Chapter 3

CW & Pulse Radar

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Continuous Wave and Pulsed Radars

Outline :

- CW Radar
- Frequency Modulation
- Pulsed Radar
- Multiple Frequency CW Radar
- Range and Doppler Ambiguities
- Resolving Ambiguities

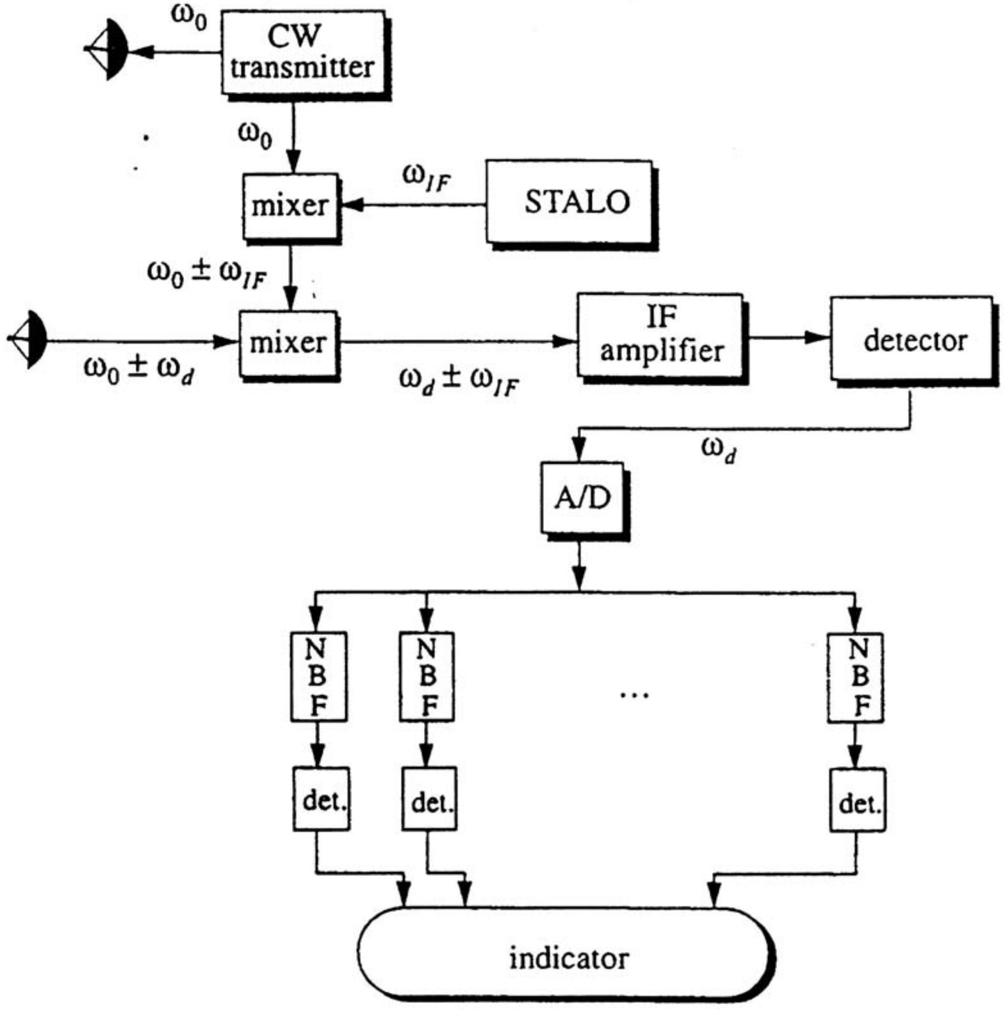
CW Radar

- CW waveforms
  - (1) CW waveforms $\cos 2\pi f_0 t$
  - (2) Stationary target and clutters – f_0
  - (3) Moving target – shifted by f_d (Doppler frequency), $f_d = 2v_R/\lambda$

- Extracts target radial velocity v_R

- Range measurement is possible with some modification to the radar operations and waveforms

CW Radar



<CW radar block diagram>

CW Radar

- Two antennas are used.
- NBF(Narrow Band Filters) – as narrow as possible.
- BW of the CW radars are that of gated CW waveform.
- NBF(Doppler filter bank)

  (1) FFT of size N_{FFT}

  (2) Effective radar Doppler BW is $N_{FFT}\Delta f/2$
- To measure target range, timing mark is necessary.

  - Linear Frequency Modulation(LFM)

CW Radar Equation

- NBF bank is implemented by FFT
  – dwell time(dwell interval) T_{Dwell} determines frequency resolution or the BW of individual NBFs

BW of NBF

$$\Delta f = 1/T_{Dwell}$$

- Maximum resolvable frequency B

$$2B = N_{FFT}\Delta f = N_{FFT}/T_{Dwell}$$

$$\boxed{T_{Dwell} = N_{FFT}/2B}$$

CW Radar Equation

▣ CW Radar Equation

$$SNR = \frac{P_{av} T_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_e F L}$$

T_i : time on target

T_e : effective noise temperature

$$\Rightarrow \quad \boxed{SNR = \frac{P_{CW} T_{Dwell} G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T_e F L L_{win}}}$$

L_{win} : loss associated with the type of window(weighting) in FFT

Frequency Modulation

- FM waveform with sinusoidal modulating signal $\cos 2\pi f_m t$

$$s(t) = A\cos\left(2\pi f_0 t + k_f \int_0^t \cos 2\pi f_m u \, du\right)$$

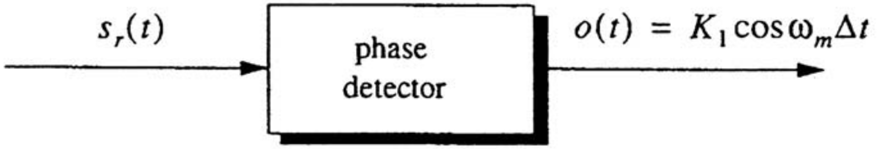
$$\psi(t) = 2\pi f_0 t + 2\pi \Delta f_{peak} \int_0^t \cos 2\pi f_m u \, du = 2\pi f_0 t + \beta \sin 2\pi f_m t, \qquad \omega = \frac{d\psi}{dt}$$

$$k_f = 2\pi \Delta f_{peak} \qquad \Delta f_{peak} = \text{peak frequency deviation}$$

FM modulation index

$$\boxed{\beta = \frac{\Delta f_{peak}}{f_m}}$$

$$s_r(t) = A_r \cos\left(2\pi f_0 (t - \Delta t) + \beta \sin 2\pi f_m (t - \Delta t)\right) \qquad \Delta t = \frac{2R}{c}$$



<Extracting range from an FM signal return K_1 is a constant>

Frequency Modulation

- FM waveform

$$s(t) = A\cos\left(2\pi f_0 t + \beta \sin 2\pi f_m t\right)$$

$$s(t) = A\,\mathrm{Re}\left\{ e^{j2\pi f_0 t}\; e^{j\beta \sin 2\pi f_m t} \right\}$$

Since $\exp(j\beta \sin 2\pi f_m t)$ is periodic with period $T=1/f_m$,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn2\pi f_m t}$$

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin 2\pi f_m t}\; e^{-jn2\pi f_m t}\, dt$$

since $$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin u - nu)} du$$

$$\Rightarrow \quad \boxed{e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t}}$$

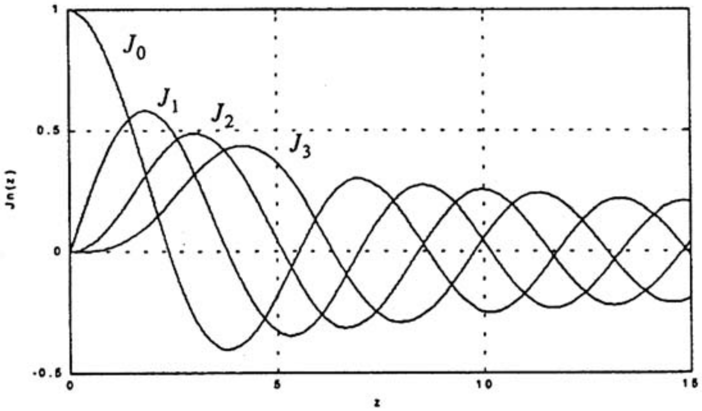
$$\Rightarrow \quad \boxed{P = \frac{1}{2}A^2 \sum_{-\infty}^{\infty} |\, J_n(\beta)\,|^2 = \frac{1}{2}A^2}$$

Frequency Modulation

- FM waveform

$$s(t) = A\,\mathrm{Re}\left\{ e^{j2\pi f_0 t} \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{jn2\pi f_m t} \right\}$$

$$\boxed{s(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi f_0 + n2\pi f_m)t}$$

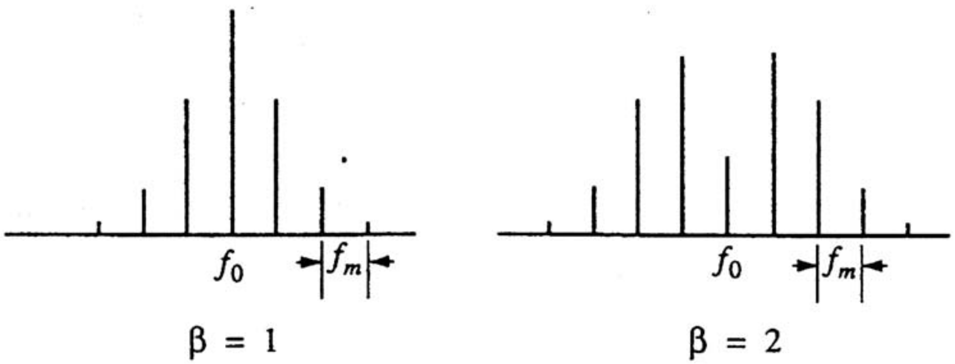


$$J_n(\beta) = J_{-n}(\beta) \text{ for } n \text{ odd}$$

$$J_n(\beta) = -J_{-n}(\beta) \text{ for } n \text{ even}$$

$$\begin{aligned} s(t) = A\{ & J_0(\beta)\cos 2\pi f_0 t + \\ & J_1(\beta)[\cos(2\pi f_0 + 2\pi f_m)t - \cos(2\pi f_0 - 2\pi f_m)t] \\ & + J_2(\beta)[\cos(2\pi f_0 + 4\pi f_m)t + \cos(2\pi f_0 - 4\pi f_m)t] \\ & + J_3(\beta)[\cos(2\pi f_0 + 6\pi f_m)t - \cos(2\pi f_0 - 6\pi f_m)t] \\ & + J_4(\beta)[\cos(2\pi f_0 + 8\pi f_m)t + \cos(2\pi f_0 - 8\pi f_m)t] + \ldots\} \end{aligned}$$

Frequency Modulation



Carson's rule: $B \approx 2(\beta + 1) f_m$ (range resolving capability)

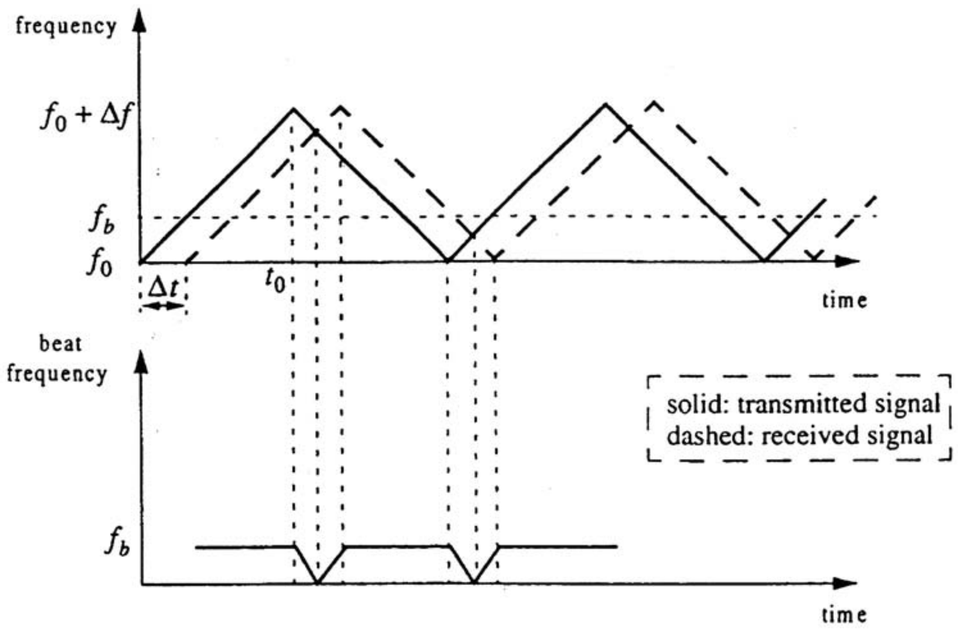
For small β,

$$J_0(\beta) \approx 1 \qquad J_1(\beta) \approx \frac{1}{2}\beta$$

$$s(t) \approx A\left\{\cos 2\pi f_0 t + \frac{1}{2}\beta[\cos(2\pi f_0 + 2\pi f_m)t - \cos(2\pi f_0 - 2\pi f_m)t]\right\}$$

Linear FM(LFM) CW Radar

- Measures both range and Doppler information.
- Triangular LFM waveform



< Transmitted and received triangular LFM signals and beat frequency for stationary target >

$$f_m = \frac{1}{2t_0}$$

$$\dot{f} = \frac{\Delta f}{t_0} = \frac{\Delta f}{(1/2 f_m)} = 2 f_m \Delta f$$

$$f_b = \Delta t\, \dot{f} = \frac{2R}{c} \dot{f}$$

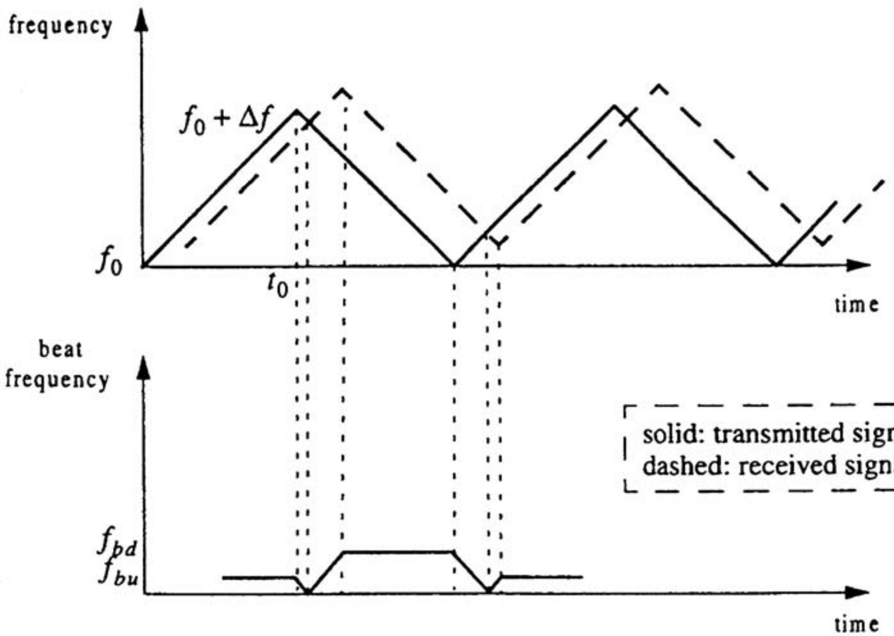
$$\dot{f} = \frac{c}{2R} f_b$$

$$\boxed{f_b = \frac{4R f_m \Delta f}{c}}$$

$$\boxed{R = \frac{c}{4 f_m \Delta f} f_b}$$

Linear FM(LFM) CW Radar

▣ Moving target



< Transmitted and received triangular LFM signals and beat frequency for moving target >

$$f_{bu} = \frac{2R}{c}\dot{f} - \frac{2v_R}{\lambda}$$

$$f_{bd} = \frac{2R}{c}\dot{f} + \frac{2v_R}{\lambda}$$

$$R = \frac{c}{4\dot{f}}(f_{bu} + f_{bd})$$

$$v_R = \frac{\lambda}{4}(f_{bd} - f_{bu})$$

$$\Delta t_{max} = 0.1 t_0$$

$$R_{max} = \frac{0.1 c t_0}{2} = \frac{0.1 c}{4 f_m}$$

Multiple Frequency CW Radar

- Multiple frequency scheme provides range measurement without frequency modulation.
- CW radar(single frequency)

$$s(t) = A\sin 2\pi f_0 t$$

Received signal $\qquad s_r(t) = A_r \sin(2\pi f_0 t - \varphi), \qquad \varphi = 2\pi f_0 \dfrac{2R}{c}$

Measuring φ, $\qquad R = \dfrac{c\varphi}{4\pi f_0} = \dfrac{\lambda}{4\pi}\varphi$

Maximum unambiguous range, $\qquad \varphi = 2\pi$

$$\boxed{R = \frac{c}{2f_0} = \frac{\lambda}{2}}$$

(single frequency CW radar)

Multiple Frequency CW Radar

- Two CW signal

$$s_1(t) = A_1 \sin 2\pi f_1 t$$

$$s_2(t) = A_2 \sin 2\pi f_2 t$$

Received signal

$$s_{1r}(t) = A_{r1} \sin(2\pi f_1 t - \varphi_1), \qquad \varphi_1 = \frac{4\pi f_1 R}{c}$$

$$s_{2r}(t) = A_{r2} \sin(2\pi f_2 t - \varphi_2), \qquad \varphi_2 = \frac{4\pi f_2 R}{c}$$

$$\varphi_2 - \varphi_1 = \Delta\varphi = \frac{4\pi R}{c}(f_1 - f_2) = \frac{4\pi R}{c}\Delta f, \qquad R = \frac{c\Delta\varphi}{4\pi \Delta f} = \frac{\lambda}{4\pi}\Delta\varphi$$

Maximum unambiguous range, $\Delta\varphi = 2\pi$

$$\boxed{R = \frac{c}{2\Delta f}} \quad (>> \frac{c}{2 f_0}) \quad \text{(two frequency CW radar)}$$

Pulsed Radar

- Train of modulated pulses
- Range : two-way time delay between transmitted and the received pulse
- Doppler measurement

  (1) From accurate measurement of range, $v_R = \dot{R} = \frac{\Delta R}{\Delta t}$

  (2) Doppler filter bank
- Pulsed radar waveform

  (1) Carrier Frequency

  (2) Pulse Width - bandwidth, range resolution

  (3) Modulation

  (4) Pulse Repetition Frequency(PRF) – Doppler and range ambiguities, maximizing average transmitted power

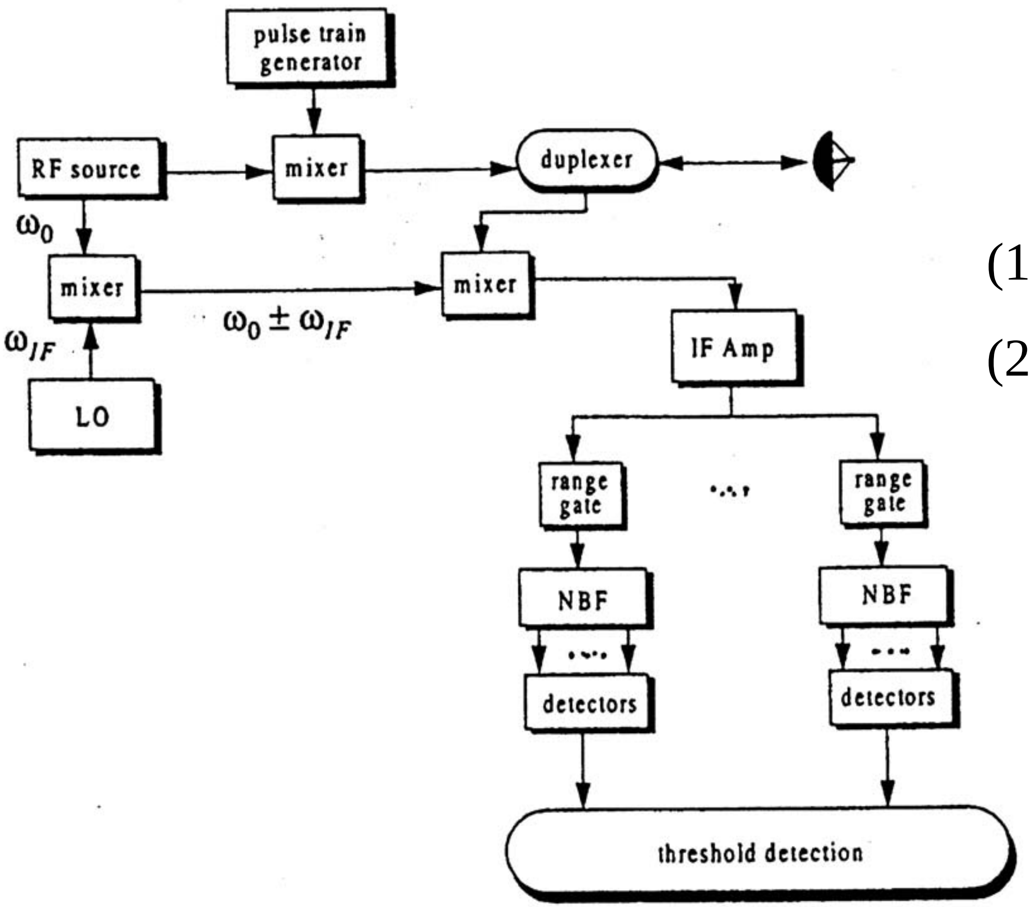
Pulsed Radar

- PRF Schemes
  - (1) Low PRF – accurate, long, unambiguous range, Doppler ambiguity
  - (2) Medium PRF – adequate average transmitted power
  - (3) High PRF – superior average transmitted power, excellent clutter rejection, ambiguous range – Pulsed Doppler Radar(PDR)

PRF	Range Ambiguous	Doppler Ambiguous
Low PRF	No	Yes
Medium PRF	Yes	Yes
High PRF	Yes	No

- Agile PRF
  - (1) Avoid blind speed (MTI) – PRF staggering
  - (2) Avoid range and Doppler ambiguities – PRF jitter
  - (3) Prevent jammers – PRF jitter

Pulsed Radar



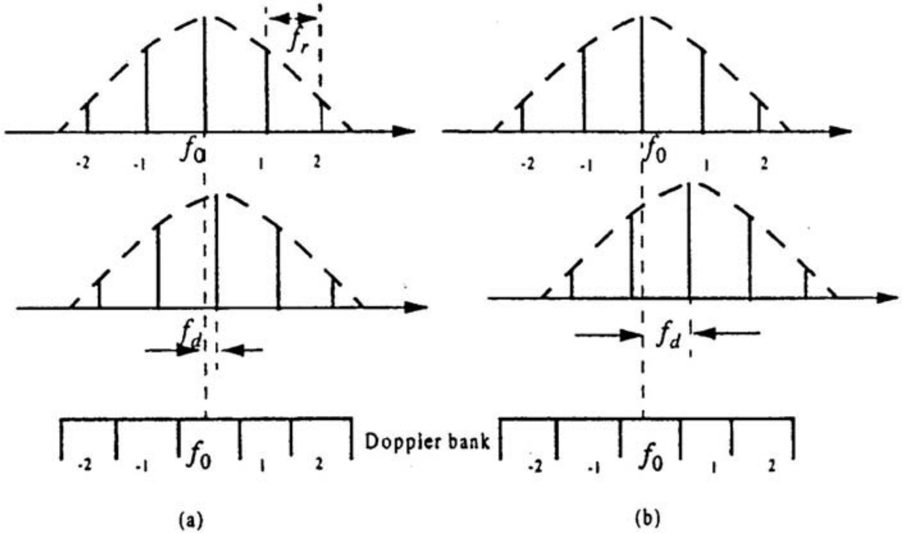
< Pulsed radar block diagram >

(1) Range Gate – range resolution

(2) NBF bank - FFT

Range and Doppler Ambiguity

- Range Ambiguity
  - (1) Second pulse is transmitted prior to the return of the first pulse
  - (2) Long range surveillance radars – low PRF
- Doppler Ambiguity



<Spectra of transmitted and received waveforms, and Doppler bank>

(1) PRF, f_r

$$f_r = 2 f_{d\,\max} = \frac{2v_{r\max}}{\lambda}$$

(2) To avoid Doppler ambiguities, high PRF or multiple PRF scheme is used.

Range and Doppler Ambiguity

- Spectrum for train of radar pulse

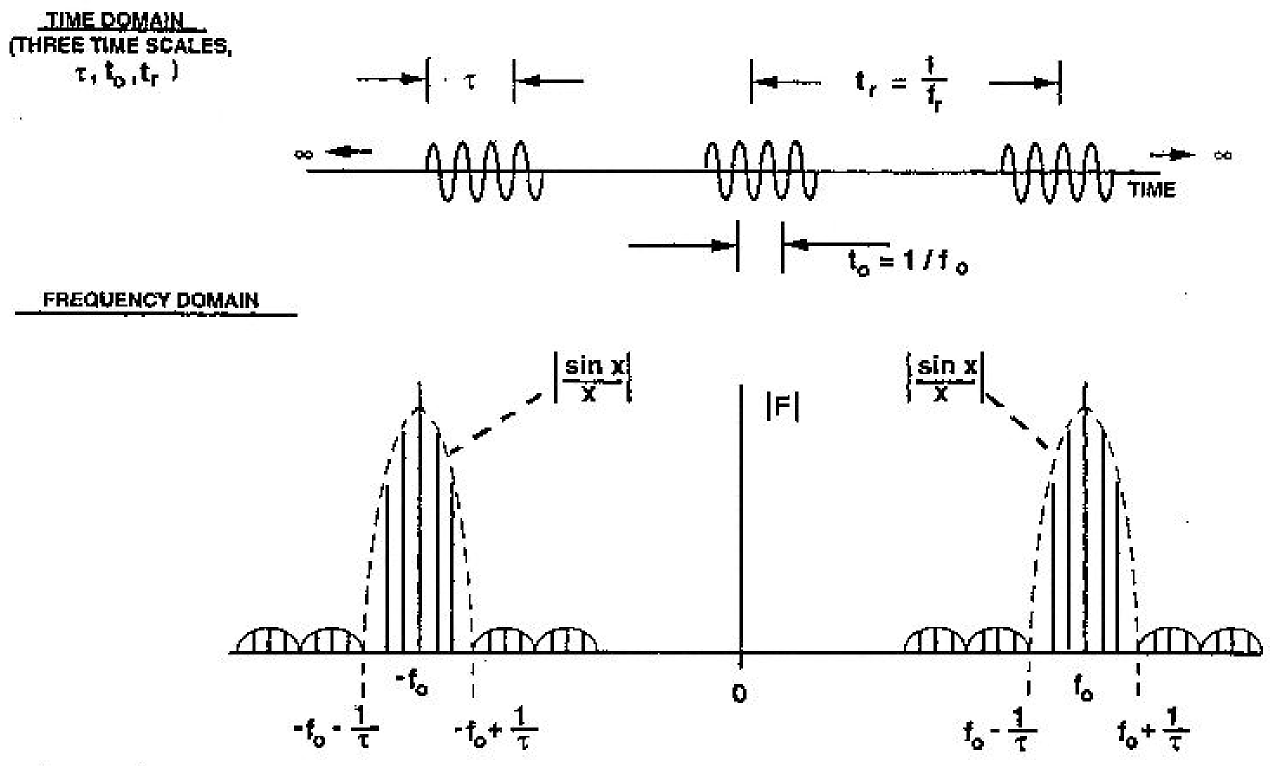


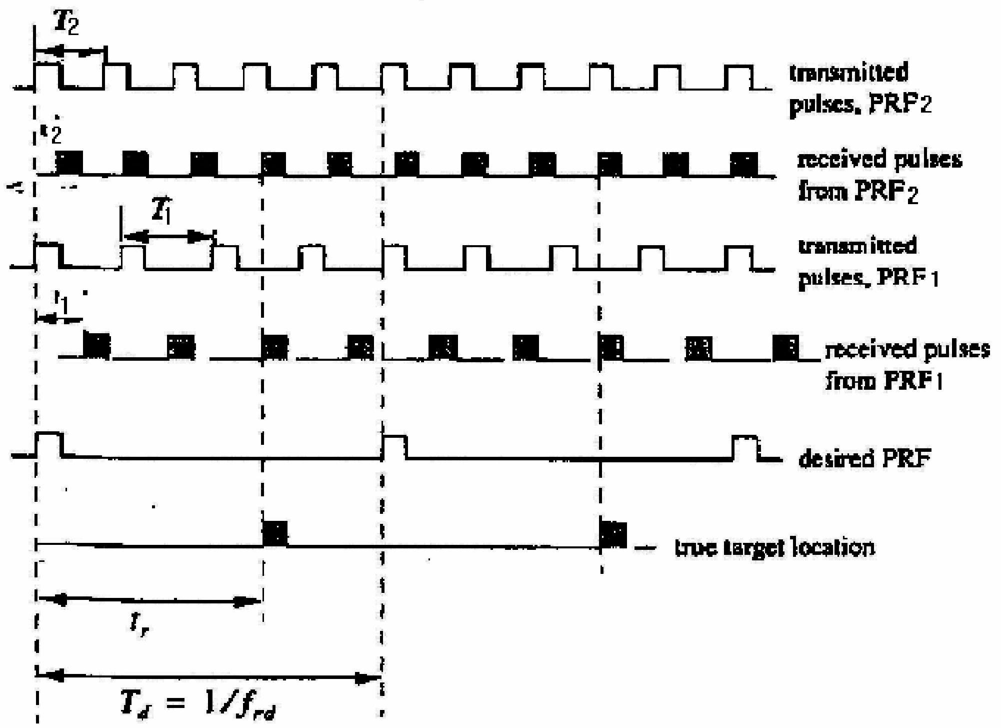
Figure 3.5 Infinite pulse (τ) train signal of PRF f_r and frequency f_0 [1].

Resolving Range Ambiguities

(1) Two PRFs, f_{r1} and f_{r2}

- Corresponding unambiguous range, R_{u1} and R_{u2} (small)

(2) Desired unambiguous range R_u (>> R_{u1}, R_{u2}) and desired PRF f_{rd}(=1/T_d)



< Resolving range ambiguity>

Resolving Range Ambiguities

$$f_{r1} = N f_{rd}, \qquad f_{r2} = (N+1) f_{rd}$$

M_1, M_2 – Number of PRF intervals between transmit and receipt of pulse

⇨

(1) $M_1 = M_2 = M \; (t_1 < t_2)$

(2) or $M_1 + 1 = M_2 \; (t_1 > t_2)$

Resolving Range Ambiguities

(1) Case $t_1 < t_2$ ($M_1 = M_2 = M$)

$$t_1 + \frac{M}{f_{r1}} = t_2 + \frac{M}{f_{r2}} \qquad (t_1 + T_1 M = t_2 + T_2 M)$$

$$\Rightarrow \quad \boxed{M = \frac{t_2 - t_1}{T_1 - T_2}} \qquad (T_1 = 1/f_{r1},\ T_2 = 1/f_{r2})$$

$$\Rightarrow \quad t_r = MT_1 + t_1$$

$$t_r = MT_2 + t_2$$

$$\Rightarrow \quad \boxed{R = ct_r / 2}$$

Resolving Range Ambiguities

(2) Case $t_1 > t_2$ $(M_1 + 1 = M_2)$

$$t_1 + \frac{M}{f_{r1}} = t_2 + \frac{M+1}{f_{r2}}$$

$$\Rightarrow \quad M = \frac{(t_2 - t_1) + T_2}{T_1 - T_2}$$

$$\Rightarrow \quad t_{r1} = MT_1 + t_1$$

$$\Rightarrow \quad \boxed{R = ct_{r1}/2}$$

Resolving Range Ambiguities

(3) If $t_1 = t_2$, target is in the first ambiguity.

$$t_{r2} = t_1 = t_2$$

$$\Rightarrow \quad \boxed{R = ct_{r2} / 2}$$

- **Blind Range –** Pulse cannot be received while the following pulse is transmitted.

  - Resolved by using third PRF

  - We may choose

$$f_{r1} = N(N+1) f_{rd}, \qquad f_{r2} = N(N+2) f_{rd}$$

$$f_{r3} = (N+1)(N+2) f_{rd}$$

Resolving Doppler Ambiguities

- Doppler Ambiguity can be resolved with the same methodology.
- f_{d1} and f_{d2} instead of t_1 and t_2.
- $f_{r1} = \frac{f_{rd}}{N}$ and $f_{r2} = \frac{f_{rd}}{N+1}$.

(1) Case $f_{d1} < f_{d2}$ ($M_1 = M_2 = M$)

$$f_{d1} + Mf_{r1} = f_{d2} + Mf_{r2}$$

$$\Rightarrow \quad M = \frac{f_{d2} - f_{d1}}{f_{r1} - f_{r2}}$$

True Doppler

$$f_d = Mf_{r1} + f_{d1}$$

$$f_d = Mf_{r2} + f_{d2}$$

Resolving Doppler Ambiguities

(2) Case $f_{d1} > f_{d2}$ $(M_1 + 1 = M_2)$

$$f_{d1} + Mf_{r1} = f_{d2} + (M+1) f_{r2}$$

$$\Rightarrow \quad M = \frac{(f_{d2} - f_{d1}) + f_{r2}}{f_{r1} - f_{r2}} \quad \Rightarrow \quad \boxed{f_d = Mf_{r1} + f_{d1}}$$

(3) If $f_{d1} = f_{d2}$, then $f_d = f_{d1} = f_{d2}$

(4) Blind Dopplers can be resolved using a third PRF.

Resolving Doppler Ambiguities

Example : Two PRFs, f_{r1} and f_{r2} , and corresponding unambiguous range, R_{u1} and R_{u2}. Desired unambiguous range R_u = 100 Km. N = 59. Compute f_{r1}, f_{r2}, R_{u1}, R_{u2}.

$$f_{rd} = \frac{c}{2R_u} = \frac{3\times10^8}{200\times10^3} = 1.5KHz$$

(desired PRF)

$$f_{r1} = Nf_{rd} = (59)(1500) = 88.5KHz$$

$$R_{u1} = \frac{c}{2f_{r1}} = \frac{3\times10^8}{2\times88.5\times10^3} = 1.695Km$$

$$f_{r2} = (N+1)f_{rd} = (59+1)(1500) = 90KHz$$

$$R_{u2} = \frac{c}{2f_{r2}} = \frac{3\times10^8}{2\times90\times10^3} = 1.667Km$$

Resolving Doppler Ambiguities

Example : Three PRFs, f_{r1} = 15 KHz, f_{r2} = 18 KHz, f_{r3} = 21 KHz. f_0 = 9GHz.

(a) For the target with v = 550 m/s, frequency of each PRF ?

$$f_d = 2\frac{vf_0}{c} = \frac{2\times 550\times 9\times 10^9}{3\times 10^8} = 33KHz$$

$$\boxed{n_i f_{ri} + f_{di} = f_d, \quad f_{ri} \geq f_{di}}$$

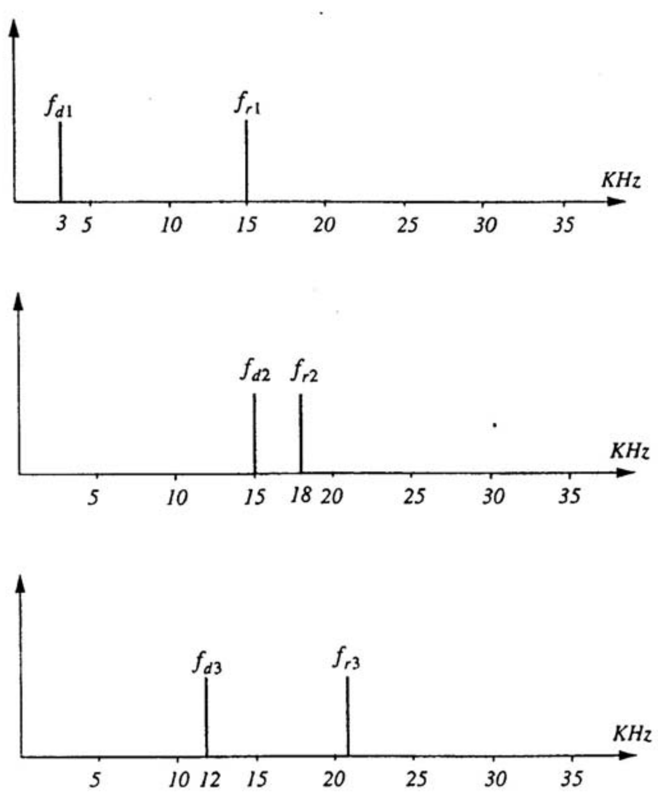
$$n_1 f_{r1} + f_{d1} = 15n_1 + f_{d1} = 33$$

$$n_2 f_{r2} + f_{d2} = 18n_2 + f_{d2} = 33$$

$$n_3 f_{r3} + f_{d3} = 21n_3 + f_{d2} = 33$$

$$n_1 = 2,\ n_1 = 1,\ n_1 = 1,$$

$$f_{d1} = 3KHz,\ f_{d2} = 15KHz,\ f_{d3} = 12KHz$$



Resolving Doppler Ambiguities

(b) For the target with frequency position of 8 KHz, 2KHz, 17KHz, f_d = ?

$$n_1 f_{r1} + f_{d1} = 15n_1 + 8 = f_d$$

$$n_2 f_{r2} + f_{d2} = 18n_2 + 2 = f_d$$

$$n_3 f_{r3} + f_{d3} = 21n_3 + 17 = f_d$$

n	0	1	2	3	4
f_d from f_{r1}	8	23	38	53	68
f_d from f_{r2}	2	20	38	56	
f_d from f_{r3}	17	38	39		

$$n_1 = 2,\ n_1 = 2,\ n_1 = 1,$$

$$\boxed{f_d = 38KHz}$$

$$\boxed{v_r = f_d \frac{\lambda}{2} = 38000 \times \frac{0.0333}{2} = 632.7(m/s)}$$

감사합니다!

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Range and Doppler Ambiguity

- Spectrum for single pulse

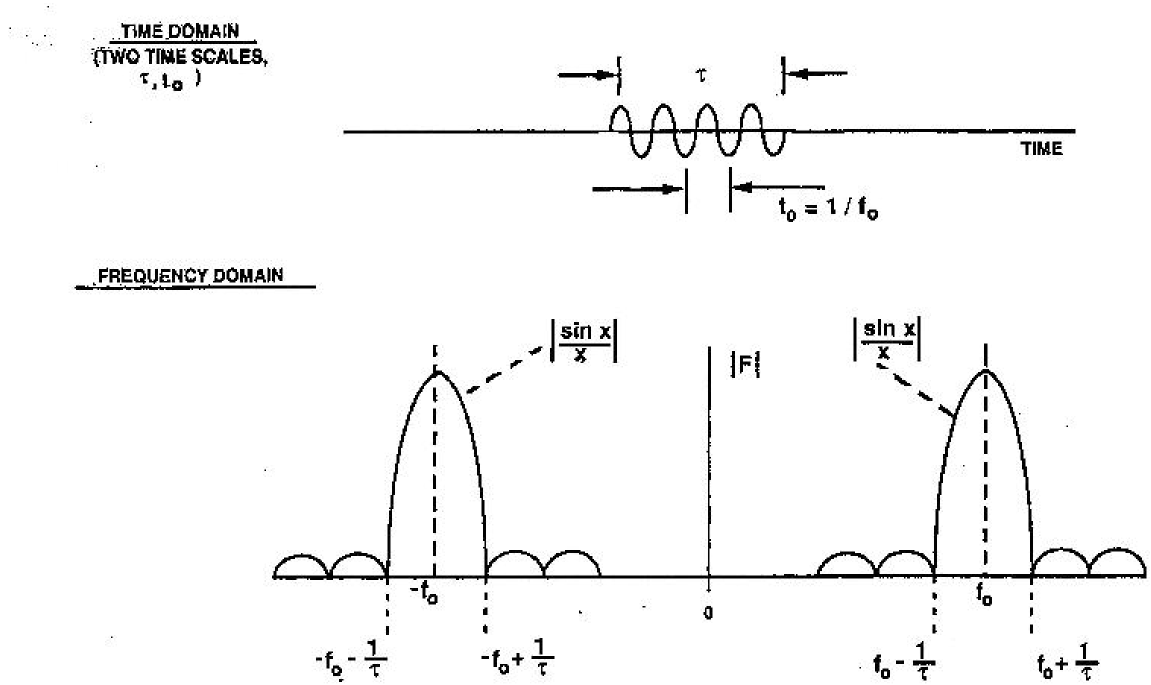

Figure 3.4 Single-pulse (τ) signal of frequency f_0 [1].

Range and Doppler Ambiguity

- Spectrum for train of radar pulse

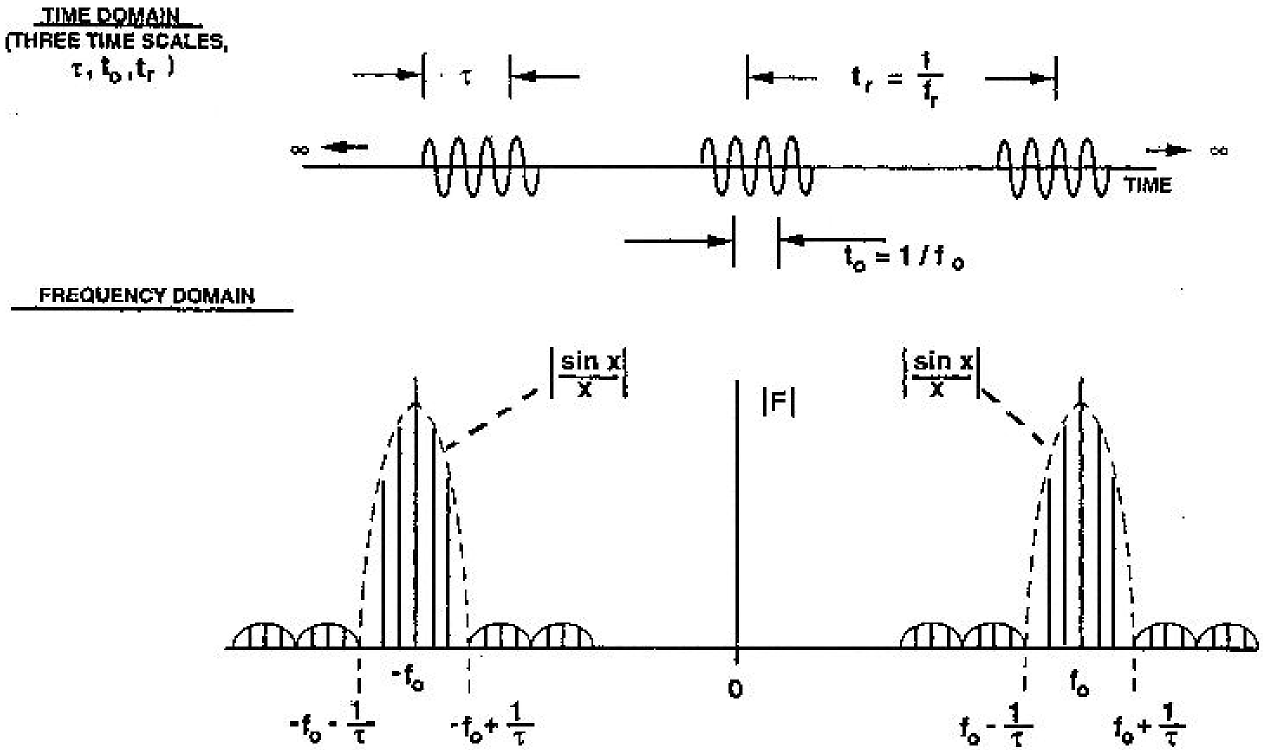

Figure 3.5 Infinite pulse (τ) train signal of PRF f_r and frequency f_0 [1].

Range and Doppler Ambiguity

- Spectrum for finite pulse train (T)

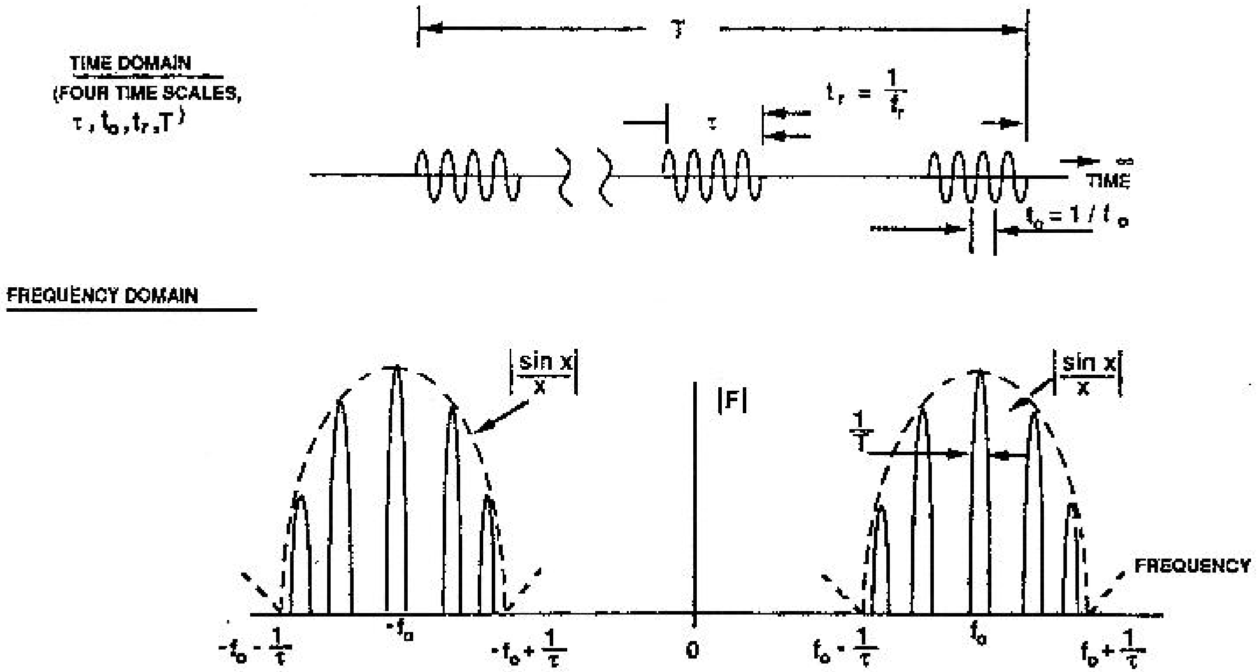

Figure 5.6 Finite (T) pulse (τ) train signal of PRF f_r and frequency f_0 [1].