SCANNING LASER DOPPLER TECHNIQUES FOR VIBRATION TESTING

by P. Sriram, J.I. Craig and S. Hanagud

The design requirements for many structural elements call for a certain level of performance under dynamic conditions and thus, accurate dynamic characterization of such structures is of paramount importance. The standard approach that has evolved is to create a preliminary design and ascertain the performance of this design through a mathematical model (usually, a finite-element description). Frequently, the model must be validated and this function is accomplished through an iterative interaction with experiments. The use of such an approach allows the designer to observe the effects of potentially destructive loads without destroying the structure itself. One of the basic building blocks in this technique is the ability to measure the dynamic characteristics of structures with sufficient accuracy in an efficient manner.

In a conventional approach, accelerometers are used to sense the response at a sequence of selected locations. The acceleration or response and input load signals are processed using various analysis techniques,[1] usually involving digital signal processing, to estimate the system dynamic characteristics. The system dynamics are defined to a good extent by the eigenvalues (natural frequencies and damping ratios) and eigenvectors (mode shapes) obtained through such analyses. The natural frequencies and damping ratios are usually obtained through an appropriate curve-fitting procedure. The mode-shape information is obtained by using multiple exciters and/or sensors or by successively moving the exciters and/or the sensors to different points in the structure. In the special case where there is little interaction between the modes, the structure can simply be excited at a selected resonant frequency and the resulting response vector used as the approximate mode shape. A similar technique can be used even in the presence of significant modal interaction by using multiple exciters and special control logic. There are many possible sources for error, and in testing lightweight structures, the local mass loading of the response sensors can often distort the measurement so that one has to seek alternate noncontact sensors like the laser doppler vibrometer (LDV).

The LDV is based on the measurement of the Doppler shift of the frequency of laser light scattered by a moving object. The magnitude of the Doppler shift is related to the optical geometry and the velocity of the scattering object. Application of the LDV to the field of vibration measurements was initially in the context of rotating systems.[2] Subsequently, the technique has found use in special situations requiring the use of a noncontacting optical sensor such as in biomedical applications and vibration studies of extremely light structures like loud-speaker diaphragms. The instrument is perceived to be useful enough that now there are several commercial LDV systems available for vibration measurements in solids, including a portable model based on a novel frequency-shifting scheme. The measurement of vibration mode shapes requires the sensing of structural response at a series of locations on the structure. When using an LDV system, the straightforward solution is to translate the test object or the complete LDV system so that various points of interest can be probed. While this is simple for small test objects and small LDV systems, it is not always convenient. Various modifications have been introduced into LDV optical systems to make multipoint measurements easier, including the use of a fiber-optic link which allows most of the LDV components and the test object to remain fixed while only a few components have to be moved. An alternate approach introduced by Bendick[3] involved the translation of a single mirror to move the probe area along the optical axis of the LDV.

There are two main types of LDV, namely, the reference or single-beam and differential or dual-beam arrangements.[4] A single-beam LDV measures velocities along the line of sight and thus scanning causes a response due to the varying distance between the surface and the sensor. Most commercial LDV systems intended for structural applications are of this type. On the other hand, a two-beam

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arrangement is sensitive only to transverse velocities and is therefore free of this range sensitivity effect. The following discussion assumes the use of a dual-beam LDV.

In the case of opaque solid surfaces, the LDV beams have to impinge on the surface and this requires the optical axis of the system to be at an angle to the surface. Then, axial motion of the probe volume is of limited use since the probe will have to be accommodated either inside the solid (which is impossible since the object is opaque) or away from the surface (in which case the velocity measured is not the solid surface velocity). The surface can be scanned only by moving the LDV probe transverse to the optical axis or in a combination of axial and transverse motion. A transverse scanning technique has been devised by Durst *et al.*[5] with scanning motion derived from an oscillating mirror in the laser beam path. The scanning rates were limited in this system by inertial effects in the mirror to about 15 Hz (lines/sec) for scan angles of about ten degrees. Chehroudi and Simpson[6] have improved upon the concept by incorporating a commercial scanning device and strip mirrors to obtain scan rates up to 150 Hz over scan lengths of about 400 mm.

A scanning LDV can cause errors in fluid-flow measurements where the scattered light signal arrives in intermittent bursts corresponding to particles crossing the sensing region.[5] In the case of solid surfaces, the scattering occurs as long as the sensing area remains on the surface of the structure and thus the Doppler signal is continuous, providing for good spatial resolution. Then, a scanning system can be used effectively on vibrating surfaces to map the spatial velocity distribution (mode shapes) accurately. The use of digital signal-processing techniques means that the velocity at a given point is required only during the sampling intervals. Between samples, the sensor system is idle. A scanning LDV can make use of this 'idle' time to measure the response at other spatial locations. The problem then is to devise an appropriate data-analysis technique that extracts the required information from the scanning LDV velocity output signal. This is dealt with in detail in a subsequent section of the paper.

EXPERIMENTAL SETUP

The experimental setup will be described only briefly here since a detailed description is available elsewhere.[7] A TSI 9100-7 high-power four-beam two-velocity component LDV operated in a single-velocity component measurement mode was used. The single-component mode was enabled by operating the laser in a single-mode configuration radiating at a 514.5-nm wavelength (green). The actual laser power output that was used during the tests was in the range of 10-20 mW which was sufficient to produce good signals from the solid surfaces. Scanning capability was added to the LDV by directing the emerging laser beams onto a scanning mechanism, as illustrated in Fig. 1. The mechanism consisted of a pivoted front surface mirror driven through a flexible lever arm by an electrodynamic shaker. The lever arrangement converted the rectilinear motion of the shaker head into angular motion of the mirror. A mirror position-sensing transducer was also incorporated into the mechanism though it was not used for the tests reported herein.

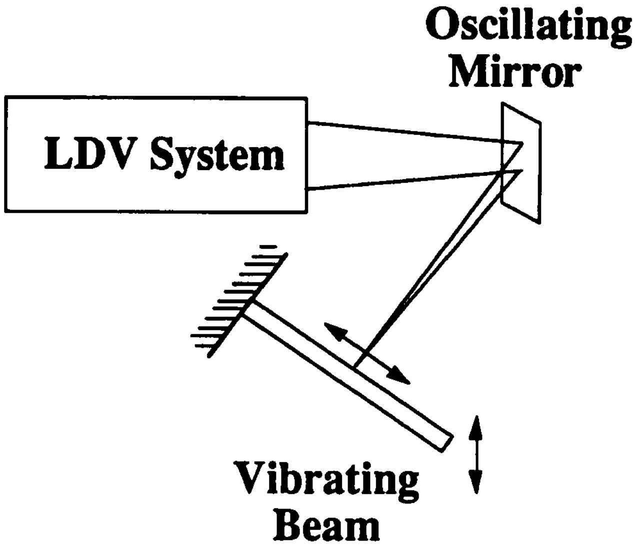

Fig. 1—*Scanning LDV experimental setup*

Using the above-described configuration, the mirror scanner can be placed on a mounting independent of the rest of the system. Thus, vibrations due to the oscillating mirror can be isolated from the rest of the optics as well as the test structure. The working distance from the scanning mirror to the test object was 1.0 m. The optics limited the scan to a ± 7.5-deg range due to defocussing effects. The maximum scan rate was restricted by inertial effects in the mirror to about 150 Hz for the full range scan, with higher rates attainable for shorter scans. These range and rate limitations can be readily overcome using correcting lenses and/or commercial scanning devices.

The test structure was an acrylic plastic rectangular bar of cross section 26 mm $\times$ 2.8 mm. One end of the bar was clamped between two aluminum blocks to produce a cantilever beam of 130 mm in length. The linear mass density of the beam was so low (86.0 g/m) that even a miniature accelerometer (1 g typical mass) caused significant mass loading. The beam was driven at its tip by a Bruel & Kjaer 4810 exciter and the force input was measured with a PCB Piezotronics 208A02 load cell. The excitation force, scanning mirror position, and velocity output signal from the scanning LDV were fed into a Preston analog-to-digital converter system controlled by a Hewlett Packard 1000 minicomputer.

The overall instrument response time was measured to be in the range of 200-250 μsec. The velocity measurement resolution and range were limited by the LDV electronics to .0004 m/sec and ± 0.5 m/sec respectively. In terms of acceleration, assuming a 250-Hz vibration frequency, the measurement resolution and range are 0.06 g and ±80 g respectively. The corresponding figures for 100-Hz vibration are 0.025 g and ±32 g.

DATA-ANALYSIS TECHNIQUES

The straightforward way to interpret the scanning LDV data is to sample the output whenever the sensing area is over a preselected location on the structure. In this way, the LDV can simulate a multitude of transducers located at these preselected locations. This forms the basis of the sorting algorithm. It is easy to recognize that due to the scanning action, the spatial velocity distribution is converted into a time-varying component of the LDV output signal. Since the scanning action is the cause of this, these components are bound to be dependent on the scanning frequency. Hence, a frequency-domain analysis of the LDV output can be used to isolate this effect from those due to the vibrations and this forms the basis of the second data-analysis technique presented here. This method is termed Chebyshev demodulation since the analysis leads to a Chebyshev series approximation of the spatial velocity distribution. The two techniques will now be described in further detail.

Sorting Algorithm

This data-analysis technique is based on associating the instantaneous velocity signal to specific points on the structure. The association is done so that the velocity measured by the scanning sensor simulates independent velocity measurements as a discrete series of locations along the scan line. Let ω_m be the frequency of the scanning sensor (rate of scanning), and assume that the velocity output from the scanning sensor is sampled at a rate denoted as ω_s. The sampling rate is assumed to be set to an integer multiple of the scanning rate. Let k be a positive integer so that

$$\omega_s = k\omega_m \tag{1}$$

Let ξ be the nondimensional scan coordinate. The sensing location of the LDV can be written as

$$\xi = \cos \omega_m t \tag{2}$$

If t_1 is the time when the first sample is digitized, then the time sequence for the sampling is

$$t_n = t_1 + (n-1)\frac{2\pi}{\omega_s} \quad \text{for } n = 1,2,3 \tag{3}$$

Let x_j^* be the value of ξ at the time instant t_j, i.e., x_j^* is the location of the scanning sensor at the time instant denoted by t_j. Hence,

$$x_j^* = \xi(t_j) = \cos \omega_m t_j \tag{4}$$

But the time sequence of sampling t_j is given by eq (3). Thus, the nth sample corresponds to the spatial location given by

$$x_n^* = \cos \omega_m \left(t_1 + (n-1)\frac{2\pi}{\omega_s}\right) \tag{5}$$

Using eq (1) gives

$$x_n^* = \cos \left(\omega_m t_1 + (n-1)\frac{2\pi}{k}\right) \tag{6}$$

Evidently, the $(n+k)$th sample corresponds to the same location but at a time k sampling periods or one scan cycle later. Thus, the result of the sampling process is a multiplexed velocity time history with the scanning frequency being the sampling frequency and the multiplexing is for data from various points along the scan. The excitation or force signal can also be digitized at the same instant as the velocity signal. Now, the first, $(k+1)$th, $(2k+1)$th, etc., data points all belong to the single location x_1. Thus, they represent the time history of velocity and force at that location, sampled at the scanning rate. Extending the argument, the sampled time history at the k points x_1, $x_2, \ldots, x_k$ can be obtained independently by sorting the multiplexed scanning sensor output signal.

Chebyshev Demodulation

The velocity distribution over the vibrating structure is assumed to be as follows.

$$v(x,t) = g(x) + \phi(x) \sin \omega_b t + \psi(x) \cos \omega_b t \tag{7}$$

where ω_b is the beam-vibration frequency. The nonoscillatory spatial velocity distribution $g(x)$ is included for generality. For a lightly damped structure excited at resonance, the ϕ and ψ functions approximate the corresponding mode shape. For real mode shapes, the two functions differ only by a scalar and in the case of complex mode shapes, they represent a set of orthogonal components.[1] The problem posed is to be able to detect $g(x)$, $\phi(x)$, and $\psi(x)$ using the scanning LDV. An LDV scanning at a frequency ω_m, with its sensing location described by eq (2) combines the spatial and temporal velocity variations and its output is governed by

$$V(t) = g(\cos \omega_m t) + \phi(\cos \omega_m t) \sin \omega_b t + \psi(\cos \omega_m t) \cos \omega_b t \tag{8}$$

This expression can be rewritten as[7]

$$V(t) = C_0 + S_0 e^{j\omega_b t} + S_0^* e^{-j\omega_b t} + \frac{1}{2}\sum_{k=1}^{\infty} \Big(C_k e^{jk\omega_m t} + C_k^* e^{-jk\omega_m t} + S_k e^{j(\omega_b \pm k\omega_m t)} + S_k^* e^{-j(\omega_b \pm k\omega_m t)}\Big) \tag{9}$$

where the * represents complex conjugate and S_k is defined as

$$S_k = \tfrac{1}{2}(B_k - jA_k) \tag{10}$$

Here, A_k, B_k and C_k are the Chebyshev series expansion coefficients of $g(x)$, $\phi(x)$ and $\psi(x)$. For example, we have

$$g(x) = C_0 + \sum_{k=1}^{\infty} C_k T_k(x) \tag{11}$$

where $T_k(x)$ is the kth Chebyshev polynomial. From eq (9), it is evident that a Fourier transform of the scanning LDV velocity output signal will exhibit peaks at the frequencies 0 (DC), $k\omega_m$, ω_b, $\omega_b \pm k\omega_m$, and $-\omega_b \pm k\omega_m$. Measurement of the complex amplitudes at these locations will therefore yield the coefficients A_k, B_k, and C_k which determine the distributions $g(x)$, $\phi(x)$ and $\psi(x)$, and hence $V(x,t)$.

The location of these peaks in the frequency domain is illustrated in Fig. 2. If discrete Fourier analysis (e.g., FFT) is used, it is essential to perform the Fourier transform such that the spectral lines of interest do not overlap. If the scan rate is set to a $1/(n + .5)$ fraction of the beam vibration frequency for some integer n, the lines are distinct and no overlap occurs. It is important to realize that as per this theory, an anti-aliasing low-pass filter cannot be used if the sorting technique is employed, as it would attenuate some of the Chebyshev terms, leading to incorrect transfer functions. The consequence is that high-frequency noise is bound to degrade the transfer functions obtained using the sorting scheme.

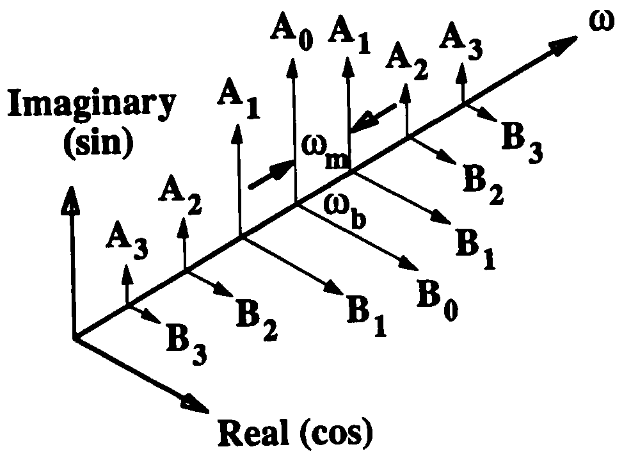

Fig. 2—*Frequency components of LDV signal*

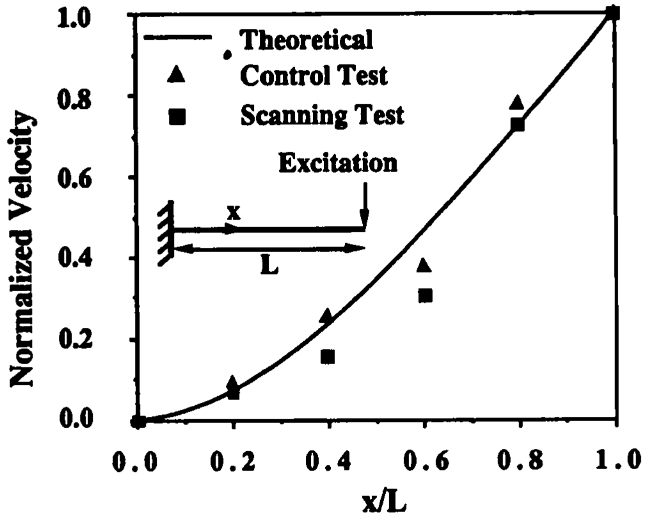

Fig. 3—*Beam mode shape (first mode, sorting technique)*

EXPERIMENTAL RESULTS

An initial sequence of tests was conducted to establish the first few natural frequencies of the beam. The LDV was used as a single point sensor (no scanning) to observe the tip of the beam and the excitation and response signals were analyzed using a GenRad 2515 modal analysis system. This provided the baseline (or control) values for the natural frequencies. The process was repeated by moving the LDV onto four more points along the beam to obtain the mode shapes. While using the sorting algorithm, since the sampling proceeds at k times the scanning rate, fixing the end point $\xi = x/L = 1$ as one response point yields a set of response points for each choice of k (here, x is the distance from the root of the beam to the point of interest and L is the length of the beam). However, if the sampling is at a constant pace, the response points are not spaced evenly along the scan length. If necessary, unequally-spaced ADC sampling can be used to avoid this situation. In this paper, only equal ADC sample spacing is considered. To obtain five equally spaced response points on the beam, the scanning was conveniently split into two regions. The two independent scan regions extended from $x/L = 0.8$ to $x/L = 1.0$ and $x/L = 0.2$ to $x/L = 0.6$. Using k of 2 and 4 respectively provided the response histories from the points $x/L = 1.0$, 0.8, 0.6, 0.4 and 0.2 as required. Thus the testing was completed effectively in the time taken to make measurements at two locations.

The beam was excited using a random waveform of band width 0-55 Hz through an electrodynamic shaker attached to the tip. The scanning frequency was set to 128 Hz. Due to the reasoning given in the theory of the analysis technique no anti-aliasing filters were used. The load and velocity signals were used to estimate the frequency-response functions (FRFs) at all the measurement locations, using 30 blocks of 256 points each. The effective sampling rate for each response point, it should be recalled, is the scanning rate (128 Hz). A significant amount of noise was evident in the measured response functions. However, the complex exponential modal analysis process employed in the GenRad analyzer was fairly insensitive to these noise spikes and consistent natural frequency and mode shapes estimates were obtained. The estimated (real) mode shapes from the control test and scanning LDV test are presented in Fig. 3, which also includes the theoretical mode shape. Though there are some discrepancies when compared with the theoretical results, there is sufficient agreement with the nonscanning LDV data, establishing the viability of the scanning LDV.

To verify the operation of the demodulation technique, the LDV was set to scan the entire length of the beam at ω_m = 7.143 Hz. The excitation was set to a sine waveform of frequency ω_b = 25 Hz (beam first mode). The autospectrum of the scanning LDV output, averaged using ten blocks of 256 samples is presented in Fig. 4. The first large peak in the autospectrum corresponds to the fundamental scan harmonic (ω_m). The largest peak in the figure is at the beam excitation frequency (ω_b = 25 Hz) and the two large peaks on either side (the second highest and third highest peaks in the figure) correspond to the $\omega_b \pm \omega_m$ terms. Further terms in the series are difficult to readily observe in the plot, although several are visible.

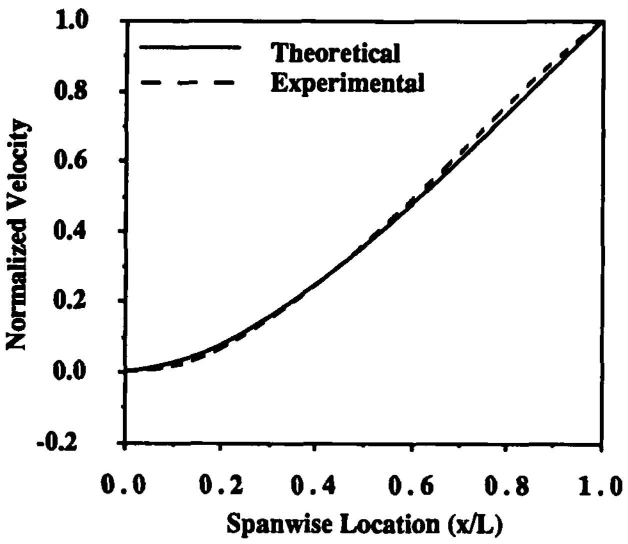

Fig. 5—*Beam mode shape (first mode, three terms)*

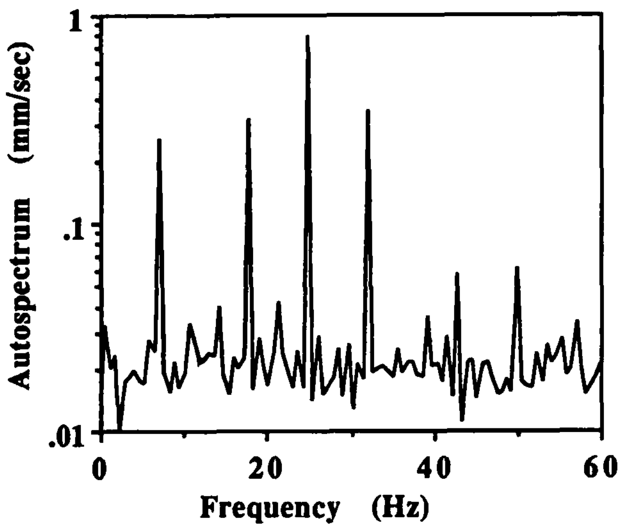

Fig. 4—*Autospectrum of scanning LDV output*

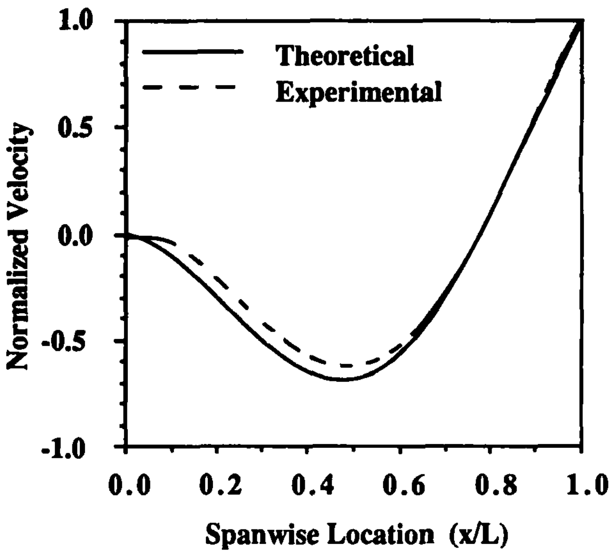

Fig. 6—*Beam mode shape (second mode, four terms)*

The Chebyshev coefficients A_0 to A_2 and B_0 to B_2 were estimated based on Fourier transforms of the scanning LDV data. The coefficients were averaged using an error minimization procedure. The spatial velocity distribution obtained from these Chebyshev coefficients is presented in Fig. 5. Assuming light damping and wide separation of the natural frequencies of the beam, this spatial distribution is an approximation to the corresponding beam mode shape. Comparison with the theoretical mode shape plotted in Fig. 5 establishes the viability of the measurement technique. In order to provide a more stringent test of the scheme, the test was repeated for the beam vibrating in the second mode and with the response limited to about 0.03 m/sec. The second mode of the beam was excited using a 255-Hz sine wave and the scan frequency was set to 56.67 Hz. This time, four terms of the Chebyshev series were extracted to capture the higher curvature present in the beam mode shape. The results from this test are presented in Fig. 6, clearly demonstrating the measurement capability.

CONCLUSION

The feasibility of using a scanning LDV to measure the mode shape of a vibrating structure has been shown. The technique of simultaneously gathering vibration velocity information from multiple locations along the scan has been demonstrated. This makes it possible to obtain the velocity simultaneously from multiple points using a single-sensor system. A new data-processing technique extracts Chebyshev approximations to the velocity profile. The scanning LDV provides a measuring tool that is noncontacting and adds no mass to the structure. The mode-shape data from measurements using the scanning LDV exhibit good agreement with theoretical predictions. Additional work is necessary to exploit the benefits of this development and to apply it to other important practical problems like separation of interacting modes and nonharmonic or random vibrations.

ACKNOWLEDGMENTS

The research described here was partially supported by the U.S. Army Research Office. The authors also wish to thank Dr. N.M. Komerath and S.G. Liou for assisting in the use of the LDV.

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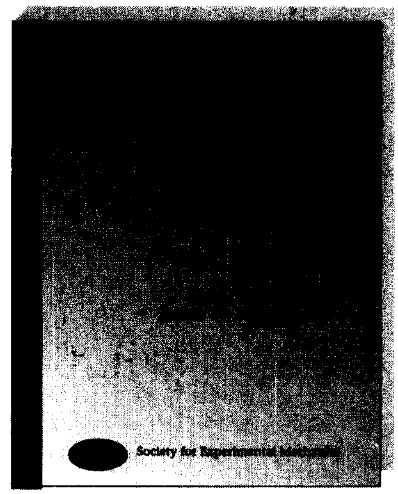

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