, das er befugt ist, diesen Laser zu erwerben und zu verwenden.

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Laser Safety Notes

LD Didactic GmbH informs the customer this is laser equipment of either class 3A, 3B or 4 according to IEC 60 825. Typically a Nd:YAG Pump Diode is class 4, a HeNe with output coupler class 3A, but a HeNe with two high reflecting is mirrors only class 1. Please see manual or attached labels for the exact specification of the laser.
Special safety precautions are necessary. Please check with local regulations. Typically the use requires a safety sign and a warning lamp that is on when the laser is activated and it might also be necessary to do and document a risk assessment.

Due to product liability, the laser must not be sold to individual persons. Companies, higher schools and universities might use it, but are notified that misuse of the laser poses a health risk, especially for the eyes.

The intended use of this equipment is for lessons, education and research in higher schools, universities or similar institutions.
Do not operate the devices outside parameters specified in the manual.
People using the laser must be properly trained and students must be supervised.

As a general guidance, the user is advised to:
- Check the laser for damages before use
- Not to look into the beam
- Take necessary measures that no people or animals can accidentally enter the beam area
- do not direct the beam on reflecting surfaces or into public areas
- do not work close to the light path with reflecting tools
- take off all jewelry and wristwatches when working with the laser to avoid reflections
- While placing or removing optical parts in the light path, switch off the laser or cover its exit
- Some of the experiments use invisible laser beams, but still might hurt the eye
- use laser protection glasses or laser adjustment glasses where necessary
- supervise students by trained personnel when they work with the laser system
- use the laser system only as described in the instruction manuals

Customer acknowledges the receipt of this information.

The customer indemnifies LD Didactic from liability for any damages that occur because of misuse of the laser.

The customer confirms that he will obey all local regulations and is allowed by law to buy and use the laser system.

Contents

1 Introduction

From practical experience we know that whenever dynamic forces are exciting vibratory structures sound and vibrations will result. Due to such vibrations intensified wear, malfunction and even persistent material damage may occur at machines, vehicles and buildings. Additionally, undesired vibration forces of machines can cause substantial damages at plants and buildings, from small cracks in walls up to complete destruction.

The dynamic forces, which excite a structure to vibrate, can be of very different kind depending on the cause of these forces. Free mass forces, which develop e.g. with the turn of the crankshaft in the engine of a car, can energize the vehicle and its body in a similar way like a bumpy roadway or imbalance at the wheels. These vibrations are radiated from the surface of the body as airborne sound and can lead to substantial comfort reduction for the passengers. The effects of vibrations and noise (i.e. unwanted sound) on humans reach from annoyance and fatigue up to decreased well-being and serious diseases. From the physical point of view noise is the part of the vibration energy of a structure which is transferred to the air of the environment. Vibrations and noise stand thus in close relation to each other. A basic condition for each investigation of vibrations is the specification of the movement procedures and the verification of them by measurements and appropriate evaluation. That applies both, to the development of machines, which are based on the utilization of vibrations (e.g. shaking filters) and the construction of oscillation-poor machines and devices (e.g. measuring instruments), and likewise to all problems of vibration insulation in buildings, vehicles and all areas, where vibrations are not desired. The most noise and vibration problems are to be due to resonance features. Resonance arises if the frequency of the exciting force matches one of the natural resonance frequencies of the swinging system. It is very important to investigate both the natural frequency spectrum of the structure and all exciter frequencies, which affect the vibratory structure. Both can be determined from measurements with vibrometers.

Conventional vibrometers are based on sensors which have to be attached to the object of interest. Especially for small and light objects a substantial damping of the oscillation amplitude or even a distortion of the whole natural frequency spectrum of the object by the attached sensor is to be expected. Further, some objects cannot be equipped with a detector because of a remote or hidden location or improper surface properties of the object. Laser Doppler Vibrometers (LDV) overcome all these difficulties: they work contactless and allow measurements on a distant object. Nowadays industrial test stands are equipped with multidimensional LDVs which are able to investigate several surface positions of vibrating objects at the same time. But the daily use of vibrometers is not only restricted to industrial investigations. Applications can be found in scientific as well as medical research of all different fields, and even at quality controls of fruits and vegetables vibrometers are involved. Two interesting examples of practical implementation of vibrometers in medicine and biology will be presented here.

1.1 Laser Audiometer

With Laser Doppler Vibrometry not only a new instrument and a new measuring technique is introduced to otoscopy, but also new diagnostic possibilities are opened. Up to now it was for instance not possible to check the performance of the ears of children bellow a certain age. With the conventional methods the infants have to be able to react obviously on acoustical stimulation what is possible at an age of several months only. On the other hand, some hearing disorders should be medicated as early as possible to retrieve a functioning sense of hearing. With LDV babies of an age of a few weeks already can be examined and if necessary medicated adequately. But examining of babies is only one example of the abilities of a so called laser audiometer. In general, the vibrometer investigation in many fields of otoscopy supplies objective results, which do not depend on subjective impressions and feelings of the patient.

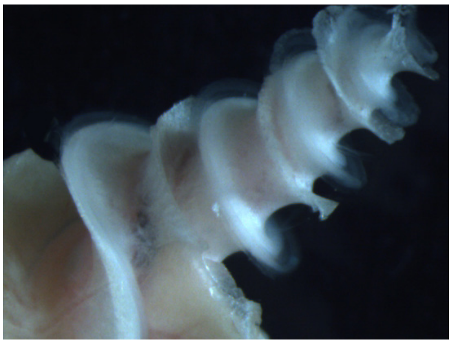


Figure 1: The cochlea of the inner ear

With the laser audiometer the matter of investigation is the vibration of the ear drum in response to acoustic stimulation. The laser light is focused onto the ear drum where the light is reflected; its frequency is (Doppler) shifted by the motion of the ear drum. From the frequency shift the velocity of the ear drum is extracted. Mechanical conditions of the middle ear and the cochlea can be determined from the dependence of ear-drum velocity on stimulus frequency and sound pressure.

1.2 Dancing bees

A very interesting topic of investigation of the life of social insects is the communication of honey bees. To inform their nest mates for example about good resources of pollen and nectar the honey bee waggle dance is the communication medium in the dark beehive.

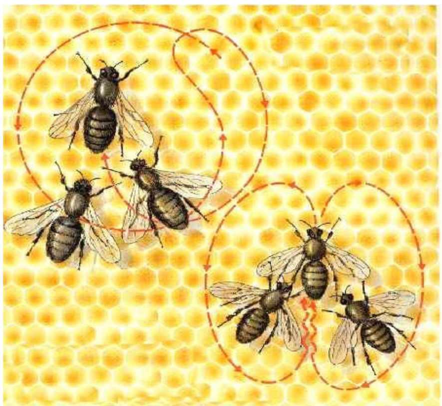


Figure 2: Communication by Bee Dance

In the temporal-spatial structure of the dance figure (Figure 2) the information about the geographical location of the resources is encoded. To pass on this information is the purpose of the dance. But first, how can the nest mates are made attentive? In order to minimize mutual disturbances and to attract not too many bees, the attracting signals which are sent by the dancer should exhibit a small range. Vibrations of the honeycomb generated by the dancers fulfil this function.
By means of contactless Laser Doppler Vibrometry vibration frequencies of the honeycombs within the range between 200 and 300 Hz could be measured. In this process pulses of approximately 30 50 milliseconds length occur. The time sequence of these pulses let's assume that an important role of the wagging dance consists of coupling the vibration pulses of the flight musculature effectively to the honeycomb. Thereby the relatively weak amplitudes of the honeycomb vibrations overlap with the detection threshold of a sensitive subgenual organ of the attracted nest mates.
If bees are attentive to the dancer, the information is transmitted during the dance movements from the dancer to the other bees by dancing in direct body contact. In this way the nest mates get information about the distance of the source of interest and its direction with respect to the position of the sun. By the way, further tests have shown that 'silent' waggle dances which perform the waggle dance but do not produce an acoustic 200–300 Hz component do not recruit bees.

2 Basics

In this experiment a Laser Doppler Vibrometer model is presented which functionality is based on heterodyne interferometry. A diode pumped solid state laser module is used as a laser source and a speaker membrane works as a vibrating object. A fast photo detector measures the interference signal and a mixer unit extracts the vibration frequency from the detected light.

2.1 Principle of Laser Vibrometry

A Laser Doppler Vibrometer is a measuring instrument for mechanical vibrations. The Doppler shift of coherent laser light which is scattered or reflected from a small area of the oscillating test object is detected. The amplitude of the frequency shift is used to measure the component of velocity which lies along the axis of the laser beam, whereas the periodicity of the frequency shift indicates the oscillation frequency of the test object.
As visible laser light has a very high frequency (approx. 5 $x10^{14}$ Hz), a direct measurement of the Doppler shifted light is not possible. An optical interferometer is therefore used to mix the scattered light coherently with a reference beam. The photo detector measures the intensity of the mixed light whose (beat) frequency is equal to the difference frequency between the reference and the measurement beam.
Due to the sinusoidal nature of the detector signal, the direction of the vibration is ambiguous, i. e. it is not possible to determine whether the object is moving towards the detector or away from it. There are two ways to introduce directional sensitivity:
a) Introduction of an optical frequency shift into one arm of the interferometer to obtain a virtual velocity offset.
b) Adding polarization components and an additional photo receiver in such a way that at the interferometer output a second homodyne signal occurs being in quadrature to the primary photo detector output.

Figure 3: Christian Doppler

While the latter technique is realized in "Michelson's Interferometer", the former is used here which is also the most common technique in vibrometry. An acousto-optic modulator (Bragg cell) is incorporated into one arm of the interferometer. The Bragg cell is driven at frequencies of several MHz or higher and generates a carrier signal at the RF drive (centre) frequency. The movement of the object frequency modulates the carrier signal. The signed object velocity determines sign and amount of frequency deviation with respect to the centre frequency. This type of interferometer is called "heterodyne interferometer".
Besides the unambiguous determination of the direction of movement, the heterodyne solution has further significant advantages for vibrometry: as only high frequency AC signals are transmitted there is no disturbance from hum and noise, introduced from all types of power supplies. Non-linear effects of the photo detector as well as of all signal pre-processing stages do not affect the integrity of the Doppler modulation content.

2.2 The Doppler Effect

Probably everybody has experienced the alarm horn of an ambulance when passing by at high speed, or the other way around when one passes a ringing bell with sufficient velocity. In both cases the tone of the acoustic source is higher than the "real" tone if the source and the observer move towards each other, whereas the tone becomes deeper if the source and the observer depart from each other.

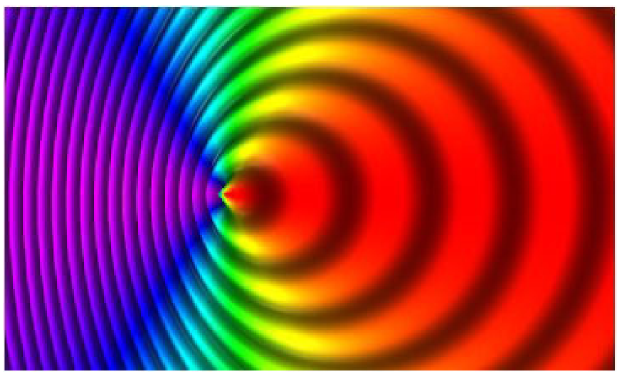


Figure 4: Wave pattern generated by moving oscillator

The *apparent* change in frequency of a wave motion when there is relative motion between the source of the wave and the observer is called the ***Doppler effect***. If the observer and the source approach each other, then the frequency of the wave increases, in the reverse case the frequency is lowered. Figure 4 shows an oscillator generating a periodical wave motion. Since the oscillator is moving to the left while oscillating, an observer on the left hand side of the oscillator sees the wave at a higher frequency than an observer on the right hand side.

The Doppler effect, named after Ch. A. Doppler (1803 1853), an Austrian mathematician and physicist, was first proposed in 1842 in his publication "*Über das farbige Licht der Doppelsterne und einige andere Gestirne des Himmels"* ("*On the colored light of binary stars and other heavenly bodies").* Interestingly, this was an essay about the Doppler Effect of light and not of acoustic waves which is more obvious in everyday life.

Nevertheless, the Doppler effect is valid for electromagnetic waves as well. It becomes obvious in astronomy, where the Doppler effect is of intense interest to astronomers who use the information about the shift in frequency of electromagnetic waves produced by moving stars in our galaxy and beyond in order to derive information about those stars and galaxies. The belief that the universe is expanding is based in part upon observations of electromagnetic waves emitted by stars in distant galaxies. Furthermore, specific information about stars within galaxies can be determined by application of the Doppler effect. Galaxies are clusters of stars which typically rotate about some centre of mass point. Electromagnetic radiation emitted by such stars in a distant galaxy would appear to be shifted downward in frequency (a "red shift") if the star is rotating in its cluster in a direction which is away from the Earth. On the other hand, there is an upward shift in frequency (a "blue shift") of such observed radiation if the star is rotating in a direction that is towards the Earth.

2.3 *Derivation of Doppler Equations*

By the above mentioned example of the ambulance car the derivation of the physical equations of the Doppler effect is illustrated here. With the explanation of the acoustic Doppler effect one must differentiate whether the acoustic source *s* or the observer *o* moves relatively to the medium. In case of light waves this differentiation has not to be done. Each of these effects is analysed separately now.

2.3.1 Moving source

First we consider a resting ambulance which emits a sound wave with frequency (normally, ambulances make a periodically frequency modulated tone, but in our case something is wrong with the siren and the tone has a constant frequency). Depending on the velocity of propagation c of the wave in the medium, the wavelength is given by

$$\lambda_s = \frac{c}{f_s} \qquad \textbf{(Eq 1)}$$

Now assuming the ambulance approaches the observer with a constant velocity v_s, i.e. the observer is placed on the left hand side in Figure 4. For simplification we consider the wave fronts (bright lines in Figure 4) of the acoustical wave only. Two successive wave fronts are emitted within a time period τ_s depending on the frequency of the wave

$$\tau_s = \frac{1}{f_s} \qquad \textbf{(Eq 2)}$$

the same period as in the case of a ambulance at rest, so the frequency of the sound that the source emits does actually not change! But what will change is the *observed* wavelength: the second wave front is emitted at a closer distance to the observer than the first, the third closer than the second and so on. The value of this decrease of distance Δs between two wave fronts depends on the speed of the car and the period τ_s

$$\Delta s = v_s \cdot \tau_s = \frac{v_s}{f_s} \qquad \textbf{(Eq 3)}$$

and has to be subtracted from the "still-stand" wavelength:

$$\lambda_0 = \lambda_s - \frac{v_s}{f_s} \qquad \textbf{(Eq 4)}$$

With (Eq 1) we get the observed frequency for a system with a moving source and a detector at rest

$$f_o = \frac{f_s}{\left(1 - \frac{v_s}{c}\right)} \qquad \textbf{(Eq 5)}$$

After the ambulance passed the observer, the tone becomes deeper; the observer has to be placed now on the right hand side of the oscillator in Figure 4. As a convention, the velocity *v* of a leaving object has to be taken negative. Therefore (Eq 4) and (Eq 5) are still valid, but due to inverting of the sign of v_s the wavelength becomes longer and the apparent frequency lower.

2.3.2 Moving detector

In the converse case of an observer moving with velocity v_0 towards a source at rest, the wavelength relative to the medium does not change. What changes is the *apparent* velocity c_s of the wave with respect to the observer

$$c_s = c + v_o \, . \qquad \textbf{(Eq 6)}$$

The period τ_0 which remains to the observer between two wave fronts is shortened by the factor c/c_s with respect to the period τ_s

$$\tau_o = \tau_s \frac{c}{c + v_o} \qquad \textbf{(Eq 7)}$$

and therefore the observed frequency is

$$f_o = f_s \frac{c + v_o}{c} = f_s \left(1 + \frac{v_o}{c}\right) \qquad \textbf{(Eq 8)}$$

Again, after the observer passed the source and moves away from it, the sign of v_s changes, and the tone of the acoustic signal becomes lower.

For sufficiently small velocities relative to the propagation velocity, formulae (Eq 5) and (Eq 8), lead to the same result. However, at moderate velocities the difference of the two results may be considerably, what is illustrated by the

following example where v is assumed to be identical to c, i. e. travelling with sonic speed.
In case of an approaching source (the engine of the ambulance car may be not powerful enough to reach c, therefore let it be an ambulance air plane) the distances between the wave fronts become zero and they all superimpose. Theoretically according to (Eq 5), the observer should hear an “infinite” frequency; practically it makes BANG!!! – the supersonic boom!
In contrary, when the observer moves with a velocity $v = c$ towards the source, there is no dramatic effect happening. He hears the tone just with the double frequency, i. e. for musicians, and one octave higher than the emitted tone. And even if you run towards an ambulance as fast as you can, you never will hear a bang (more precisely: if there *is* a bang, it is not caused by Doppler effect for sure!).

2.3.3 Relativistic Doppler Effect

Up to now we considered acoustic waves or in general waves which require a medium to propagate. But how is the situation for waves which do not require a medium, such as light waves? Let us once again walk through the example given above: assume the observer and an ambulance space ship are moving towards each other in space with a relative velocity v which may come close to the speed of light c. Due to a lack of a medium acoustic sound is not transported in space, but each ambulance has an optical alarm signal and our space ambulance as well. Let us consider the problem from the reference frame of the observer who wants to measure the time period τ between two successive wave fronts of the optical signal. Without relativistic effects we end up with (Eq 5), expressed in terms of τ:

$$\tau_{o,classical} = \tau\left(1-\frac{v}{c}\right) \quad \textbf{(Eq 9)}$$

However, from the special relativity theory we know that clocks in moving systems are running slower than in still standing frames. Therefore the relativistic time dilation has to be taken into account, i. e. the period has to be scaled by the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \textbf{(Eq 10)}$$

Therefore the observer will measure the period to be

$$\tau_{o,\,relativistic} = \tau\cdot\gamma\cdot\left(1-\frac{v}{c}\right) \quad \textbf{(Eq 11)}$$

and hence the frequency of the observed alarm light as

$$\mathrm{f}_{o,\,relativistic} = \mathrm{f}\cdot\sqrt{\frac{1+v/c}{1-v/c}} \quad \textbf{(Eq 12)}$$

resulting in a blue shift for positive and a red shift for negative velocities, respectively.
Now the other way around: we want to consider what an observer measures while we are fixed to the reference frame of the source, and an observer approaches with the relative velocity v. The classical Doppler formula for this case is according to (Eq 7)

$$\tau_{o,classical} = \tau\frac{c}{c+v} \quad \textbf{(Eq 13)}$$

Since the time at the moving observer passes slower than the time at the source this time period has to be corrected by

$$\tau_{o,\,relativistic} = \frac{1}{\gamma}\tau_{o,classical} = \frac{1}{\gamma}\tau\frac{c}{c+v} \quad \textbf{(Eq 14)}$$

and for the frequency of the observed alarm light we get

$$\mathrm{f}_{o,\,relativistic} = \mathrm{f}\cdot\sqrt{\frac{1+v/c}{1-v/c}} \quad \textbf{(Eq 15)}$$

which is identical to (Eq 12)! Indeed, for waves which do not require a medium, such as light or gravity, only the relative difference in velocity between the observer and the source needs to be considered.

2.4 Interferometry

A Laser-Doppler vibrometer works on the basis of optical interference, requiring two coherent light beams (signal and reference), with their respective light intensities to overlap. A photo detector measures the time dependent intensity *I(t)* at the point where the signal and reference beams interfere. To calculate the overall intensity at the detection point, we describe the electric fields of the two beams E_1 and E_2 as a function of time t and position x:

$$E_i = E_{0i}\cos\left(2\pi\cdot\mathrm{f}_i\cdot t+\frac{2\pi\cdot\mathrm{f}_i\cdot x_i}{c}\right),\ i=1,2 \quad \textbf{(Eq 16)}$$

with the field amplitude E_{01} and the wave frequency f_i. Since relative rather than absolute path differences are considered here, an eventual phase shift by 180° at ideal reflecting surfaces is neglected. At the detection point the two beams sum up to

$$E = E_1 + E_2 \quad \textbf{(Eq 17)}$$

However, physical systems are able to measure the luminous intensity instead of the field intensity, which calculates as the square of the electrical field strength $I=E^2$

$$\begin{aligned} I = E^2 &= (E_1+E_2)^2 \\ &= {E_{01}}^2\cos^2\left(2\pi\mathrm{f}_1\left(t+\frac{x_1}{c}\right)\right) \\ &+{E_{02}}^2\cos^2\left(2\pi\mathrm{f}_2\left(t+\frac{x_2}{c}\right)\right) \\ &+2E_{01}E_{02}\cos\left(2\pi\mathrm{f}_1\left(t+\frac{x_1}{c}\right)\right)\cos\left(2\pi\mathrm{f}_2\left(t+\frac{x_2}{c}\right)\right) \end{aligned} \quad \textbf{(Eq 18)}$$

The frequencies of visible light are in the order of $5\cdot10^{14}$ Hz which cannot be resolved by any detector yet. Therefore the temporal average of the two quadratic addend can be written as:

$${E_{0i}}^2\cos^2\left(2\pi\mathrm{f}_i\left(t+\frac{x_i}{c}\right)\right) = \frac{1}{2}{E_{0i}}^2 \quad \textbf{(Eq 19)}$$

With the two multiplied cosine we have to be careful: depending on their argument they might result in a low frequency which may be detectable. Therefore we apply the following addition theorem

$$2\cos\alpha\cos\beta = \left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right) \quad \textbf{(Eq 20)}$$

and get:

$$I = \frac{1}{2}E_{01}{}^2 + \frac{1}{2}E_{02}{}^2$$
$$+E_{01}E_{02}\cos\left(2\pi \mathrm{f}_1\left(t+\frac{x_1}{c}\right)+2\pi \mathrm{f}_2\left(t+\frac{x_2}{c}\right)\right) \quad \textbf{(Eq 21)}$$
$$+E_{01}E_{02}\cos\left(2\pi \mathrm{f}_1\left(t+\frac{x_1}{c}\right)-2\pi \mathrm{f}_2\left(t+\frac{x_2}{c}\right)\right)$$

Again, the first cosine term averages out because of the high frequency. But this time it averages to zero, and finally we get:

$$I = \frac{1}{2}E_{01}{}^2 + \frac{1}{2}E_{02}{}^2$$
$$+E_{01}E_{02}\cos\left(2\pi \mathrm{f}_1\left(t+\frac{x_1}{c}\right)-2\pi \mathrm{f}_2\left(t+\frac{x_2}{c}\right)\right) \quad \textbf{(Eq 22)}$$

If we express the electric field amplitudes in terms of luminous intensity

$$I = \frac{1}{2}E_{0i}{}^2,\ i = 1,2 \quad \textbf{(Eq 23)}$$

we can formulate eq. 22 as the sum of the two intensities of the original beams plus an interference term:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(2\pi \mathrm{f}_1\left(t+\frac{x_1}{c}\right)-2\pi \mathrm{f}_2\left(t+\frac{x_2}{c}\right)\right) \quad \textbf{(Eq 24)}$$

From this general expression for two beam interference we will derive a few special cases.

2.4.1 Homodyne interferometry

If the two interfering beams have the same frequency we refer to such an interferometer as a homodyne interferometer. The most popular representative of this type is a Michelson interferometer, usually used to measure short scale distances and displacements.

With $\mathrm{f}_1=\mathrm{f}_2=\mathrm{f}_3$ eq. 24 simplifies to:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(2\pi \mathrm{f}\frac{\Delta x}{c}\right), \Delta x = x_1 - x_2 \quad \textbf{(Eq 25)}$$

The Intensity is time-independent now but depends on the path difference of the two beams only. Moreover, if the two beam intensities are the same one gets the biggest modulation depth possible

$$I = 2I_0 + 2I_0\cos\left(2\pi \mathrm{f}\frac{\Delta x}{c}\right) \quad \textbf{(Eq 26)}$$

for $I_1 = I_2 = I_0$. Depending on the path difference Δx the intensity alternates as follows: if Δx is a multiple of the laser wavelength, the overall intensity is four times the single beam intensity. Correspondingly, the overall intensity is zero if the two beams have a path length difference of a multiple and a half of one wavelength. In the former case the two beams interfere constructively, in the latter case it is referred to as destructive interference.

2.4.2 Heterodyne interferometry

Assuming the two interfering beams have different frequencies but a constant path difference, then we get a heterodyne interferometer. A Laser Vibrometer is based on this heterodyne technique which allows determining not only the amplitude of a movement, but also the direction. This will become clear by the further calculations. Taking the path difference of the two beams constant and using $\Delta\mathrm{f}=\mathrm{f}_1 - \mathrm{f}_2$, (Eq 24) can be written as:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(2\pi\Delta\mathrm{f}\cdot\mathrm{t}+\varphi\right) \quad \textbf{(Eq 27)}$$

with the phase factor

$$\varphi = \frac{2\pi}{c}\left(f_1 x_1 - f_2 x_2\right) \quad \textbf{(Eq 28)}$$

Here the interference signal intensity is time dependent and alternates with the carrier frequency Δf provided that the phase factor is time independent. Also here the biggest possible modulation depth can be achieved with two beams of the same intensity $I_1 = I_2 = I_0$:

$$I = 2I_0 + 2I_0\cos\left(2\pi\Delta\mathrm{f}\cdot\mathrm{t}+\varphi\right) \quad \textbf{(Eq 29)}$$

Now let us play a little bit with the phase factor *φ*. We assume the second interferometer arm to be the reference arm, fixed to a constant path length x_2, preferably at a multiple of the wavelength λ; thus we can drop the second addend of φ in (Eq 28). The other arm x_1 (signal arm) is assumed to be extended by moving a mirror in an interferometer at a constant velocity *v*. Since the light beam is reflected at the mirror its frequency is Doppler shifted twice: when reaching the mirror the light wave appears frequency shifted (the mirror acts like a moving detector). When reflecting the beam the mirror acts as a moving source and the light reaches the photo detector again frequency shifted. Therefore we have to apply (Eq 12) and (Eq 15), and f_1 will be transformed to:

$$\mathrm{f}_1^* = \mathrm{f}_1\cdot\left(\sqrt{\frac{1+v/c}{1-v/c}}\right)^2 = \mathrm{f}_1\cdot\left(1+\frac{2v/c}{1-v/c}\right) \quad \textbf{(Eq 30)}$$

and since

$$v/c \ll 1$$

$$\mathrm{f}_1^* \cong \mathrm{f}_1 + 2\frac{v}{c}\mathrm{f}_1 \quad \textbf{(Eq 31)}$$

Inserted in (Eq 28) and (Eq 29) the time dependent intensity becomes

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(2\pi t\left(\Delta\mathrm{f}+2\frac{v}{c}\mathrm{f}_1\right)\right) \quad \textbf{(Eq 32)}$$

Here the advantage of the heterodyne technique becomes clear: the beat frequency Δf of the two interfering beams experiences an upward shift, if *v* is positive (mirror moves towards the detector), or a downward shift, if the mirror moves away from the detector. The direction of movement of the mirror – or in general the object of interest – is clearly determined by the direction of the spectral shift.

A helpful way to extract the velocity of the moving object is to superimpose electronically the detected signal by a cosine signal with a frequency Δf. Taking only the interference term of (Eq 32) into consideration we get:

$$V \propto \cos\left(2\pi t\left(\Delta\mathrm{f}+2\frac{v}{c}\mathrm{f}_1\right)\right)+\cos\left(2\pi t\Delta\mathrm{f}\right) \quad \textbf{(Eq 33)}$$

Dr. Walter Luhs - Dec. 2012

According to (Eq 20) we can transform:

$$V \propto 2\cos\left(2\pi t\left(\Delta f + \frac{v}{c} f_1\right)\right)\cos\left(2\pi t \cdot 2\frac{v}{c} f_1\right) \quad \textbf{(Eq 34)}$$

Usually Δf is greater than

$$\frac{v}{c} f_1,$$

which results in a beat signal with the carrier frequency of:

$$\left(\Delta f + \frac{v}{c} f_1\right)$$

and a beat frequency of:

$$2\frac{v}{c} f_1.$$

If we go one step further and allow to the object to perform not a linear but an oscillating motion we will end up with the basic aim of a Laser Doppler Vibrometer: the detection of a vibration! But how do we have to change now the upper formulae to describe the response of a vibrating membrane? We start with the velocity *v* of the membrane and it is now a function of time

$$v(t) = u\cos(\omega t) \quad \textbf{(Eq 35)}$$

with the vibration frequency ω and the vibration amplitude *u*. When inserting (Eq 35) in (Eq 34) everything is done already:

$$V \propto 2\cos\left(2\pi t\left(\Delta f + \frac{u\cos(\omega t)}{c} f_1\right)\right)\cdot\cos\left(2\pi t \cdot 2\frac{u\cos(\omega t)}{c} f_1\right) \quad \textbf{(Eq 36)}$$

The general behaviour is the same as in the case of a linear motion (Eq 34): in the oscilloscope we can see a beat signal, now with the carrier frequency f_1:

$$\left(\Delta f + \frac{u\cos(\omega t)}{c} f_1\right)$$

and the beat frequency

$$2\frac{u\cos(\omega t)}{c} f_1.$$

For small amplitudes *u* the carrier frequency is slightly changing only. The beat frequency however is periodically changing, what will be observed in a pattern of a periodically squeezed and stretched waving oscilloscope trace.
At the temporal points

$$\tau = n\cdot\frac{\pi}{\omega}, n = 1,2,3,....$$

the membrane oscillates with maximum velocity . This is to observe at the sections of the trace at which the wave shows a fast oscillation, similar to the linear moving mirror. In between of these sections, at n=1, 2, 3 ... , the trace shows a much slower, almost still standing oscillation. At these sections the membrane reaches the turning points where the direction of its motion changes. Here the velocity reaches zero as well as the beat frequency.
The time interval from one membrane turning point to the next corresponds to a half period of the membrane vibration, therefore the vibration frequency can be calculated. Also the fast oscillation in between of the turning points bears information: at these sections the cosine term of (Eq 35) has a value of around 1, therefore the beat frequency simplifies to:

$$f_{beat} \cong 2\frac{u}{c} f_1 \quad \textbf{(Eq 37)}$$

from which the vibration amplitude of the membrane can be extracted.

2.5 Principle of Operation

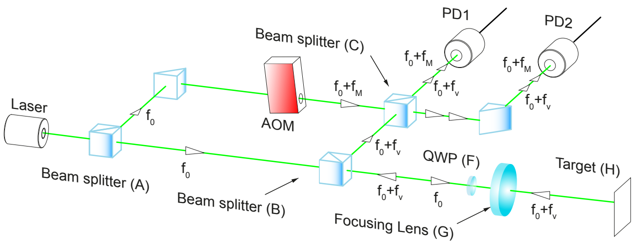


Figure 5: Principle of Laser Doppler Vibrometer

A schematic diagram of the Laser Doppler Vibrometer experiment is shown in Figure 5. The beam of the laser is divided into two beams at the beam splitter (A). One part is reflected and the other part transmitted. Both beams have the same frequency, namely that of the laser f_o. The reflected beam passes an acousto-optic modulator (AOM) whereby its frequency is changed to f_o+f_m. At the beam splitter cube (C), this beam is directed to the photo detector (PD1 and PD2). The beam which is transmitted at the beam splitter cube of (A) passes through the polarising beam splitter cube B and is focused on the vibrating target. The frequency of the scattered beam is Doppler shifted, caused by the vibration of the object. The frequency of the returning radiation is therefore $f_o \pm f_v$. The return beam are passes the quarter wave plate (QWP) where the polarisation is changed in such a way that it is now reflected at the beam splitter (B) and diverted to the non-polarising beam splitter (C). In the same way as the beam from the AOM the beam is divided into two and hit the photo detector (PD1 and PD2). Due to its characteristic the photo detector produces the difference of both frequencies $f_m \pm f_v$. To obtain the desired Doppler frequency, which is proportional to the speed v of the target, this frequency is mixed with the modulation frequency of the AOM f_m. The periodicity of the Doppler shift finally leads to the vibration frequency and the amplitude of the Doppler shift itself gives the velocity of the target.
There exist a variety of other optical arrangements for such a vibrometer, however this one brings the maximum of intensity to the target and uses two photodetector exploiting the 180° phase shift between both signal to boost the sensitivity by a factor of two.

2.6 Acousto-optical modulation

A central part of a heterodyne interferometer (and hence for the LDV) is the acousto-optic modulator (AOM). Therefore the principles of acousto-optics will be discussed in the following section, as well as frequency shifting by acoustic wave interaction.

2.6.1 Acousto-optical effect

The acousto-optical (AO) effect can be categorized under the generic term of photoelasticity. By photoelasticity one understands the change of the refractive index due to mechanical stress. In case of the AO effect this stress is produced by an ultrasonic transducer in a solid or liquid medium. An acoustic wave spreading in the medium produces a periodic modulation of the medium density. This in turn generates a periodic modulation of the refractive index and therefore modifies the effect of the optical medium on a light wave. This periodic modulation can be considered as a grating at which the light wave is diffracted.

2.6.2 Diffraction at a grating

With the theoretical treatment of a grating one has to differentiate between two cases: The Raman Nath range with small, and the Bragg case with large thickness of a grating. In the first case the diffraction takes place into a multiplicity of diffraction orders. On the other side with diffraction on thick gratings only one diffraction order with strong intensity arises. The angle of incidence is no longer arbitrary, but must be equal to the half of the diffraction angle. In the following we are concerned only with the diffraction on thick gratings, because this is decisive for an AOM as a frequency shifter.
In the case of an AO wave propagating in a crystal the regions of increased refractive index can be regarded as layers perpendicular to the direction of propagation of the acoustic wave. On those layers a light beam incident the crystal seems to be partially reflected. Since the portions of light reflected on the stack of layers have to interfere constructively, a macroscopic diffraction signal results only at a specific intersection angle, the so called glancing angle or Bragg angle. In the following a mathematical description of the Bragg condition is deduced.

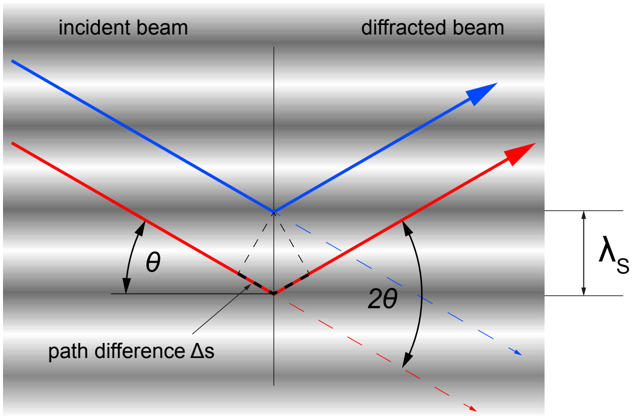


Figure 6: Bragg Diffraction of two Rays

Regarding Figure 6, two rays are representative for a bundle of rays, a light beam. The periodically changing density generated by an acoustic wave is depicted by bright and dark stripes spaced in intervals of λ_S, the wavelength of the acoustic wave. The light rays intersect this stack of layers in an angle θ. On two neighboured layers the rays are reflected what results in a path difference Δs of the two rays. This difference can be expressed as

$$\Delta s = 2 \cdot \lambda \cdot \sin\theta \qquad \textbf{(Eq 38)}$$

The condition for constructive interference of the two rays is

$$\Delta s = m \cdot \lambda \qquad \textbf{(Eq 39)}$$

The path difference has to be a multiple of the optical wavelength. With $m = 1$ we get the first diffraction order signal expressed by the diffraction equation:

$$2 \cdot \lambda \cdot \sin\theta = \frac{\lambda}{2\lambda_s} = \lambda \frac{f}{2c_s} \qquad \textbf{(Eq 40)}$$

which is identical to the famous Bragg equation for X ray diffraction.

2.7 Photons and Phonons

As in all physical processes also here at light diffraction the conservation of energy and of momentum have to be fulfilled. For clarification we describe the process in the particle picture: light rays are represented by a stream of photons, each photon with the energy hν and momentum

$$\frac{h}{2\pi} \cdot \vec{k}$$

$\vec{k}$ stays for the wave vector of a light or acoustic wave and indicates the propagation direction of the wave as well as its wavelength. In the same way the acoustic wave can be described by quasi-particles, so called phonons with the energy $h\nu_S$ and momentum

$$\frac{h}{2\pi} \cdot \vec{k}_s$$

Phonons in general describe the irregular thermal vibrations of a crystal lattice. An arbitrary vibration mode of the crystal can be expressed as a superposition of harmonic eigen vibrations, quantised as phonons.
The acousto-optic interaction has to be understand as the impact between photons and phonons, for which the conservation laws has to be fulfilled. For the momentum conservation we formulate

$$\overrightarrow{p}_P^{\,in} + \overrightarrow{p}_{phonon} = \overrightarrow{p}_{photon}^{\,out} \qquad \textbf{(Eq 41)}$$

An intersecting photon collides with a phonon and is scattered by the phonon which is absorbed:

$$h\vec{k}_{in} + h\vec{k}_s = h\vec{k}_{out} \qquad \textbf{(Eq 42)}$$

But a formulation of momentum conservation like

$$h\vec{k}_{in} = h\vec{k}_s + h\vec{k}_{out} \qquad \textbf{(Eq 43)}$$

is correct as well: a photon is scattered by emission of a phonon. A similar description holds for the energy conservation. In case of phonon absorption it is written

$$h\vec{\nu}_{in} + h\vec{\nu}_s = h\vec{\nu}_{out} \qquad \textbf{(Eq 44)}$$

i. e. the energy of the light quantum is increased. In case of phonon emission we write

$$h\vec{\nu}_{in} = h\vec{\nu}_s + h\vec{\nu}_{out} \qquad \textbf{(Eq 45)}$$

and the energy of the light quantum is lower. But this change of the frequency of the photons, either a frequency upshift or downshift, is nothing else than the Doppler Effect. However, the frequency shift is usually rather small (10^7 - 10^9 Hz) compared to the frequency of light ($10^{14\ Hz}$).

2.8 Crystal optics

How this theory does looks like in practice when a light beam interacts with an acoustic wave in a medium, e.g. a crystal? For a better visualization we describe all waves in the picture of wave vectors. A wave vector is a vector which indicates the propagation direction of the wave by its orientation and the frequency (or wavelength) by its length. The three waves of interest are therefore described as:

$$\vec{k}_{in}, \vec{k}_{diff} \text{ and } \vec{k}_{AW},$$

representing the incident light, the diffracted light and the acoustic wave. From (Eq 42) and (Eq 43) we know that

$$\vec{k}_{in} = \vec{k}_{AW} \pm \vec{k}_{diff},$$

i. e. the three vectors have to build a closed triangle. Since all these waves are propagating in a medium their absolute value of the wave vector shows the following relation to the index of refraction of the medium n and the velocity of sound in the medium v

$$\left|\vec{k}_{in}\right| = 2\pi \frac{n_{in}}{\lambda_0}$$

$$\left|\vec{k}_{diff}\right| = 2\pi \frac{n_{diff}}{\lambda_0}$$

$$\left|\vec{k}_{AW}\right| = 2\pi \frac{f}{v}$$

with frequency f of the acoustic wave. Note that n_{diff} not necessarily has to be equal to n_{in}! In fact, if the propagation medium is a crystal, it is more likely that they are different rather than the same. For understanding this we have to look at the isotropy of the medium.

In acousto-optics one distinguishes between isotropic and anisotropic interaction. In the first case, the light beam sees the same index of refraction on its way through the medium, n_{in} is the same as n_{diff} what is fulfilled in optically isotropic media (n is the same in all directions), or in crystals of high symmetry, the right crystal cut and a proper orientation of the beam with respect to the crystal.

2.8.1 Isotropic interaction

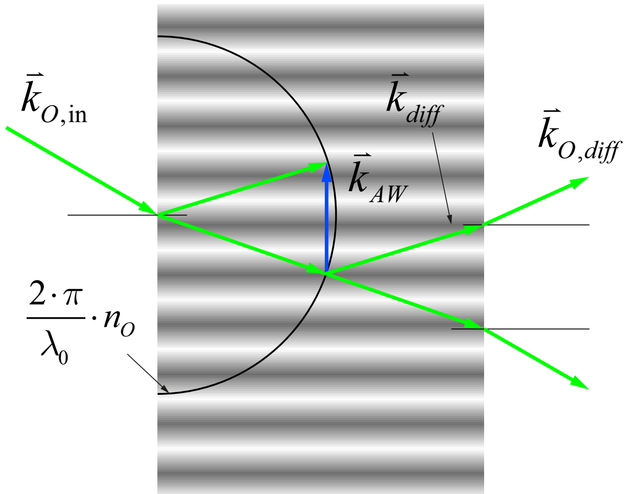


Figure 7: Isotropic acousto-optic interaction

In Figure 7 the case of isotropic interaction is depicted. A light beam represented by its wave vector enters an optical medium, let's say an isotropic crystal. On the surface of the crystal it is refracted according to Snell's law. Inside the crystal it interacts with the AO wave and is partially diffracted according to Bragg's law, described above already (Figure 6). In the crystal the index of refraction is independent from the direction of light propagation which is indicated by the "index circle". The tips of both, the incident and the diffracted wave vector have to lay on this circle and span – together with the acoustic wave vector – the interaction triangle. When leaving the crystal Snell's law applies again for both, the diffracted and the unchanged beam (so called 1st and 0th order of diffraction). All together the incident and the diffracted beam show a symmetric behaviour travelling through the crystal. Especially the polarisation state of incident and diffracted beam is preserved (what will be important later on), however, the frequency of the diffracted beam is shifted by the acoustic wave frequency.

2.8.2 Anisotropic interaction

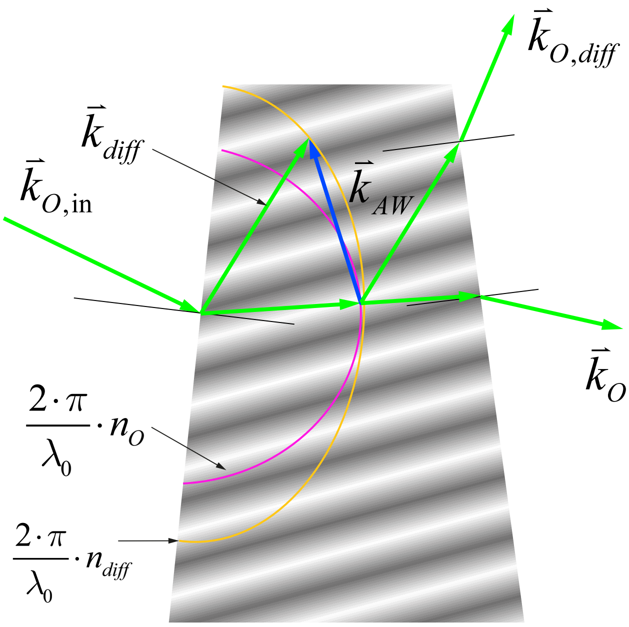


Figure 8: Anisotropic acousto-optic interaction

In case of anisotropic crystals the behaviour is different. Since these crystals have at least one optical axis they show the effect of birefringence, i. e. different indices of the diffracted refraction for different polarisation states. Moreover, the index depends on the light's propagation direction. Therefore, we have to draw two index lines in Figure 8, one for each polarisation state. Further, these lines are in general no circles any more, but ellipses, indicating the direction dependency of the index.

When entering the crystal, the incident wave vector points to the index line which is in agreement with the beam's polarisation state. The acoustic wave vector points from one index line to the other. When interacting with the acoustic wave, the incident beam becomes diffracted in direction of k_{diff} which ends up at the index line for the diffracted beam. This line represents another state of polarisation, i. e. the polarisation of the diffracted beam is rotated with respect to the polarisation of the incident beam.

Since the acoustic wave “shears” the incident beam from one index line to the other, the type of acoustic wave is called *shear mode*. Further, the vector of the diffracted beam changed its length indicating a different index of refraction. Due to this anisotropy the simple Bragg law cannot be applied anymore and to calculate the right diffraction angle, crystal cut and surface orientation is a demanding task of the crystal manufacturer.

2.8.3 TeO_2 for AOM

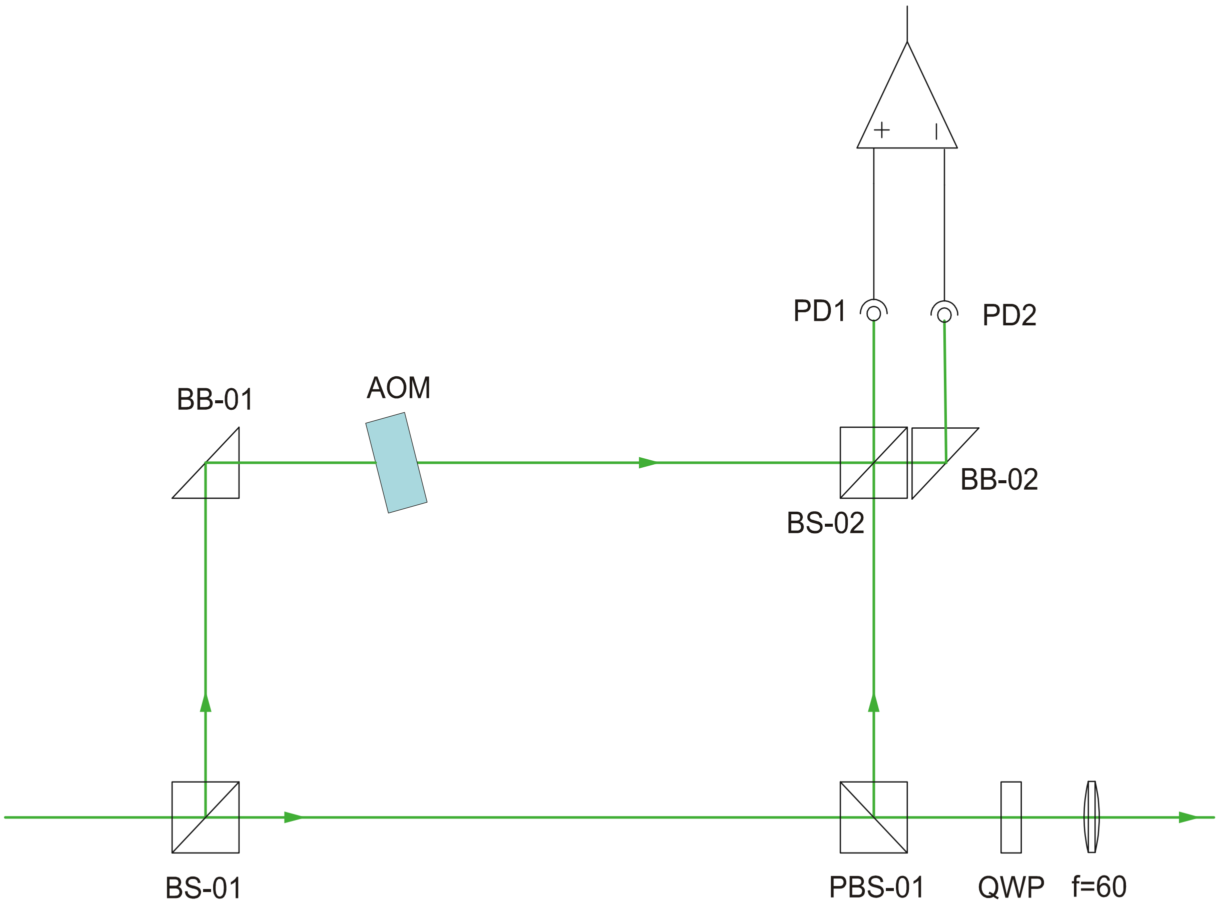
Because of its high optical homogeneity, low light absorption and scattering, and high optical power capability Tellurium Dioxide crystals are commonly used in acousto-optical devices. TeO_2 belongs to the tetragonal crystal system and is therefore optically uniaxial. As AOM it can be used either in an isotropic interaction mode (so called longitudinal mode) or in the shear mode. In the longitudinal mode the acoustic wave travels along the [001] axis of the crystal. Since the values for the index of refraction are axially symmetric around this axis an isotropic interaction (although the crystal is anisotropic) is possible and the incident beam is symmetrically diffracted.
The AOM used in this experiment contains a TeO_2 crystal which is optimised for shear mode operation. Here the acoustic wave vector is not parallel to any basic crystal axes but rather in an orientation pointing from one index line to the other what leads to anisotropic interaction with an incident light beam. The crystal cut and orientation is chosen for best performance in the green light region. The resulting angles of incident, 0^{th} and 1^{st} order beams are as follows:

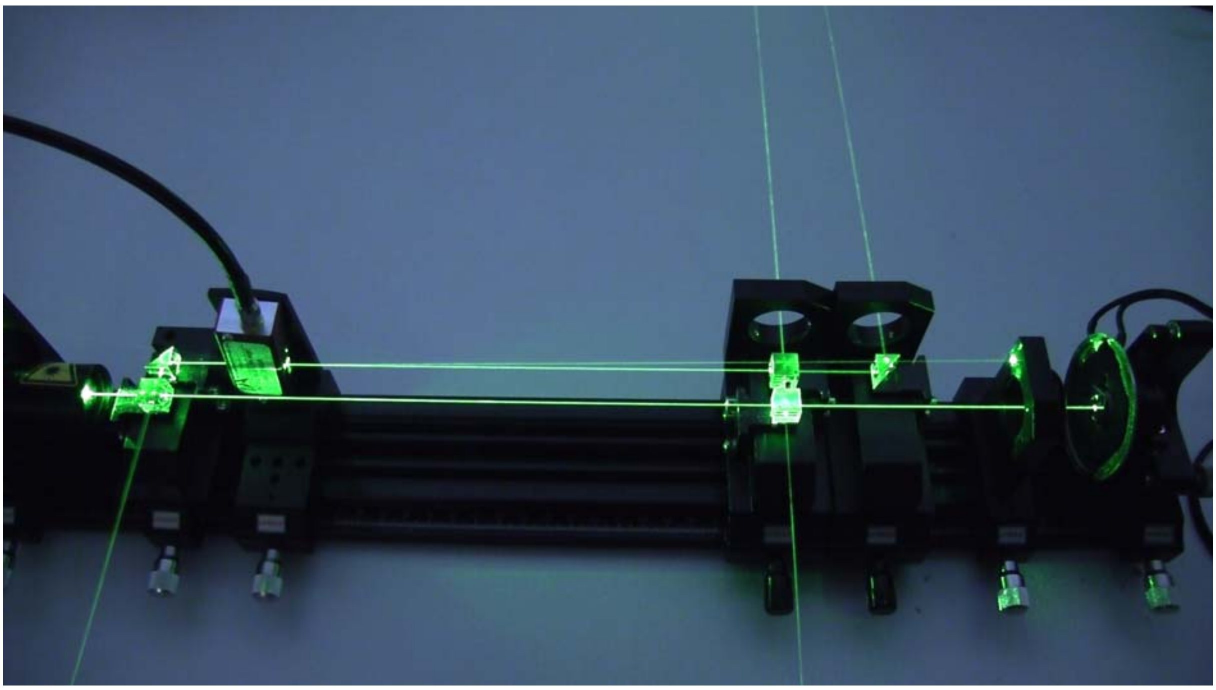
Incident direction:	+ 6,7°
0^{th} order direction:	+ 1,2°
1^{st} order direction:	– 0,8°

The maximum diffraction efficiency is better than 90% and the polarization of the diffracted beam is rotated by 90 degree.

Theory

The Mach-Zehnder Interferometer.





Light coming in from the laser on the left is split (50:50) by the (not polarisation dependent) beam splitter BS-01. The split path will be the reference signal (top) and measurement signal (bottom).

PBS is a polarizing beam splitter, where all the light with the E vector vertical will be reflected while light with horizontal E-field vector will pass through.
The measurement signal passes through BS-01 and is intended to pass through PBS-01. The laser diode is not fully polarized, but has a certain degree of linear polarization. It is oriented in such a way that the reflection out of PBS-01 (downwards in the pictures) is minimized.
The now fully polarized light passes through a quarter wave plate ($\lambda/4$) and comes out as circular polarized light.
After reflection at the target (loudspeaker), the light comes back with opposite circular polarisation. The quarter wave plate turns it into linear polarised light again, but with rotated polarisation. The PBS-01 will now reflect the light into the direction of BS-02 and the photo detectors.

The reference beam comes out of BS-01 and is bend by BB-01 into approximately the same direction as the measurement beam.
Next element is the AOM, an acousto-optic modulator.
There is an acoustic wave of 40 MHz inside a transparent crystal and the laser beam passes through. We can either say, we have interaction of photons and phonons or the laser light wave is diffracted in Bragg-style at the layers of the acoustic wave.
The AOM works like a grating and splits the incoming beam into several orders. The zero order beam is not affected, but the first order beam we use is shifted in frequency by 40 MHz.

Green light has a wavelength of 532nm or a frequency of 563 THz and the reference beam is now 40 MHz higher or lower than the measurement beam.

BS-02 is now the combiner. We have a laser beam from the loudspeaker coming from PBS-01 and a reference beam from the AOM. Both meet inside BS-02.

The idea of an interferometer is now to have both beams not only meet each other at one point in space, but also to have parallel wavefronts. Or in practical words, to be parallel.
This is later done through alignment.
Then, both wavefronts can interfere with each other.

50% of each beam will exit BS-02 on top and right side, and go towards the photo detectors PD1 and PD2.

The photodiode:

The AOM generates a reference light beam with 40 MHz frequency offset.
The target is illuminated with the unshifted light at 532 nm or lets say a frequency of
$\omega 0$= 563 909 774 MHz * 2 π.
The AOM shifted light has then a frequency of
$\omega 1$ = 563 909 7**3**4 MHz * 2 π

Right in front of the silicon photodiode, both light rays pass through air without any interaction, and we have two separate electromagnetic fields with frequencies in the 563 THz range.
As the light travels different distances inside our interferometer, the target ray will have a certain phase difference ϕ.
The sum of both waves has an electric filed of
$E_{target} * \sin(\omega 0\ t) + E_{ref} * \sin(\omega 1\ t + \phi)$

A real fast antenna could detect these two THz frequencies and nothing else; there is no mixing or interference yet.

The silicon diode is no antenna and does not respond to such THz frequencies, and it does not even respond to an electric or magnetic field.
It detects incoming power of light. This is called a square law detector, with the output being proportional to power and therefore the square of the E field.
The intensity is proportional to

$I \sim E^2 = [E_{target} * \sin(\omega 0\ t) + E_{ref} * \sin(\omega 1\ t+\phi)]^2$

Using the good old binominal formula (a+b)^2 = a^2 + b^2 + 2 ab
this is equal to
$E_{target}^2 * \sin(\omega 0\ t)^2 + E_{ref}^2 * \sin(\omega 1\ t+\phi)^2 + 2\ E_{target} * E_{ref} * \sin(\omega 0\ t) \sin(\omega 1\ t+\phi)$

Applying the addition theorems of the trigonometric functions,
the product of two sines is equal to a sum of cosines with sum and difference frequencies.

The sum frequencies of two light beams are not detectable by our photodiode and average to zero.
The difference frequencies are the interesting ones here,
Multiplying two sines of the same frequency just gives a zero (DC) frequency with amplitude 1/2,
and multiplying two sines with different frequencies gives the difference frequency.

We end up with

$I_{diode} \sim 1/2\ E_{target}^2 + 1/2\ E_{ref}^2 + 2/2\ E_{target} * E_{ref} * \cos((\omega 0-\omega 1)\ t + \phi)$

Calling
$I_{target} \sim 1/2\ E_{target}^2$ and the same for E_{ref}

we can rewrite this as
$I_{diode} = I_{target} + I_{ref} + 2\ \mathrm{sqrt}(I_{target} * I_{ref}) * \sin((\omega 0-\omega 1)\ t+\phi)$

So our photo detector sees a DC component and a 40 MHz frequency.
If $I_{target} = I_{ref}$, we have 100 % Modulation of the signal at 40 MHz,
any imbalance will reduce the modulation efficiency and leave a DC offset.

As we can see, the initial phase difference ϕ is conserved, the phase difference of the light beams in the interferometer is now converted into a phase difference of the photodiode signal versus the AOM reference frequency.
If our target moves 133 nm, the light path will change for 266 nm and the light takes 8.8 E-16 seconds or half a period longer until it arrives at the detector.
This phase shift ϕ = 180° is conserved during the detection process, and in the end, our 40 MHz signal again shifts by 180°, but this time it is an easily measurable 12.5 ns

Looking at the 40 MHz output of the photodiode versus the AOM frequency is a very sensitive Interferometer.
Normally, one can see a fringe shift of about a quarter wavelength with the eye. But with the AOM heterodyning, the oscilloscope can display tiny phase shift in the percent range of a fringe.
This is equivalent to a movement of the target below the nanometre range.

So far we discussed a stationary target, moving just at a speed of nanometres per second

If the target moves faster, the phase shift ϕ will vary with time.
Let s be the position of the target and v the velocity:
Assuming v= const

$\phi = s / \lambda * 2\pi = v * t / \lambda * 2\pi$

the lightwave is then
$\sin(\omega 0\ t + \phi) = \sin(\omega 0\ t + v / \lambda * 2\pi * t) = \sin((\omega 0 + v/\lambda\ 2*\pi)\ t)$
completely ignoring special relativity.

A signal changing its relative phase at a constant speed is obviously a signal at a different frequency.
$\omega 2 = \omega 0 + v/\lambda\ 2*\pi$

Putting it the other way round, as the target moves the reflected light is shifted in frequency and this is well know as Doppler shift.

In our setup, the 563 THz frequency of the light ray is no longer 40 MHz away from the AOM, but additionally Doppler shifted.
When the target moves at a constant speed of 13 cm / s, the Doppler shifted light is 1 MHz above or below the normal light frequency, depending on the direction of movement.

The output of the photodiode will consequently no longer be 40 MHz, but 39 or 41 MHz, again depending on direction of movement.

The heterodyne mixer 474313

In the end, the purpose of such an vibrometer is to record the position of a moving object versus time. We know, the information about the velocity of the target is encoded in the frequency of the photodiode signal. With some electronics, this information can be recovered.

We just need a FM receiver tuned to 40 MHz and integrate the output signal to get the position of the object.

A FM receiver can be built in several ways:
If we multiply the electrical 40 MHz reference signal from the AOM with the output of the photodiode,

sin (40 MHz *2pi *t) * sin (41 MHz *2pi *t) = 1/2 cos (1 MHz *2pi *t) - 1/2 cos (81 MHz *2pi *t)

we have again the product of two sines being equivalent to the sum of cosines at sum and difference frequency.
The sum frequencies around 80 MHz are filtered out in the end and ignored,
the difference frequency is an indication of the speed of our target.
The process is called downmixing.

In the example above, Doppler shifted light by 1 MHz will create an output of 41 MHz and mixing this with the 40 MHz AOM reference will leave a 1 MHz Signal again. This frequency is the Doppler shift and equivalent to the target speed.

sin (40 MHz *2pi *t) * sin (41 MHz *2pi *t) = 1/2 cos (1 MHz *2pi *t) - 1/2 cos (81 MHz *2pi *t)

But a target moving away from the detector at the same speed will shift the light by -1 MHz and create an intermediate frequency of 39 MHz, which will then be downmixed to 1 MHz again. The information on direction is present at the photodiode (39 vs 41 MHz) but lost in a single downmixing process, both resulting in 1 MHz.

sin (40 MHz *2pi *t) * sin (39 MHz *2pi *t) = 1/2 cos (-1 MHz *2pi *t) - 1/2 cos (79 MHz *2pi *t)

Since cos (x) = cos(-x) the direction information is lost.

To preserve the direction information,
a second mixer is set up, again multiplying the photodiodes signal, but this time not with the AOM reference signal itself, but with a time delayed version of the reference signal. Delayed by 7.5 ns, equal to a 90° phase shift, this is like replacing sin with -cos in the formulas.
The delayed mixer output is called Q for quadrature, while the undelayed mixer is the I (in-phase) output.

41 MHz = target travelling towards the detector
I: sin (40 MHz *2pi *t) * sin (41 MHz *2pi *t) = 1/2 cos (1 MHz *2pi *t) - 1/2 cos (81 MHz *2pi *t)
Q:-cos (40 MHz *2pi *t) * sin (41 MHz *2pi *t) = -1/2 sin (1 MHz *2pi *t) - 1/2 sin (81 MHz *2pi *t)

39 MHz, travelling away
I: sin (40 MHz *2pi *t) * sin (39 MHz *2pi *t) = 1/2 cos (1 MHz *2pi *t) - 1/2 cos (79 MHz *2pi *t)
Q:-cos (40 MHz *2pi *t) * sin (39 MHz *2pi *t) = +1/2 sin (1 MHz *2pi *t) - 1/2 sin (79 MHz *2pi *t)

Downmixing a 39 or 41 MHz signal, both mixers will give out 1 MHz signals,
with the Q mixer putting out a 90° out of phase 1 MHz signal.
But in one case the Q output will be the negative (=180° out of phase) then in the other,
or relative to the I channel it is either +90° or -90°

To restore the position, we have to integrate over velocity,
(equivalent to counting fringes), for example zero crossings of the I signal.
Whether to count up or down at an I zero crossing is indicted by the sign of the Q channel.

The whole process can alternatively be described without the Doppler shift,
directly count fringes, which manifest themselves as a phase shift of the 40 MHz signal.
Counting is done with a quadrature counter, looking at the I and Q downmix signals.
This will omit the derivative in s -> v and the integration of the FM signal.
Straight forward mathematics, but less intuitive.

Finally, we might compare this heterodyne setup to the homodyne Michelson interferometer from experiment P5881.
In both setups, we finally count fringes.

The homodyne Michelson uses no AOM, both interfering light rays have the same frequency and mix down to DC.
Motion is seen as a sinusoidal variation of this nearly DC signal.
The 90° shifted signal for the quadrature counter is generated optically with a second set of photo detectors.

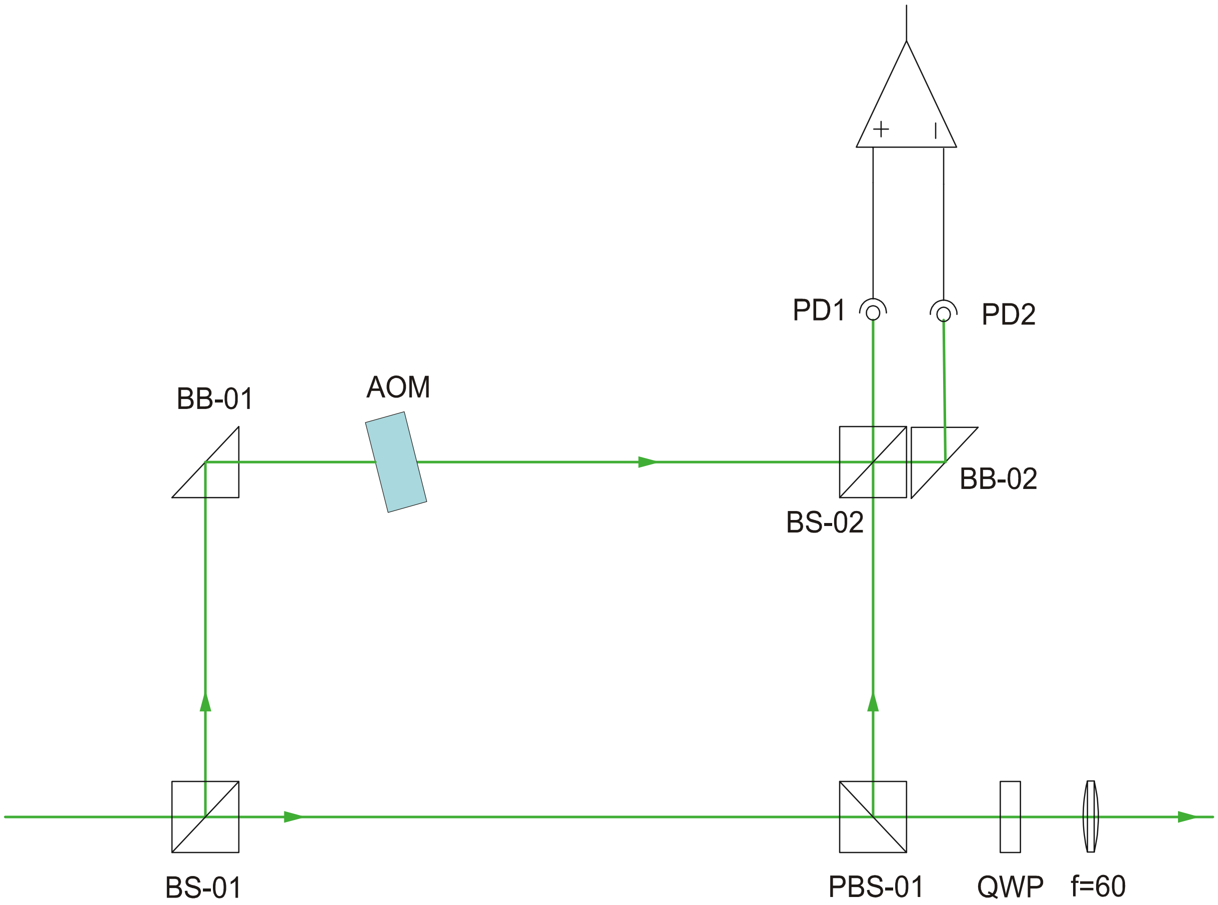
The heterodyne Vibrometer uses an intermediate frequency of 40 MHz, an AOM but only one photo detector.
The light rays interfere into a 40 MHz signal that is then electrically downmixed to DC in quadrature.

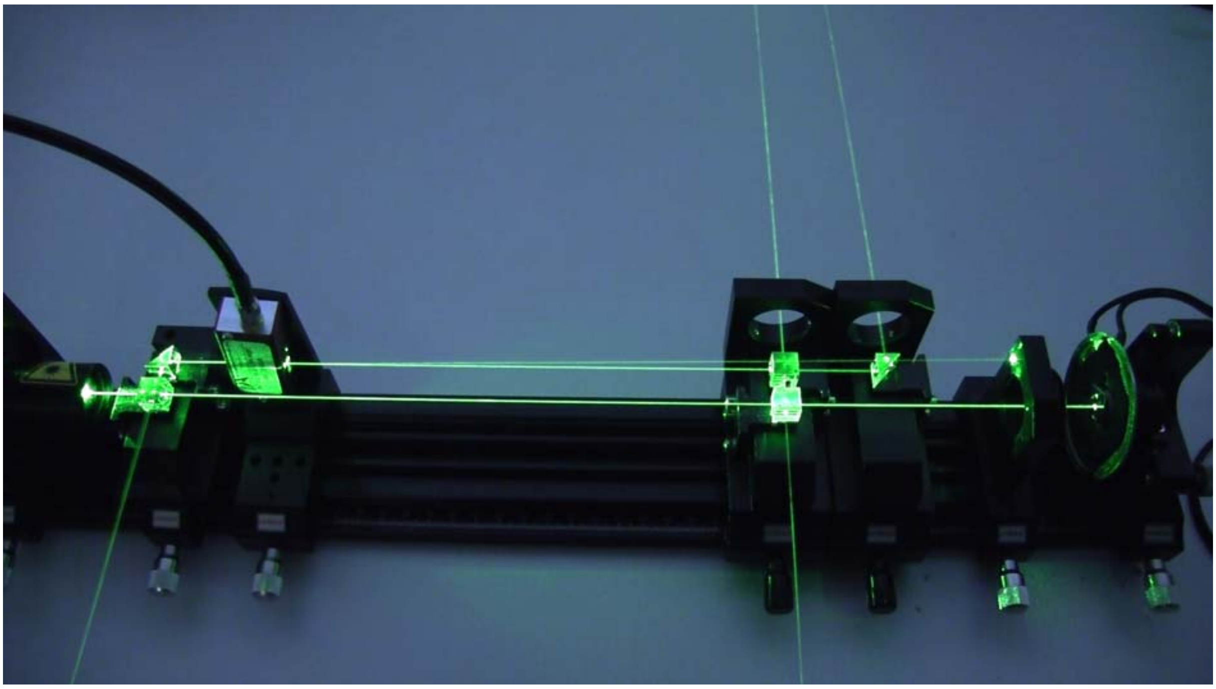
Both start with an interferometer setup and have a quadrature counter in the end.
The intermediate MHz frequency of the Vibrometer has the advantage of easy amplification at that stage.

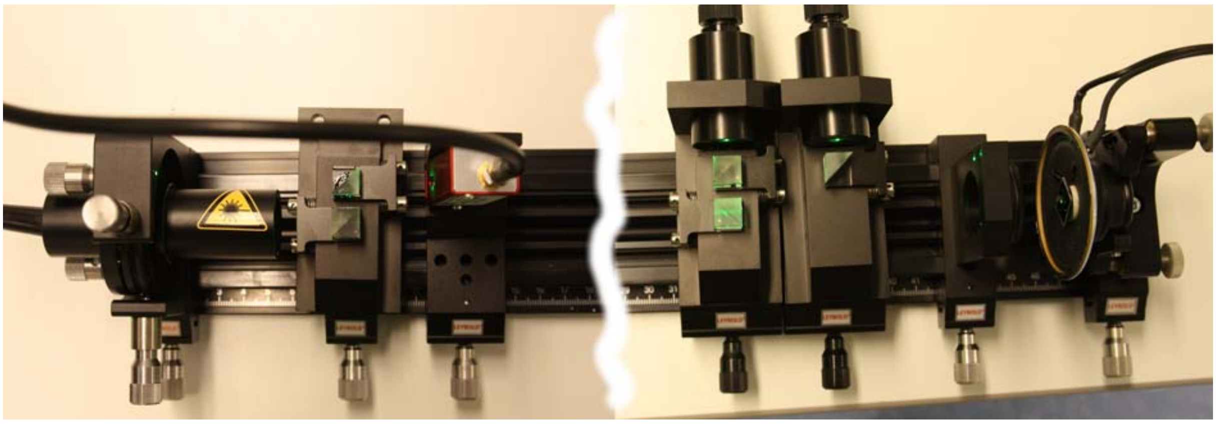
As we have seen above, if the intensities of the reference and the target beam are not equal, the interference contrast will drop.
In a heterodyne Interferometer, we can amplify a 30 .. 50 MHz signal by several decades without technical problems.
Most sources of noise, including stationary reflected light are close to DC.

In a homodyne Interferometer, such an amplification of a signal buried in noise is hard.

Experiment Setup







Description of the parts:

Laser 474 5430

As a laser source a frequency doubled Nd:YVO_4 laser of 30 mW output power (class 3B !) at 532 nm is used. The laser head is mounted in an X,Y – □,φ alignment holder for quick and easy beam adjustment.

The laser with the surrounding tilt adjustment disc can be easily removed from the XY setting screws. To remove the laser out of the tilting disc, there is a screw on the side of the disc.

Beam Splitter 474 208

The laser beam is split into a signal and a reference beam by a non-polarizing beam splitter cube BS-01. A 90 degree reflection prism BB-01 directs the reference beam towards the acousto-optic Modulator (AOM). Since both parts and especially the prism are mounted on a tuneable stage the reference beam can be tilted a small angle necessary for angular matching of the AOM crystal.

Acousto-Optic Modulator 474 411

Inside the AOM the reference beam is diffracted on the supersonic standing wave generated by the HF driven Piezo crystal. The first order diffracted signal is frequency shifted by 40 MHz and its polarisation is rotated by 90 degree.

Since the crystal is shaped a bit like a prism, the zero order beam will bend upwards, while the first order beam will exit the AOM with only a minor angular change to the incoming beam.

The AOM Driver supplies the AOM with a RF voltage of 40 MHz and a power of 0.5 W necessary to generate the supersonic standing wave in the TeO_2 crystal of the AOM. On an additional BNC output the AOM controller provides the same signal for use as a trigger reference or for mixing with the detected vibration signal. Do not power the AOM driver without the AOM crystal connected!

Beam Combiner 474 207

The beam combining unit consists of two different beam splitter cubes: a polarizing cube PBS-01, and a non-polarizing cube BS-02, each mounted on a small adjustment holder. The BS-02 is used for combining the reference and the signal beam, while the PBS-01 directs the laser beam first to the target and then reflects it towards BS-02.

By turning the adjustment holder the reference beam can be aligned for optimal overlap with the signal beam.

A photo detector 474 331 can be inserted into the clickmount.

Beam Bender 4742071

This is just a prism to mirror the second output of BS-02 on 474 207 towards a second detector.

When interferences occur, conservation of energy suggests that both detectors will generate a 180° phase shifted signal.

Focusing Optics 474 104

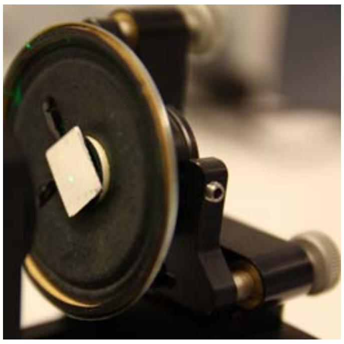
A focusing lens (f = 60 mm) focuses the signal beam on the vibrating object. Reflected or scattered light from the object is collimated on its way back to the photo detector.

Quarter Wave Plate 474 5320

An optical part that converts linear light into circular polarized light and backwards.
Used here to steer the laser beam through PBS-01 and later let it

be reflected.
The quarter wave plate can be rotated in the holder to achieve maximum or minimum reflection of the beam in PBS-01.

Mounted Membrane Speaker 474 206



As a vibrating object a small membrane speaker is used.

To increase the portion of reflected light a thin layer of reflecting tape is stuck to the surface of the speaker
Two screws allow the Speaker to be precisely tilted, such that a laser beam can be reflected precisely without focusing optics. To reduce the amplitude of the speaker, an 82 Ohm resistor is connected in one of the wires.

Fast Photo Detector 474 331

With a fast photo diode the time dependent intensity of the interfering signal and reference beam is detected. The detector is clicked in the 25 mm mount attached to the beam-combining unit
Please note that the small black box contains a 9 V battery and a switch
The output voltage will have an offset of –4.5 Volts, so any measurement should be done with AC coupling. If the –4.5 Volts are not present, the battery is empty.

Heterodyne mixer 474 313

The Heterodyne Mixer does the entire electronic signal processing steps of the Vibrometer.

There are three inputs in the left field: Two incoming detector signals can be subtracted, or one of these can be left open. The gain of the input signals can be adjusted.
The reference input takes the BNC Signal from the AOM box.

In the right field we have several outputs to observe the signal at different stages of processing.
In a first step the detector signal is multiplied by the reference channel,
So we will get the sum and difference frequencies.
Then the sum frequency is filtered out, so we have only the Doppler shift frequency.
To detect the direction, an IQ demodulation is used with a 90° phase shifted reference signal, so the Q channel will be out of phase by either plus or minus 90° depending on direction.

I and Q signal are fed into a counter that will output signal proportional to the position of the target.
That counter is not a digital one, but simply an analogue integrator fed by a charge pulse for each count.

Experimental set-up

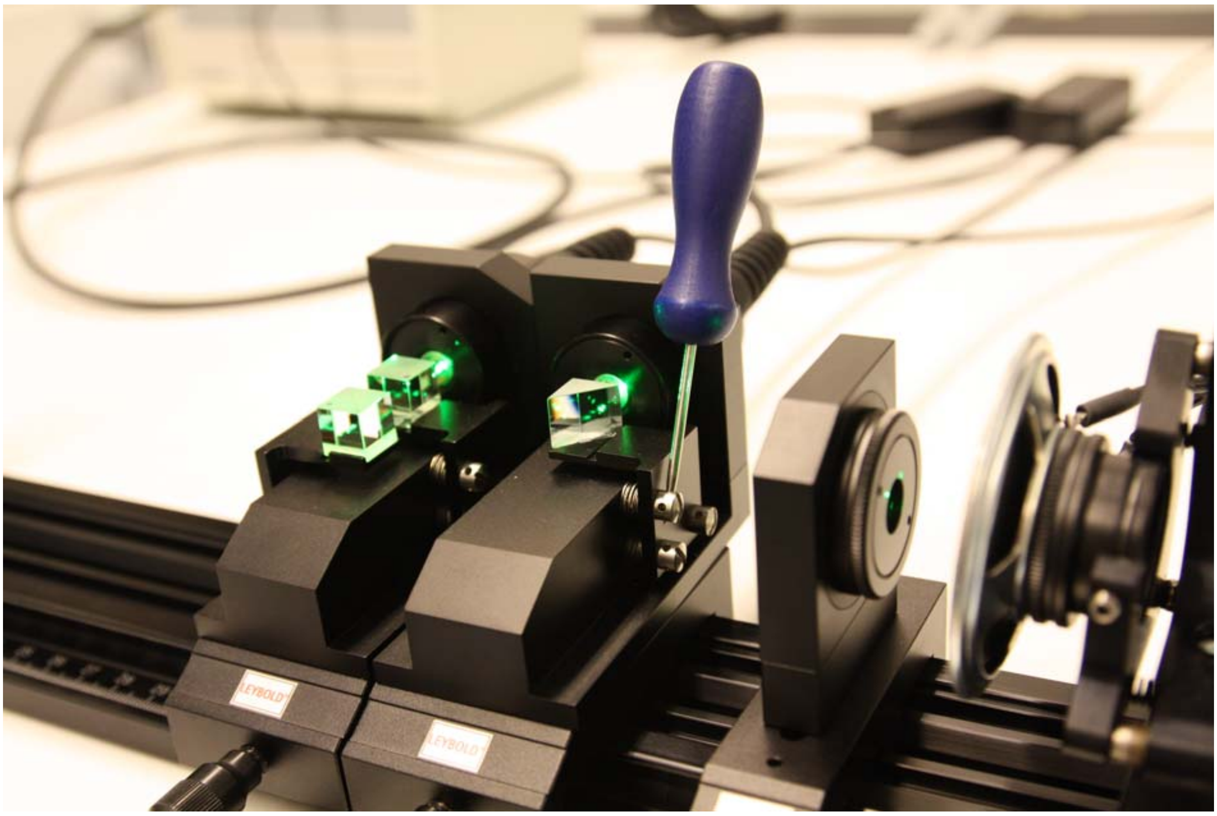
Laser alignment:

In a first step the laser source has to be adjusted to be parallel to the rail. Put the laser in its X,Y – φ, □ adjustment holder on one end of the optical rail. Switch the laser power supply on. Take care of the emerging 30 mV (class 3B) laser beam.
We use the quarter wave plate as a target, having a small diameter so it is easy to point at the centre:

A. Place the quarter wave plate close to the laser
B. Adjust the laser with the X and Y translation screws to hit the centre of the QWP with the beam
C. Place the quarter wave plate on the far end of the optical rail
D. Adjust the laser with the φ and □ tilting screws to hit the centre of the QWP with the beam

Proceed with steps A – D until the beam is aligned properly.

Adjusting the optics, general:



All the beamsplitter cubes are mounted on small tables, which can be tilted by three screws on the side. As the space is limited, these screw are to be operated by a needle as shown in the picture. Please take care of laser beam reflections from the needle.

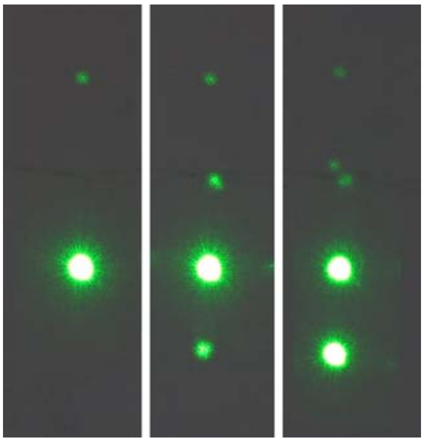
Beam splitting procedure

Insert the beam splitting unit 474 208 at the far end of the optical rail. The beam splitting cube BS-01 will create a reflection of the laser beam. Align the beam splitter cube until the laser beam is reflected back into itself.
Move the beam splitting unit 474 208 close behind the laser. The non-polarizing beam splitter cube splits the beam into a signal (transmitted) and a reference beam (reflected). The reference beam is

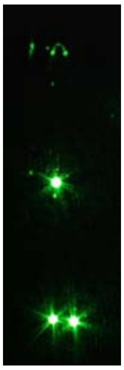
again reflected by a reflection prism. Align the reflection prism for parallel beams. Especially the horizontal beam-to-beam distance of 15 mm should be kept within an mm over a distance of 3 meters.

AOM Alignment

Connect the AOM with the RF connection lead to the AOM RF output socket of the AOM driver and place it on the optical rail.



Without the RF driver switched on, there should be only one laser beam coming out of the AOM and it is tilting upwards.(left part of image on the left)
Switch the driver on. Now we'll see three laser beams, zero order and plus minus first order.
The lower beam is the one we are interested in, and it has to be parallel to the reference beam.
Internally to the AOM, the Bragg condition has to be fulfilled. The lower first order beam has to be at least as bright as the zero order beam (right part of image). If it is to dark (middle part of image), the tilting angle of the AOM needs later readjustment. Release the upper screw on the back of the AOM, vary the tilting angle until the first order beam has maximum brightness and tighten the screw again. But first we will align BB-01 for parallel beams and then realign the AOM.



Align the Prism BB-01 on 474 208 for exactly parallel beams. Both vertical and with 15 mm separation as precise as possible, say 1 mm on 3 meters. The image on the right still shows some height variations that need to be cancelled. The AOM first order beam is a bit too high with respect to the measurement beam.
If this alignment is done sufficiently, it is time to look at the intensity of zero and first order beam. If the first order beam is darker than the first order beam, there is room for improvement. Tilt the AOM as described above, then realign BB-01.

Beam combining procedure

Place the beam-combining unit 474 207 somewhere in the middle of the optical rail and shift it until the zero order beam from the AOM just passes above the BS-02 cube. The first order beam will hit the BS-02 cube exactly.

First adjust PBS-01 by looking at the back reflection towards the laser and aligning PBS-01 until the reflection goes precise back into the laser.
BS-02 will be adjusted later.

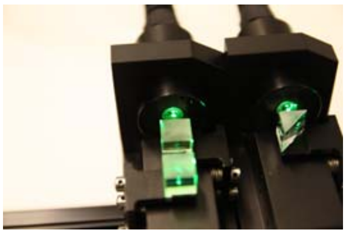
Signal Beam

Mount the quarter wave plate and the loudspeaker onto the optical rail, no lens, and leave some space for the later to be inserted 474 2071.
Tilt the loudspeaker until the reflection from the metal mirror on top of it goes back into the laser
If necessary rotate the quarter wave plate until some part of the reflected light passes through PBS-01 onto the laser.
Rotate the quarter wave plate such that the light is reflected in PBS-01 and exits through BS-02.
Remove any detector in the beam combiner 474 207.

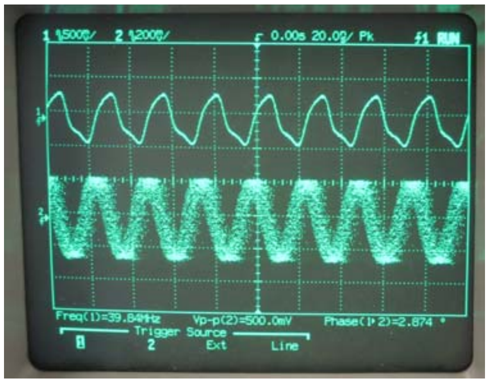
On a distant wall, there will now be two laser dots visible.

One is coming from the loudspeaker through reflection in PBS-01 and passing through BS-02,
The other one is the reference beam from the AOM, reflected in BS-02.
Tilt BS-02 until both beams are parallel:
Start by bringing them to the same point in far distance, 3 meters or so.
Then check if both laser beams come to the same point in close distance on a black sheet of paper.
Usually they will not hit the same spot close to BS-02, because the measurement and reference beam will not meet inside the BS-02.

Align BB-01 (part of 474 108) on the left side of the rail to overlap both beams in close distance and BS-02 to align them in parallel in far distance.



Now the Fast Photo Detector 474 331 can be placed in its mount at the Beam Combining Unit.
As both beams are overlapping, there should be only one bright spot be visible on the glass, as seen in the photo.
Using a two-channel oscilloscope use the AOM drivers 40 MHz as a reference signal, and look at the photo detector output. The heterodyne mixer is not used yet.
With aligned beams the photo detector will get a signal of about 50 mVpp at a frequency of 40 MHz.
With the previously described aligning steps, the photo detector should at least be able to see something, which can then be optimised carefully.



Of course, this signal is an interferometer output and as such very susceptible to mechanical noise.
In contrast to a rigid interferometer, we have a loose loudspeaker here that will pick up any noise.
While touching any adjustment screw, there will be no clean sine on the scope at all; only the envelope of the “noise” signal will increase with good alignment. After not touching anything for a second, the interferometer should stabilize.
Of course, people walking around in a few meters distance or talking loud will create enough movement of the loudspeaker to make the alignment a hard job.

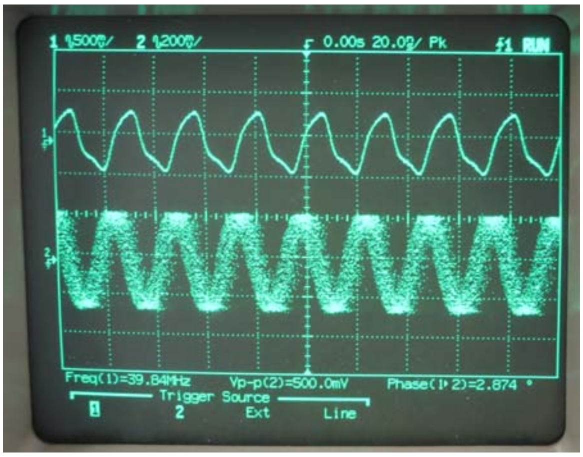
For fine adjustment of the modulation signal the laser beam can be adjusted very carefully by means of the □ and φ adjustment screws.

If no signal can be found,
put the beam bender 474 2071 on its place on the rail close the beam combiner, without a detector in here, we can now observe the electrical signal of the detector in the beam combiner 474 207 and the optical alignment of the beams coming out of the beam bender 474 2071.
A search for faint electrical signals by tilting BS-02 is only useful when the two beams out of 474 2071 are close to each other in far distance.

Experiments

The very first experiment is of course the optical setup of the interferometer until the first 40 MHz signal comes out of the photo detector.

Experiment 1a: 40 MHz domain



After setting up the optical interferometer, we can observe the 40 MHz heterodyne signal and observe the inteferometric phase shift:

Connect input 1 of the oscilloscope to the 40 MHz reference of the AOM driver,
Connect Input 2 of the oscilloscope to the output of the photo detector in 474 207
Use Channel 1 for trigger
Channel 1: 500 mV / div, AC coupled, trigger on zero crossing
Channel 2: 50 mV / div, AC coupled
Timebase: 20 ns per division

Connect the function generator 522 621 to the loudspeaker, and use the slowest possible settings:
Frequency = 1 X 0.1 Hz, sine, Amplitude 1 Volt

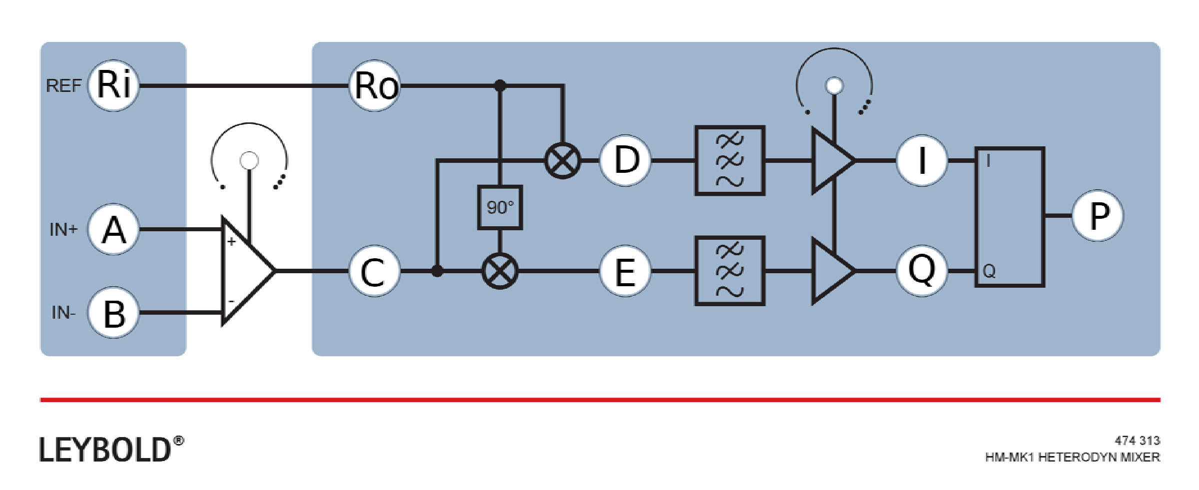
The lower trace will move t the left and right according to the motion of the loudspeaker, in the photo indicated by the dotted shifted trace.

This is in total about 20 fringes peak to peak, so we see a movement of 20 x 266 nm, about 5 µm.
Vary the amplitude setting of the function generator to get a larger and faster movement, until the eye cannot follow the signal on the oscilloscope.

As we are looking at an interferometer output, any mechanical noise will distort the lower trace.
For comparison, set the trigger to Input 2.
Now we have the reference moving, but the photo detector signal will remain absolutely stable. Therefore we see that the mechanical noise does only influence the phase and make it vary wildly, but it does not affect the quality of the photo detector signal.

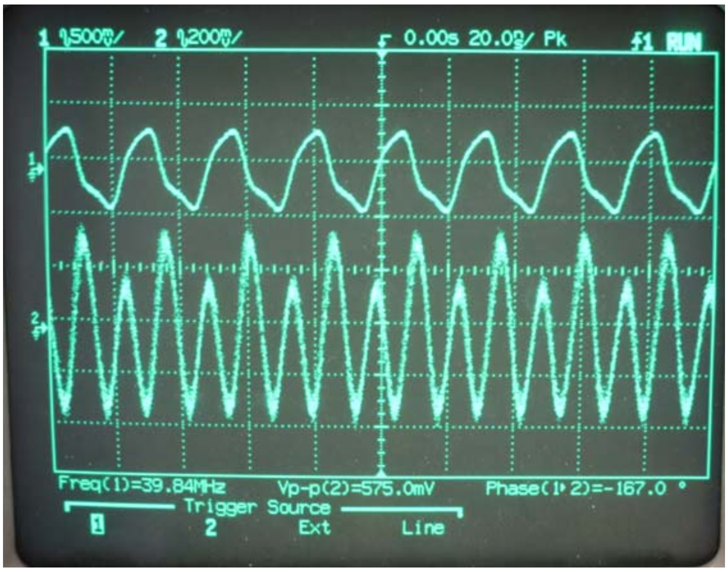
Experiment 1b: 40 MHz domain

We had not used the Heterodyne Mixer 474 313 so far.



Remove the cables from the oscilloscope and connect the AOM driver reference to the Mixer reference input (Ri). Connect one or both photo detectors to the signal input of the mixer (A,B). Connect input 1 of the oscilloscope to the reference output Ro and input 2 of the oscilloscope to output C. The same screen as in experiment 1 should appear on the oscilloscope, and we can adjust the signal gain by the trimmer of the mixer.

Now move input 2 of the oscilloscope to output D or E and we will see the output of the mixer. After multiplying the photo detector signal with the 40 MHz reference, we have sum and difference frequencies, with the sum of about 80 MHz clearly visible.
The difference frequency is just a few Hertz and can be seen by the vertical wobbling of the signal on screen.



Experiment 2: Downmix domain

Connect input 1 and 2 of the oscilloscope to outputs I and Q of the mixer,

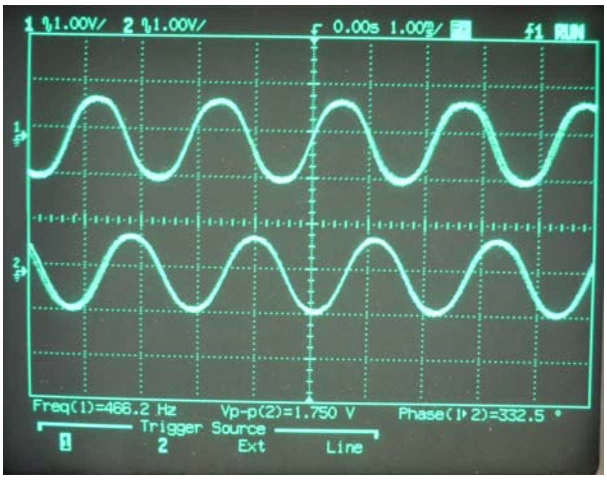
Set 1 V/div for both,
Timebase 1 ms/div

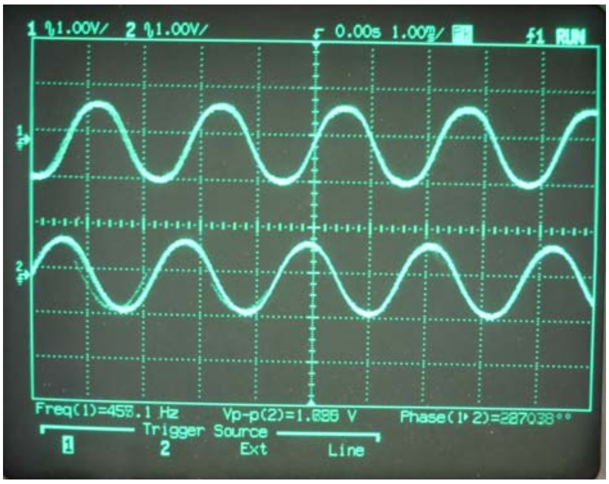
The function generator is set to
1 Hz
Triangular waveform
Full amplitude

The oscilloscope records the low pass filtered signals after the mixer, so we see only the difference frequency between AOM Ref and photo detector output.
This is the Doppler-shift induced frequency difference.
As can be seen in the photos, in this example we have about 500 Hz of Doppler shift in our reflected light from the loudspeaker.

With triangular waveform, this frequency is stable most of the time, except when the loudspeaker reverses direction.
With sinusoidal waveform, the frequency will vary continuously.

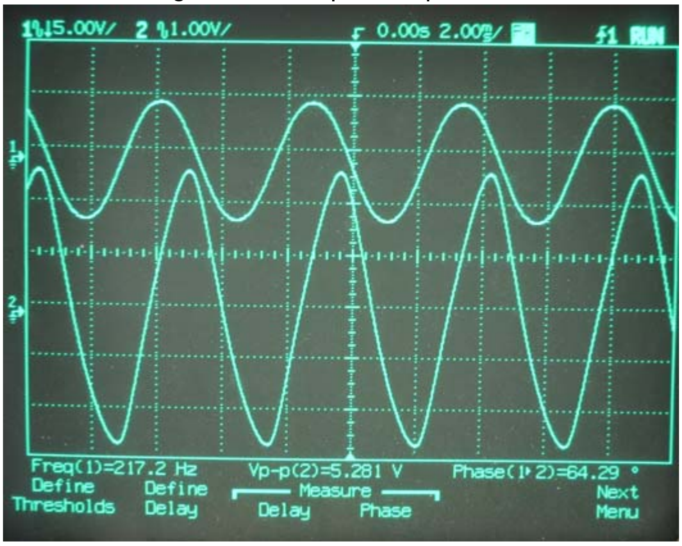
The speed of the loudspeaker can be calculated from the I channel alone.
But the direction of movement of the loudspeaker is shown in the relative phases of I and Q.
The two photos shown on this page show the I channel in the upper trace and the Q channel in the lower trace, in one case having +90° phase shift and in the other one –90° phase shift.





Experiment 3: Loudspeaker motion

Use the supplied 4 mm to BNC cable to connect the function generator output to input 1 of the oscilloscope and connect input 2 of the oscilloscope to output P of the mixer.



Set the oscilloscope to
Input 1: 5 V / div, Trigger
Input 2: 1 V/div
Time: 2 ms/div

And the function generator
2 x 100 Hz
Sine
Full amplitude

The upper trace shows the voltage supplied to the loudspeaker (8 Ohm coil + 82 Ohm resistor) and the lower trace shows the output of the IQ integrator.
For each zero crossing of the I signal the output is moved a fixed voltage step up or down, depending on the polarity of the Q signal. So we count fringes.

The scaling factor is approximately 100 fringes per volt.

Since the loudspeaker carries a rather heavy metal mirror, we created a nice mechanical resonance.
When varying the frequency from 150 to 250 Hz., we see that at low frequencies the electrical feeding of the speaker and the Vibrometer signal are in phase. If they are in anti-phase, please swap the connections of the loudspeaker.
When the frequency is increase, the resulting amplitude of the speakers motion increases suddenly, as seen in the photo in this setup at 217 Hz and the phase is about 90°, as expected for a resonance.
Even higher frequencies will result in lower amplitude of the speaker and a 180° phase shift relative to the electrical input of the speaker.

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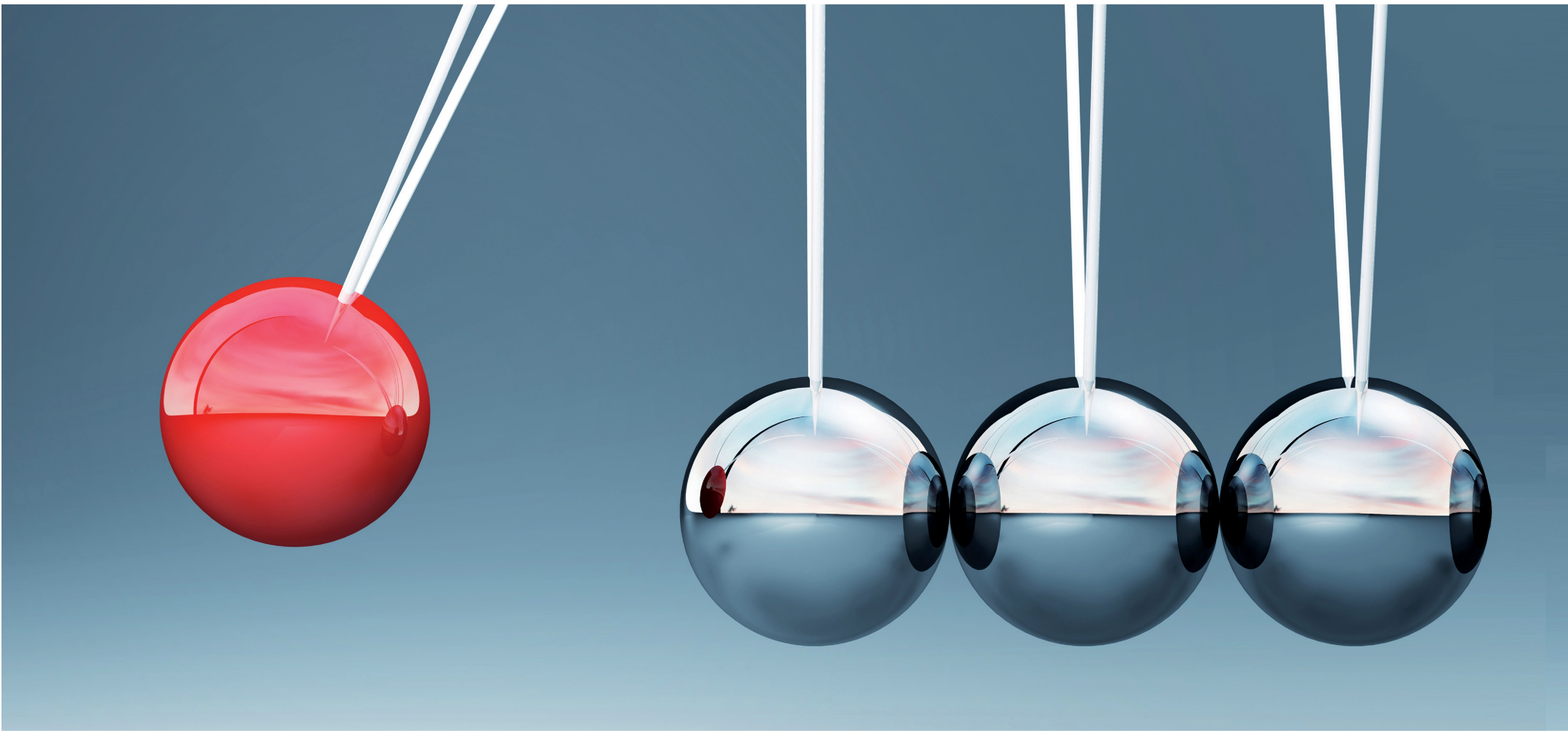

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