Naval Postgraduate School *Distance Learning*

Microwave Devices & Radar

LECTURE NOTES
VOLUME I

by Professor David Jenn

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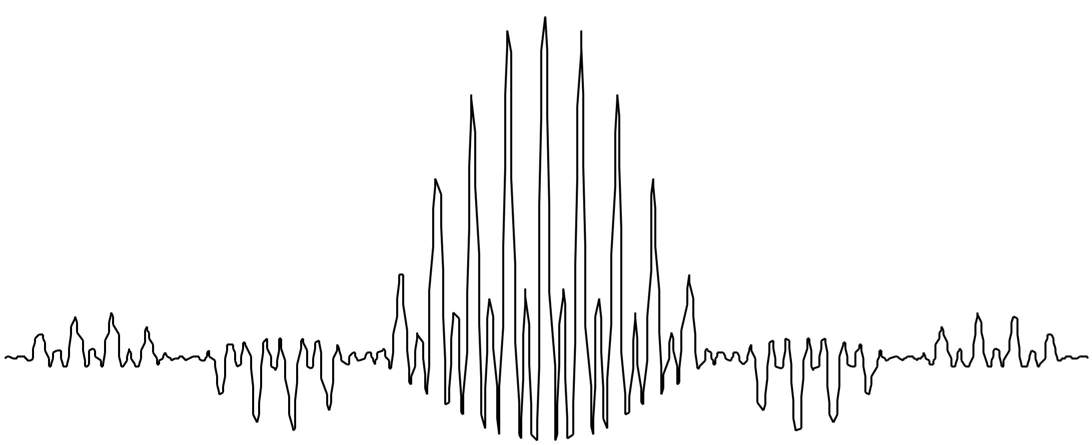

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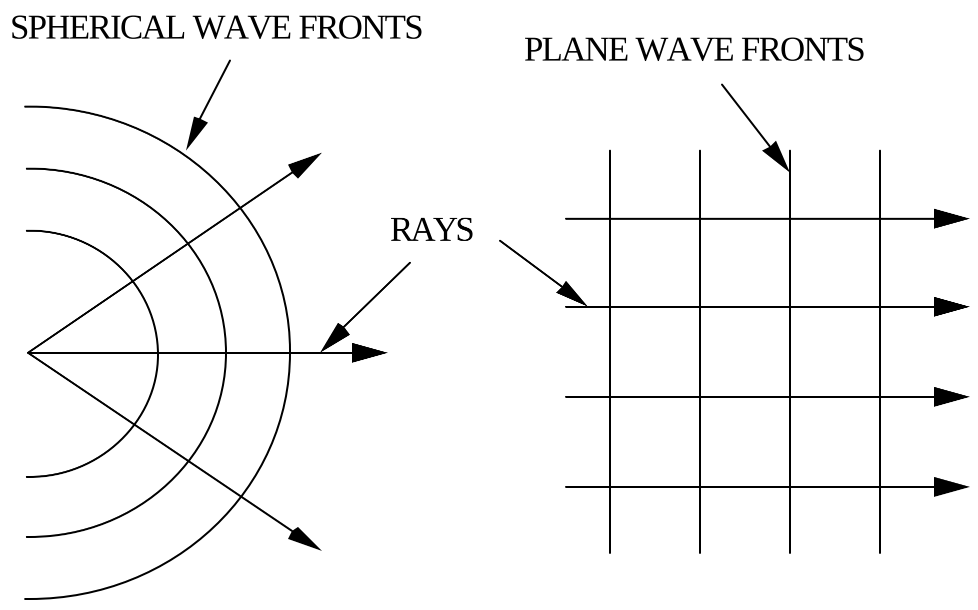
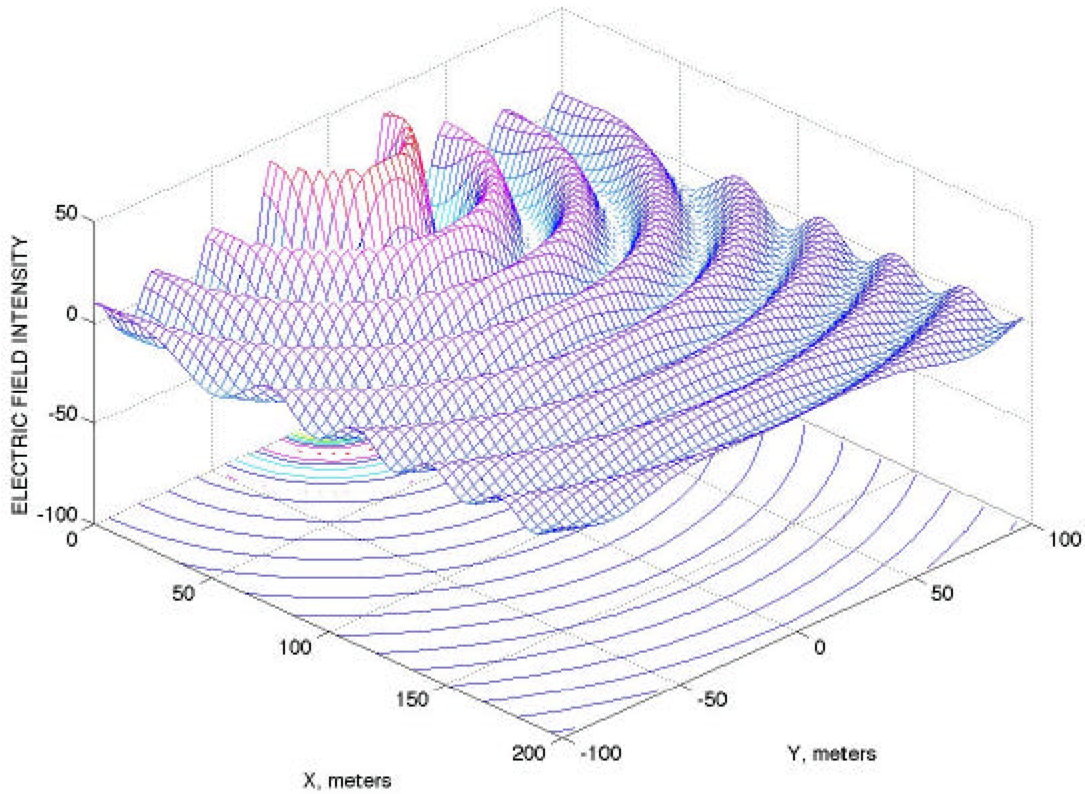
Electromagnetic Fields and Waves (1)

Radar is based on the sensing of electromagnetic waves reflected from objects. Energy is emitted from a source (antenna) and propagates outward. A point on the wave travels with a phase velocity u_p, which depends on the electronic properties of the medium in which the wave is propagating. From antenna theory: if the observer is sufficiently far from the source, then the surfaces of constant phase (wavefronts) are spherical. At even larger distances the wavefronts become approximately planar.



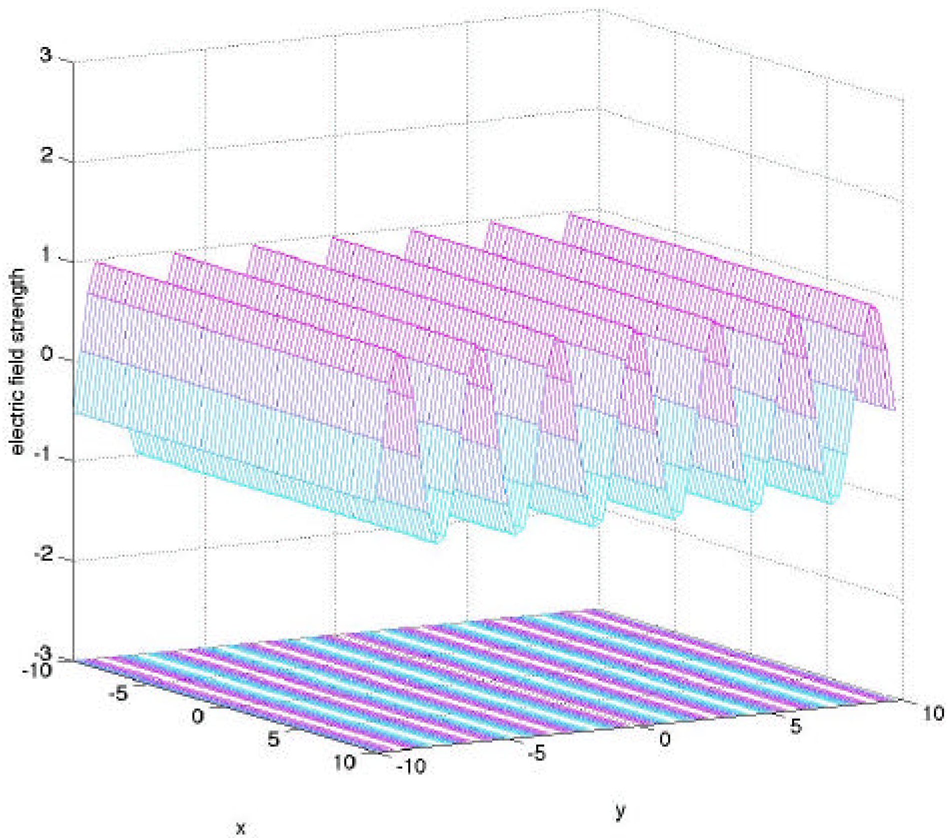
Electromagnetic Fields and Waves (2)

Snapshot of a spherical wave propagating outward from the origin. The amplitude of the wave $\vec{E}(R,t) = \hat{q}\dfrac{E_o}{R}\cos(wt - bR)$ in the x-y plane is plotted at time $t = 0$



Electromagnetic Fields and Waves (3)

Snapshot of a plane wave propagating in the $+y$ direction $\vec{E}(y,t) = \hat{z}E_o \cos(wt - by)$ at time $t = 0$



Electromagnetic Fields and Waves (4)

Electrical properties of a medium are specified by its <u>constitutive parameters</u>:

- permeability, $\mu = \mu_o \mu_r$ (for free space, $\mu \equiv \mu_o = 4\pi \times 10^{-7}$ H/m)
- permittivity, $\varepsilon = \varepsilon_o \varepsilon_r$ (for free space, $\varepsilon \equiv \varepsilon_o = 8.85 \times 10^{-12}$ F/m)
- conductivity, σ (for a metal, $\sigma \sim 10^7$ S/m)

Electric and magnetic field intensities: $\vec{E}(x,y,z,t)$ V/m and $\vec{H}(x,y,z,t)$ A/m

- vector functions of location in space and time, e.g., in cartesian coordinates

$$\vec{E}(x,y,z,t) = \hat{x}E_x(x,y,z,t) + \hat{y}E_y(x,y,z,t) + \hat{z}E_z(x,y,z,t)$$

- similar expressions for other coordinates systems
- the fields arise from current $\vec{J}$ and charge ρ_v on the source ($\vec{J}$ is the volume current density in A/m^2 and ρ_v is volume charge density in C/m^3)

Electromagnetic fields are completely described by Maxwell's equations:

$$(1)\ \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \qquad (3)\ \nabla \cdot \vec{H} = 0$$

$$(2)\ \nabla \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad (4)\ \nabla \cdot \vec{E} = \rho_v / \varepsilon$$

Electromagnetic Fields and Waves (5)

The wave equations are derived from Maxwell's equations:

$$\nabla^2 \vec{E} - \frac{1}{u_p^2}\frac{\P^2 \vec{E}}{\P t^2} = 0 \qquad \nabla^2 \vec{H} - \frac{1}{u_p^2}\frac{\P^2 \vec{H}}{\P t^2} = 0$$

The phase velocity is $u_p = w\sqrt{me}$ (in free space $u_p = c = 2.998\times 10^8$ m/s)

The simplest solutions to the wave equations are plane waves. An example for a plane wave propagating in the z direction is:

$$\vec{E}(z,t) = \hat{x}\, E_o e^{-az} \cos(w t - bz)$$

- a = attenuation constant (Np/m); $b = 2p / l$ = phase constant (rad/m)
- l = wavelength; $w = 2p\ f$ (rad/sec); f = frequency (Hz); $f = \dfrac{u_p}{l}$

Features of this plane wave:

- propagating in the $+z$ direction
- x polarized (direction of electric field vector is $\hat{x}$)
- amplitude of the wave is E_o

Electromagnetic Fields and Waves (6)

Time-harmonic sources, currents, and fields: sinusoidal variation in time and space. Suppress the time dependence for convenience and work with time independent quantities called phasors. A time-harmonic plane wave is represented by the phasor $\vec{E}(z)$

$$\vec{E}(z,t) = \mathrm{Re}\left\{\hat{x}E_o e^{-(a+jb)z} e^{jwt}\right\} = \mathrm{Re}\left\{\vec{E}(z)e^{jwt}\right\}$$

$\vec{E}(z)$ is the phasor representation; $\vec{E}(z,t)$ is the instantaneous quantity

$\mathrm{Re}\{\cdot\}$ is the real operator (i.e., “take the real part of”)

$j = \sqrt{-1}$

Since the time dependence varies as e^{jwt}, the time derivatives in Maxwell’s equations are replaced by $¶\ /\partial t \equiv jw$:

$$(1)\ \nabla\times\vec{E} = -jwm\vec{H} \qquad (3)\ \nabla\cdot\vec{H} = 0$$

$$(2)\ \nabla\times\vec{H} = \vec{J} + jwe\vec{E} \qquad (4)\ \nabla\cdot\vec{E} = r_v/e$$

The wave equations are derived from Maxwell’s equations:

$$\nabla^2\vec{E} - g^2\vec{E} = 0$$

$$\nabla^2\vec{H} - g^2\vec{H} = 0$$

where $g = a + jb$ is the propagation constant.

Electromagnetic Fields and Waves (7)

Plane and spherical waves belong to the to a class called transverse electromagnetic (TEM) waves. They have the following features:

1. $\vec{E}$, $\vec{H}$ and the direction of propagation $\hat{k}$ are mutually orthogonal
2. $\vec{E}$ and $\vec{H}$ are related by the intrinsic impedance of the medium

$$h = \sqrt{\frac{m}{(e - js/w)}} \quad \Rightarrow \quad h_o = \sqrt{\frac{m_o}{e_o}} \approx 377\ \Omega \text{ for free space}$$

The above relationships are expressed in the vector equation $\vec{H} = \dfrac{\hat{k} \times \vec{E}}{h}$

The time-averaged power propagating in the plane wave is given by the Poynting vector:

$$\vec{W} = \frac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\} \text{ W/m}^2$$

For a plane wave: $\vec{W}(z) = \dfrac{1}{2}\dfrac{|E_o|^2}{h}\hat{z}$

For a spherical wave: $\vec{W}(R) = \dfrac{1}{2h}\dfrac{|E_o|^2}{R^2}\hat{R}$ (inverse square law for power spreading)

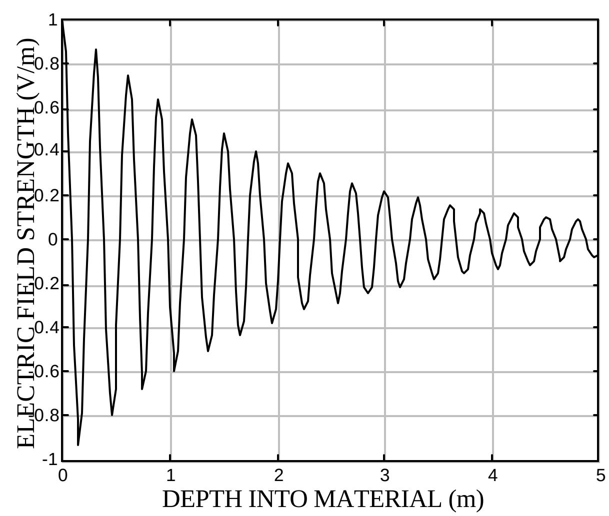
Electromagnetic Fields and Waves (8)

A material's conductivity causes attenuation of a wave as it propagates through the medium. Energy is extracted from the wave and dissipated as heat (ohmic loss). The attenuation constant determines the rate of decay of the wave. In general:

$$a = w\left\{\frac{me}{2}\left[\sqrt{1+\left(\frac{s}{we}\right)^2} - 1\right]\right\}^{1/2} \qquad b = w\left\{\frac{me}{2}\left[\sqrt{1+\left(\frac{s}{we}\right)^2} + 1\right]\right\}^{1/2}$$

For lossless media $s = 0 \Rightarrow a = 0$. Traditionally, for lossless cases, k is used rather than b. For good conductors ($s / we >> 1$), $a \approx \sqrt{p m f s}$, and the wave decays rapidly with distance into the material.

Sample plot of field vs. distance



Electromagnetic Fields and Waves (9)

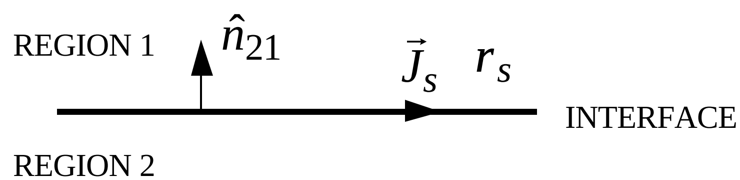
For good conductors the current is concentrated near the surface. The current can be approximated by an infinitely thin current sheet, or <u>surface current</u>, $\vec{J}_s$ A/m and <u>surface charge</u>, r_s C/m

Current in a good conductor Surface current approximation

At an interface between two media the boundary conditions must be satisfied:

$$(1)\ \hat{n}_{21}\times(\vec{E}_1-\vec{E}_2)=0 \qquad (3)\ \hat{n}_{21}\cdot(\vec{E}_1-\vec{E}_2)=r_s/e$$

$$(2)\ \hat{n}_{21}\times(\vec{H}_1-\vec{H}_2)=\vec{J}_s \qquad (4)\ \hat{n}_{21}\cdot(\vec{H}_1-\vec{H}_2)=0$$



Wave Reflection (1)

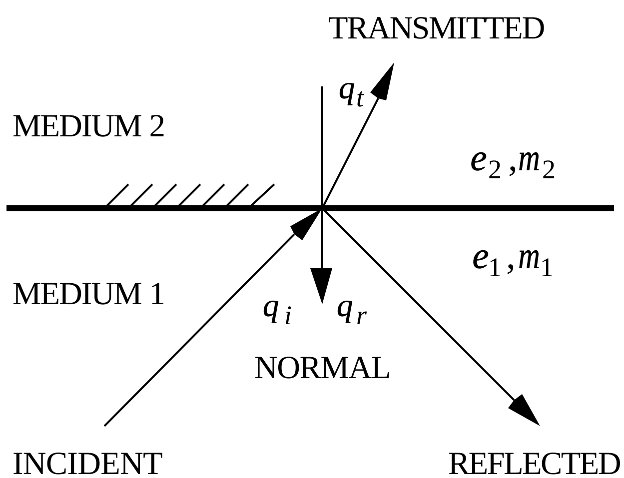
For the purposes of applying boundary conditions, the electric field vector is decomposed into parallel and perpendicular components $\vec{E} = \vec{E}_{\perp} + \vec{E}_{\parallel}$

$\vec{E}_{\perp}$ is perpendicular to the plane of incidence

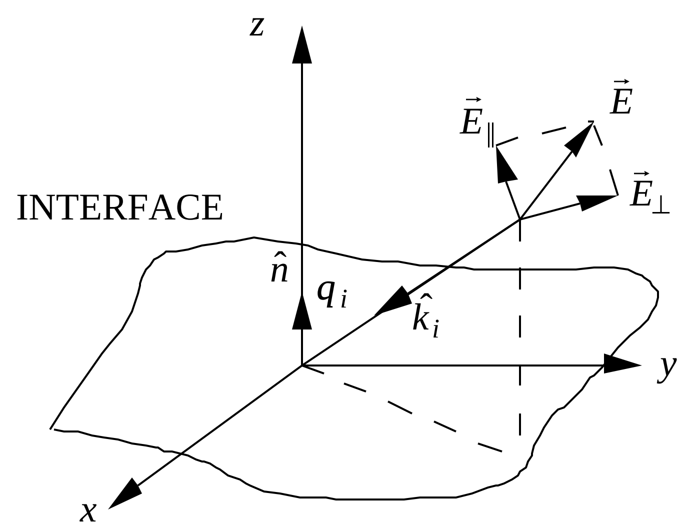
$\vec{E}_{\parallel}$ lies in the plane of incidence

The plane of incidence is defined by the vectors $\hat{k}_i$ and $\hat{n}$

PLANE WAVE INCIDENT ON AN INTERFACE BETWEEN TWO DIELECTRICS

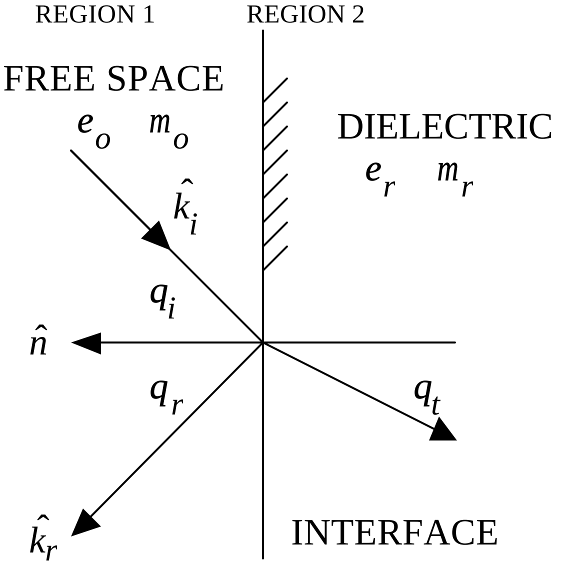


DECOMPOSITON OF AN ELECTRIC FIELD VECTOR INTO PARALLEL AND PERPENDICULAR COMPONENTS



Wave Reflection (2)

Plane wave incident on an interface between free space and a dielectric



$$\sin\theta_i = \sin\theta_r = \sqrt{\varepsilon_r\mu_r}\,\sin\theta_t$$

$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \text{ and } \eta = \sqrt{\frac{\mu_r\mu_o}{\varepsilon_r\varepsilon_o}} = \eta_o\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

Reflection and transmission coefficients:

Perpendicular polarization:

$$\Gamma_\perp = \frac{\eta\cos\theta_i - \eta_0\cos\theta_t}{\eta\cos\theta_i + \eta_0\cos\theta_t}$$

$$\tau_\perp = \frac{2\eta\cos\theta_i}{\eta\cos\theta_i + \eta_0\cos\theta_t}$$

$$E_{r\perp} = \Gamma_\perp E_{i\perp} \text{ and } E_{t\perp} = \tau_\perp E_{i\perp}$$

Parallel polarization:

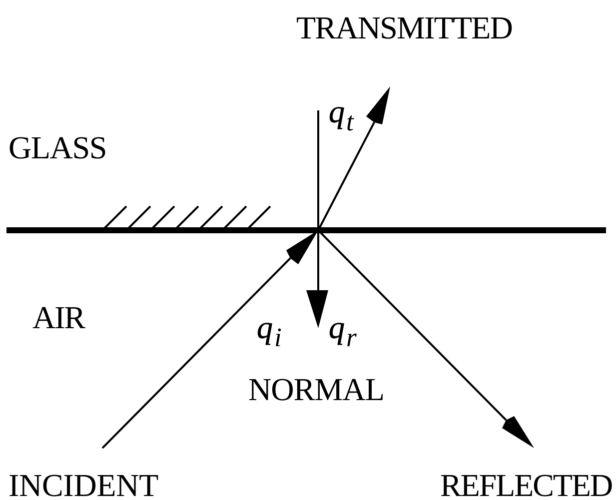
$$\Gamma_\parallel = \frac{\eta\cos\theta_t - \eta_0\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i}$$

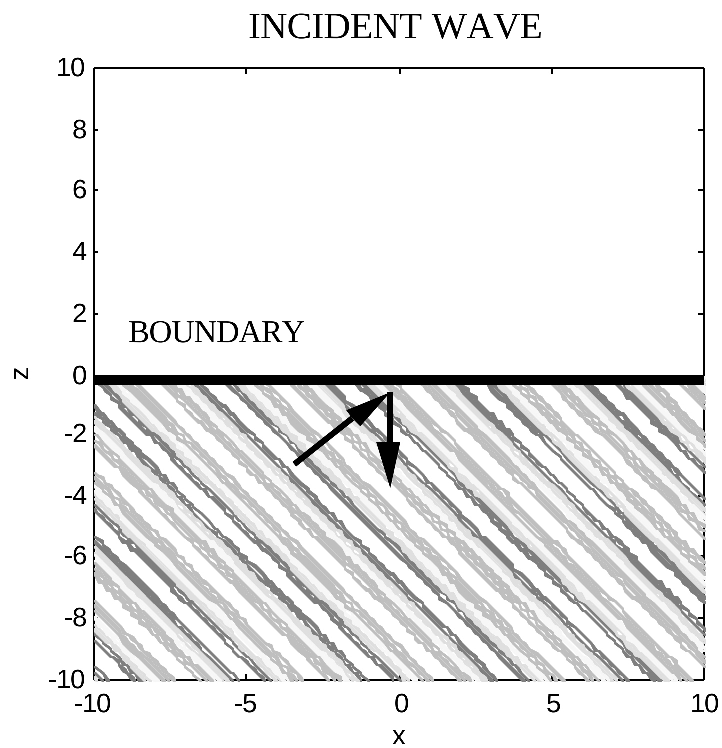
$$\tau_\parallel = \frac{2\eta\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i}$$

$$E_{r\parallel} = \Gamma_\parallel E_{i\parallel} \text{ and } E_{t\parallel} = \tau_\parallel E_{i\parallel}$$

Wave Reflection (3)

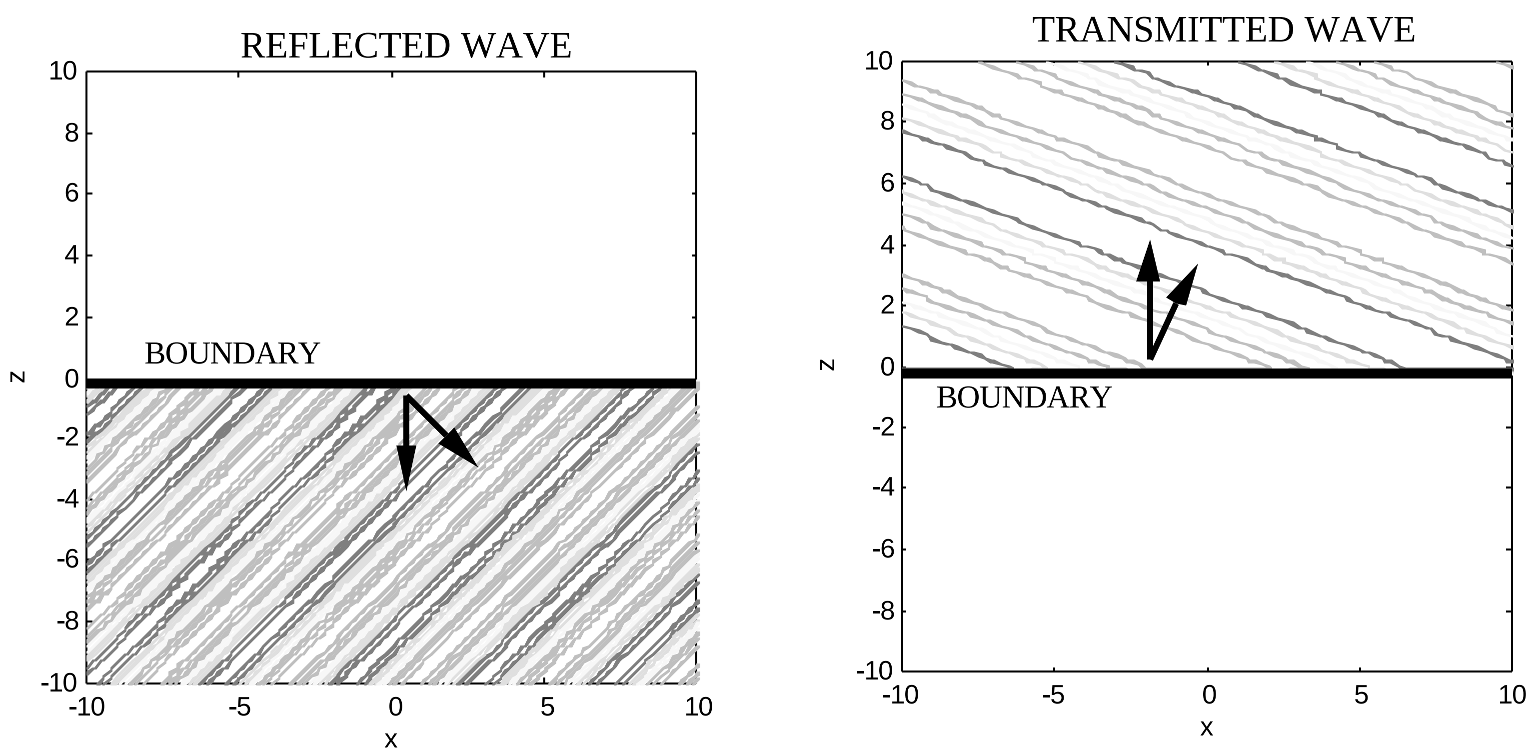
Example of a plane wave incident on a boundary between air and glass ($e_r = 4, q_i = 45^\circ$)





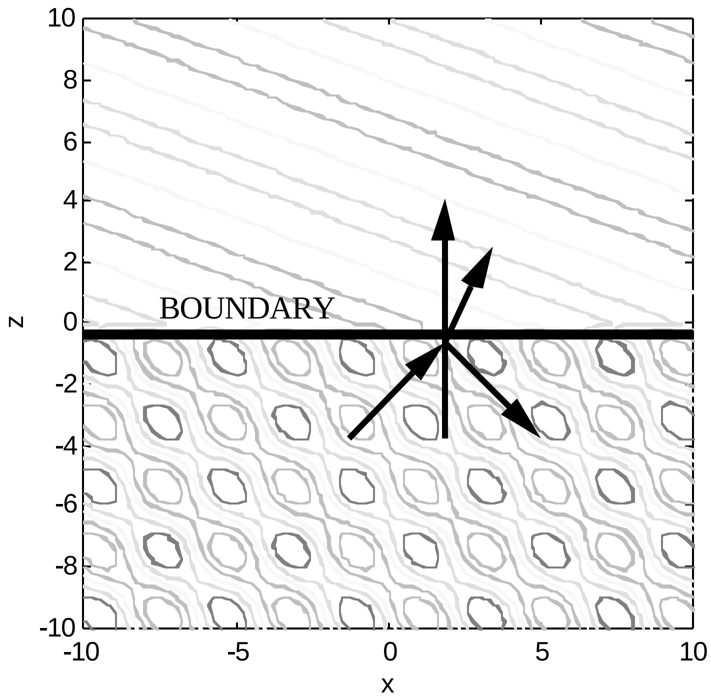
Wave Reflection (4)

Example of a plane wave reflection: reflected and transmitted waves ($e_r = 4, q_i = 45^\circ$)



Wave Reflection (5)

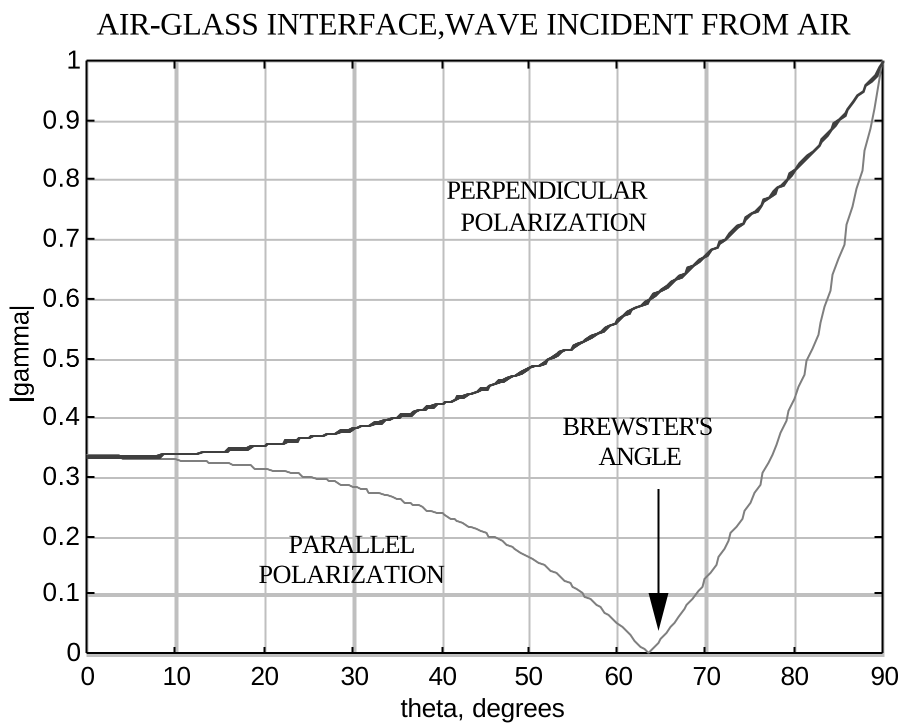
Example of a plane wave reflection: total field

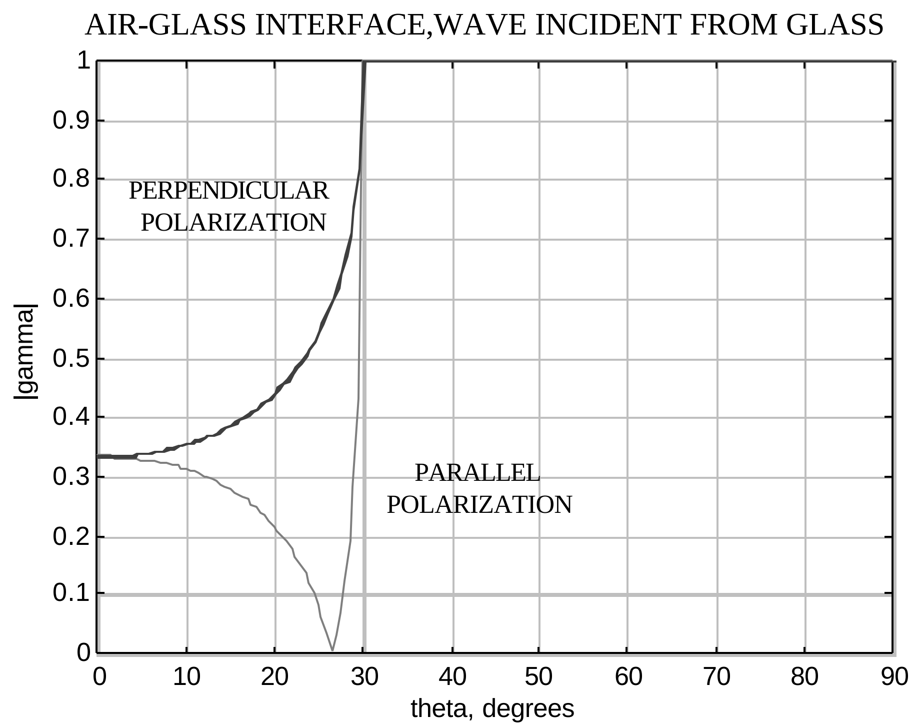


- The total field in region 1 is the sum of the incident and reflected fields
- If region 2 is more dense than region 1 (i.e., $e_{r2} > e_{r1}$) the transmitted wave is refracted towards the normal
- If region 1 is more dense than region 2 (i.e., $e_{r1} > e_{r2}$) the transmitted wave is refracted away from the normal

Wave Reflection (6)

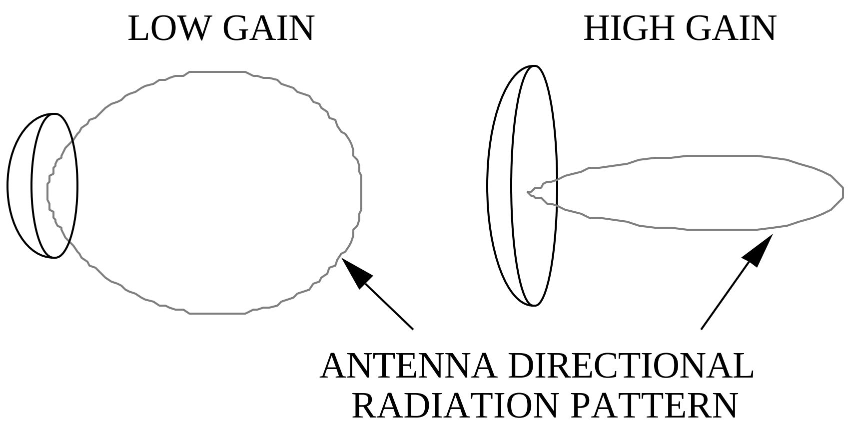
Boundary between air ($e_r = 1$) and glass ($e_r = 4$)





Antenna Patterns, Directivity and Gain

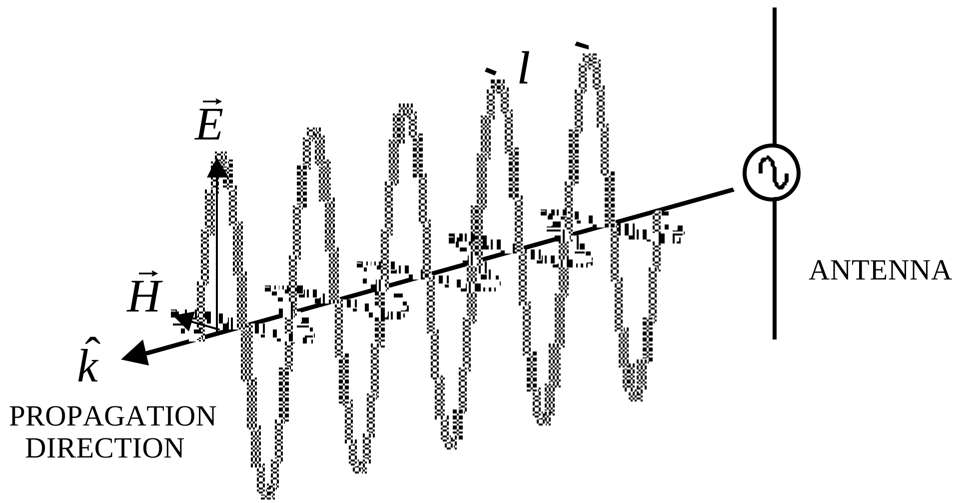
- The antenna pattern is a directional plot of the received or transmitted) signal
- From a systems point of view, two important antenna parameters are gain and beamwidth
- Both gain and beamwidth are measures of the antenna's capability to focus radiation
- Gain includes loss that occurs within the antenna whereas directivity refers to a lossless antenna of the same type (i.e., it is an ideal reference)
- In general, an increase in gain is accompanied by a decrease in beamwidth, and is achieved by increasing the antenna size relative to the wavelength
- With regard to radar, high gain an narrow beams are desirable for long detection and tracking ranges and accurate direction measurement



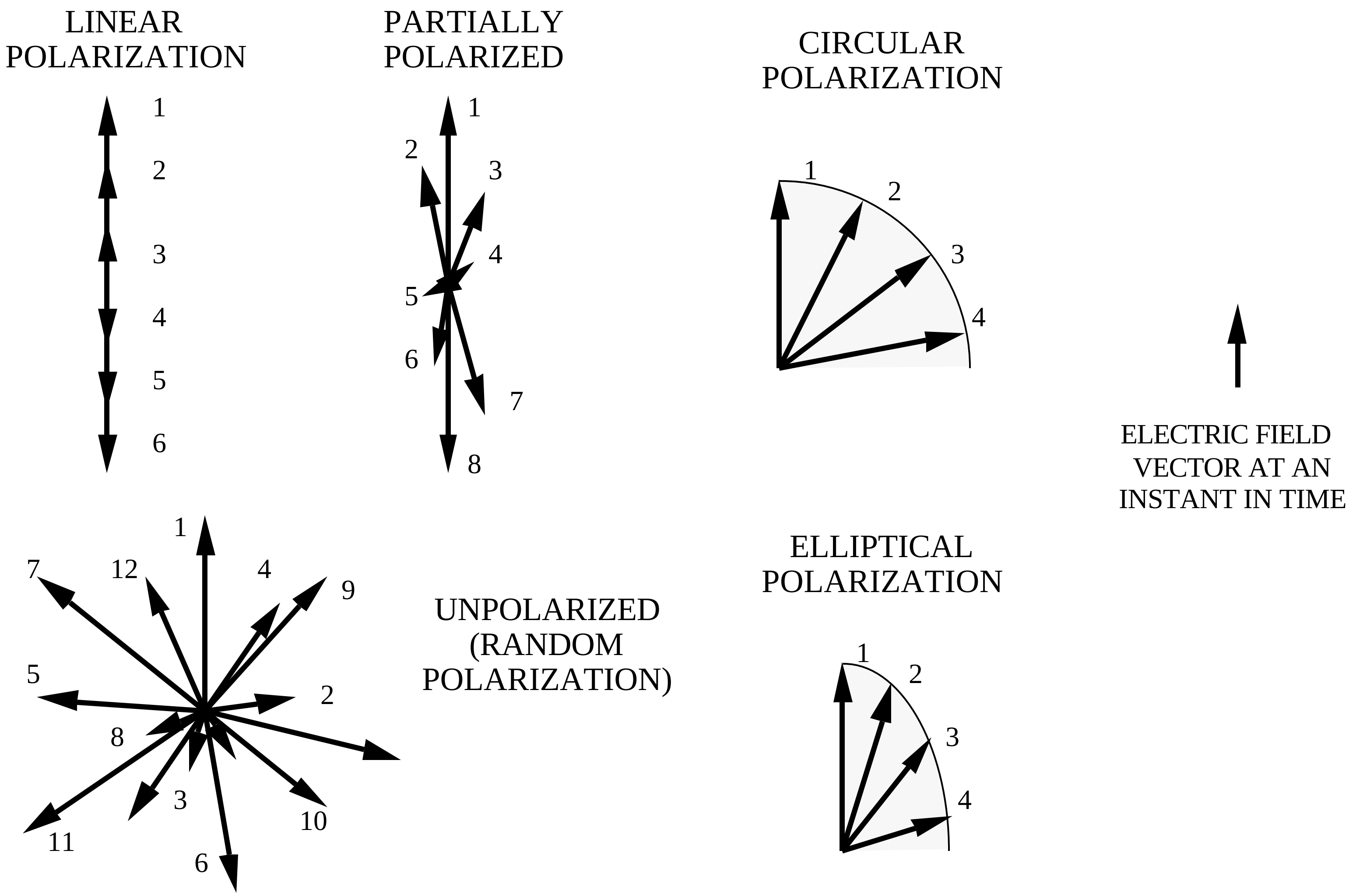
Polarization of Radiation

Example of a plane wave generated by a linearly polarized antenna:

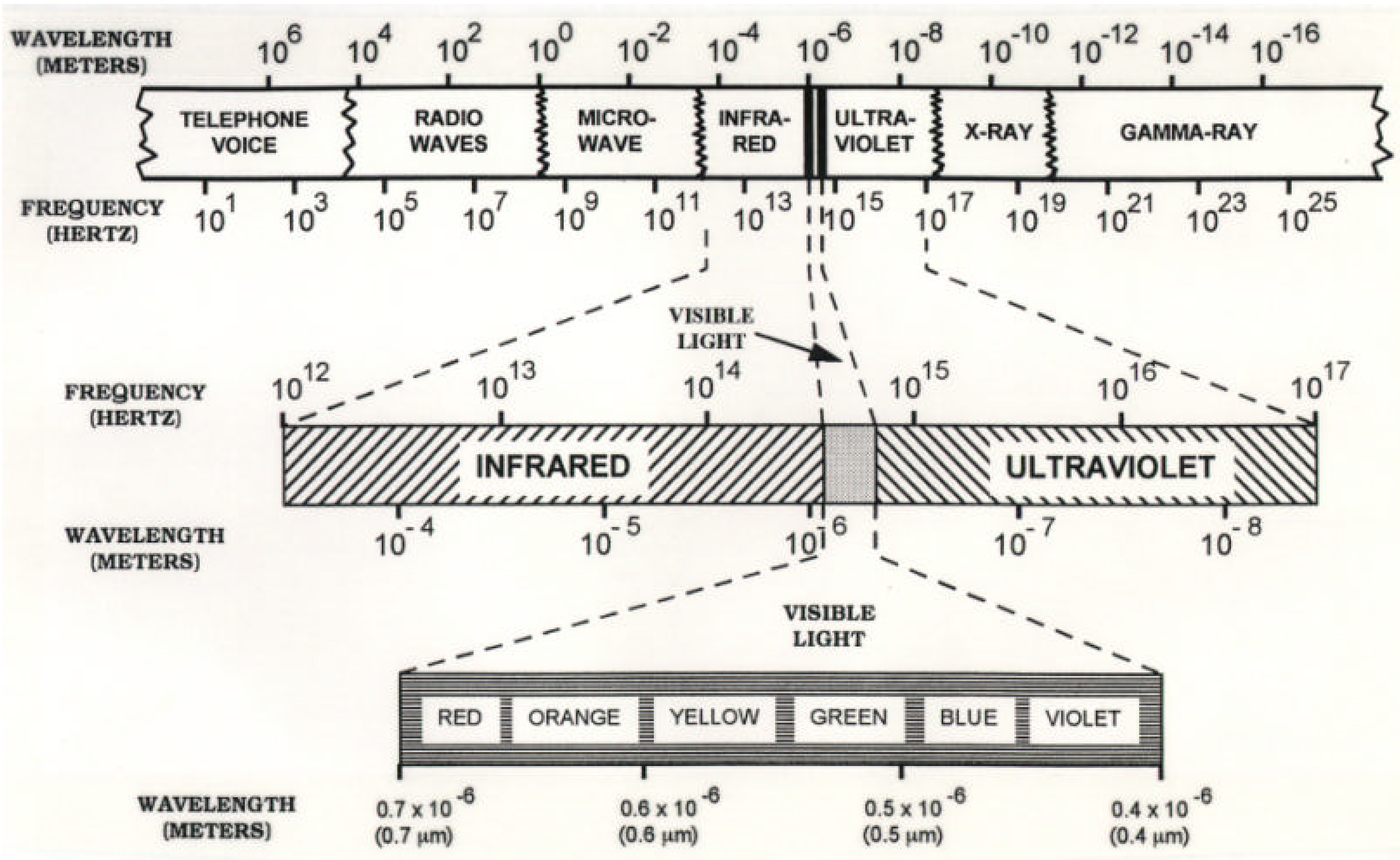
1. Finite sources generate spherical waves, but they are locally planar over limited regions of space
2. Envelopes of the electric and magnetic field vectors are plotted
3. $\vec{E}$ and $\vec{H}$ are orthogonal to each other and the direction of propagation. Their magnitudes are related by the intrinsic impedance of the medium (i.e., TEM)
4. Polarization refers to the curve that the tip of $\vec{E}$ traces out with time at a fixed point in space. It is determined by the antenna geometry and its orientation relative to the observer



Wave Polarization



Electromagnetic Spectrum



Radar and ECM Frequency Bands

Standard Radar Bands[1]			ECM Bands[2]	
Band Designation[3]	**Frequency Range (MHz)**		**Band Designation**	**Frequency Range (MHz)**
HF	3–30		Alpha	0–250
VHF[4]	30–300		Bravo	250–500
UHF[4]	300–1,000		Charlie	500–1,000
L	1,000–2,000		Delta	1,000–2,000
S	2,000–4,000		Echo	2,000–3,000
C	4,000–8,000		Foxtrot	3,000–4,000
X	8,000–12,000		Golf	4,000–6,000
K_u	12,000–18,000		Hotel	6,000–8,000
K	18,000–27,000		India	8,000–10,000
K_a	27,000–40,000		Juliett	10,000–20,000
millimeter[5]	40,000–300,000		Kilo	20,000–40,000
			Lima	40,000–60,000
			Mike	60,000–100,000

[1] From IEEE Standard 521-1976, November 30 1976.

[2] From AFR 55-44 (AR105-96, OPNAVINST 3420.9B, MCO 3430.1), October 27, 1964.

[3] British usage in the past has corresponded generally but not exactly to the letter-designated bands.

[4] The following *approximate* lower frequency ranges are sometimes given letter designations: P-band (225–390 MHz), G-band (150–225 MHz), and I-band (100–150 MHz).

[5] The following *approximate* higher frequency ranges are sometimes given letter designations: Q-band (36–46 GHz), V-band (46–56 GHz), and W-band (56–100 GHz).

Radar Bands and Usage

Band Designation	Frequency Range	Usage
HF	3–30 MHz	OTH surveillance
VHF	30–300 MHz	Very-long-range surveillance
UHF	300–1,000 MHz	Very-long-range surveillance
L	1–2 GHz	Long-range surveillance En route traffic control
S	2–4 GHz	Moderate-range surveillance Terminal traffic control Long-range weather
C	4–8 GHz	Long-range tracking Airborne weather detection
X	8–12 GHz	Short-range tracking Missile guidance Mapping, marine radar Airborne intercept
K_u	12–18 GHz	High-resolution mapping Satellite altimetry
K	18–27 GHz	Little use (water vapor)
K_a	27–40 GHz	Very-high-resolution mapping Airport surveillance
millimeter	40–100+ GHz	Experimental

Joint Electronics Type Designation

First Letter

- A – Airborne (installed and operatied in aircraft)
- B – Underwater mobile, submarine
- C – Air transportable (inactivated, do not use)
- D – Pilotless carrier
- F – Fixed
- G – Ground, general ground use (includes two or more ground-type installations)
- K – Amphibious
- M – Ground, mobile (installed as operating unit in a vehicle which has no function other than transporting the equipment
- P – Pack or portable (animal or man)
- S – Water surface craft
- T – Ground, transportable
- U – General utility (includes two or more general installation classes, airborne, shipboard, and ground)
- V – Ground, vehicular (installed in vehicle designed for functions other than carrying electronic equipment, etc., such as tanks)
- W – Water surface and underwater

Second Letter

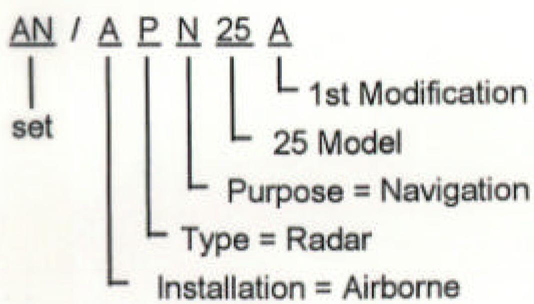
- A – Infrared, heat radiation
- B – Pigeon
- C – Carrier (wire)
- D – Radiac
- E – Nupac
- F – Photographic
- G – Telegraph or teletype
- I – Interphone and public address
- J – Electromechanical (not otherwise covered)
- K – Telemetering
- L – Countermeasures
- M – Meteorological
- N – Sound in air
- P – Radar
- Q – Sonar and underwater sound
- R – Radio
- S – Special types, magnetics, etc., or combinations of types
- T – Telephone (wire)
- V – Visual and visible light
- W – Armament (peculiar to armament, not otherwise covered)
- X – Facsimile or television
- Y – Data processing

Third Letter

- A – Auxiliary assempblies (not complete operating sets used with or part of two or more sets or sets series
- B – Bombing
- C – Communications
- D – Direction finder and/or reconnaissance
- E – Ejection and/or release
- G – Fire control or search light directing
- H – Recording and/or reproducing (graphic meteorological and sound)
- L – Searchlight control (inactivated, use G)
- M – Maintenance and test assemblies (including tools)
- N – Navigational aids (including altimeters, beacons, compasses, racons, depth sounding, approach, and landing)
- P – Reproducing (inactivated, do not use)
- Q – Special, or combination of purposes
- R – Receiving, passive detecting
- S – Detecting and/or range and bearing
- T – Transmitting
- W – Control
- X – Identification and recognition

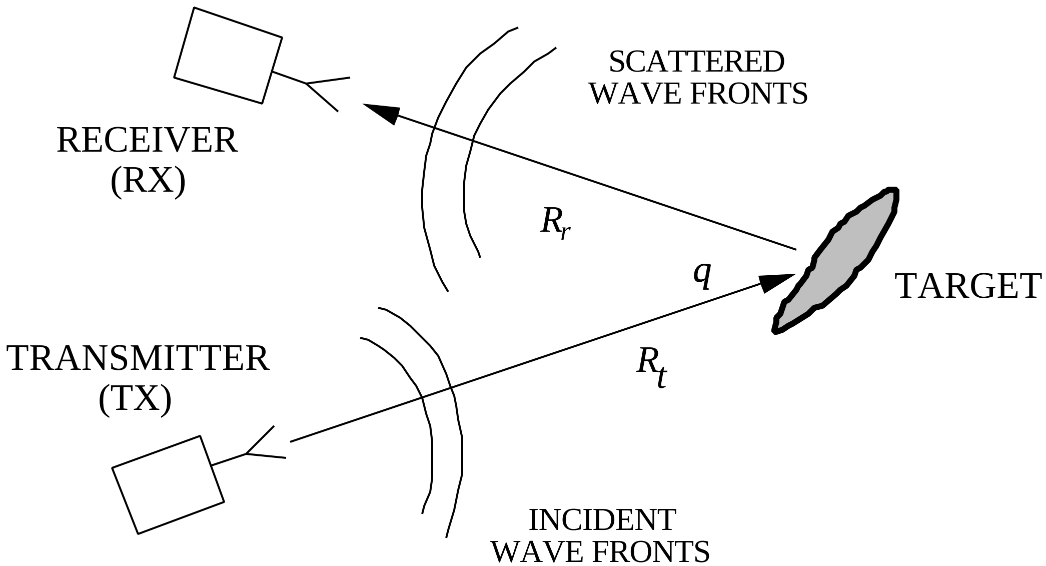
Examples of EW Systems

Examples



SYSTEM	PURPOSE	CONTRACTOR
ALQ–137(V)4	Repeater Jammer	Sanders
ALQ–144(V)	IR Countermeasures Set	Sanders
ALQ–153	Tail Warning Set	Westinghouse
ALQ–155	B–52 ECM Power Management Systems	Northrop
ALQ–157(V)	Navy Infrared Jammer	Loral
ALQ–161	B–1B Integrated ECM System	AIL
ALQ–162(V)1	Navy Countermeasures Set	Northrop
ALQ–162(V)2	Army Countermeasures Set	Northrop
ALQ –165	Self-Protection Jammer	ITT/Westinghouse
ALQ –171	Tactical countermeasures	Northrop
ALQ –172	B–52G/H, C–130 Countermeasures Set	ITT
ALQ –172/155	B–52G/H Sensor Integration	—
ALQ –176(V)	ECM Pod	Hercules
ALQ –184(V)	ECM Pod (Modified ALQ–119)	Raytheon
ALQ –188	Countermeasure System	Northrop QRC Division
ALQ –196	Jammer	Sanders
ALR–20A	B–52 Panoramic receiver	RCA
ALR–46	Radar Warning Receiver (4 Versions)	Dalmo Victor
ALR–56	Radar Warning Receiver (4 Versions)	Loral
ALR–62(V)	Radar Warning, Countermeasures Sets	Dalmo Victor
ALR–69	Radar Warning receiver	ATD/Dalmo/Litton
ALR–72	Threat Panel	Dalmo Victor
ALT–6B	Jammer	GE
ALT–16	Jammer	Motorola
ALT–28	Jammer	Northrop
ALT–32	Jammer	Multiple
APM–427	Radar Simulator	AAI
APR–38	Radar Attack Warning System	WJ/IBM/Microphase
APR–39(V)1	Radar Warning Receiver	Dalmo Victor
APR–39(V)2	Radar Signal Detecting Set	Dalmo Victor
APR–44(V)	Radar Warning System	AEL

Radio Detection and Ranging (RADAR)



Bistatic: the transmit and receive antennas are at different locations as viewed from the target (e.g., ground transmitter and airborne receiver, $q \neq 0$)

Monostatic: the transmitter and receiver are colocated as viewed from the target (i.e., the same antenna is used to transmit and receiver, $q = 0$)

Quasi-monostatic: the transmit and receive antennas are slightly separated but still appear to be at the same location as viewed from the target (e.g., separate transmit and receive antennas on the same aircraft, $q \approx 0$)

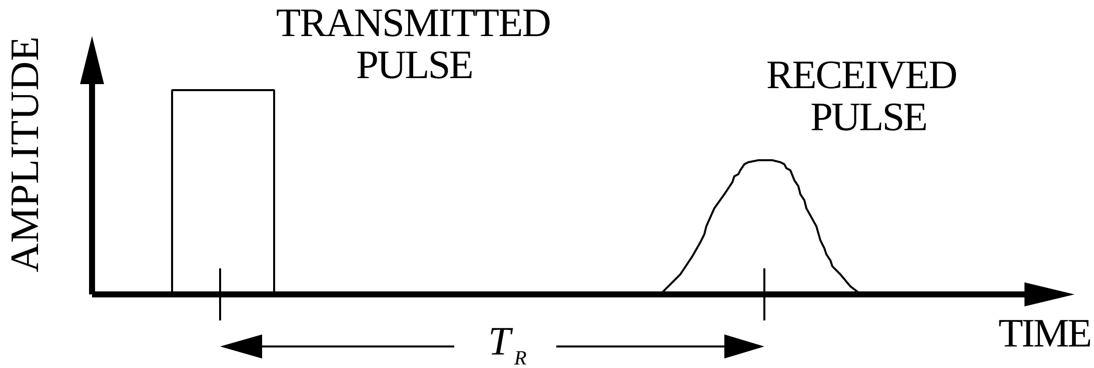
Time Delay Ranging

Target range is the fundamental quantity measured by most radars. It is obtained by recording the round trip travel time of a pulse, T_R, and computing range from:

bistatic: $R_t + R_r = cT_R$

monostatic: $R = \frac{cT_R}{2} \quad (R_r = R_t \equiv R)$

where $c \approx 3\times10^8$ m/s is the velocity of light in free space.



Information Available From the Radar Echo

"Normal" radar functions:

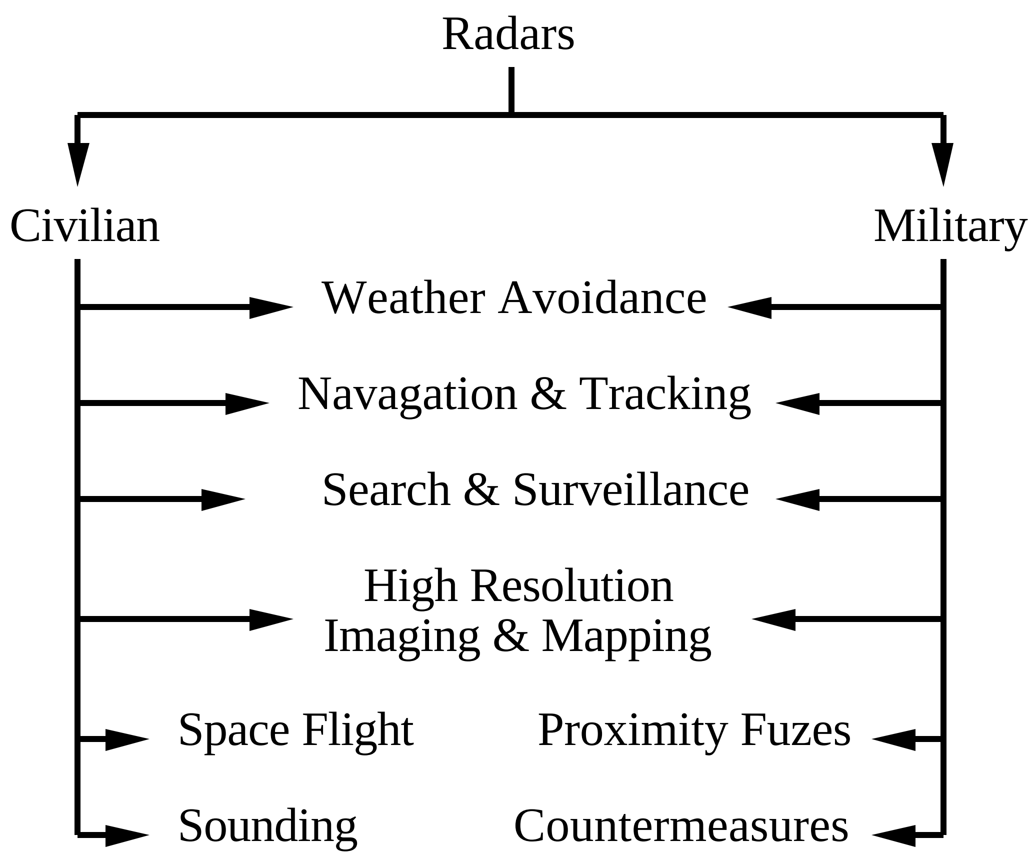
1. range (from pulse delay)
2. velocity (from doppler frequency shift)
3. angular direction (from antenna pointing)
4. target size (from magnitude of return)

Signature analysis and inverse scattering:

5. target shape and components (return as a function of direction)
6. moving parts (modulation of the return)
7. material composition

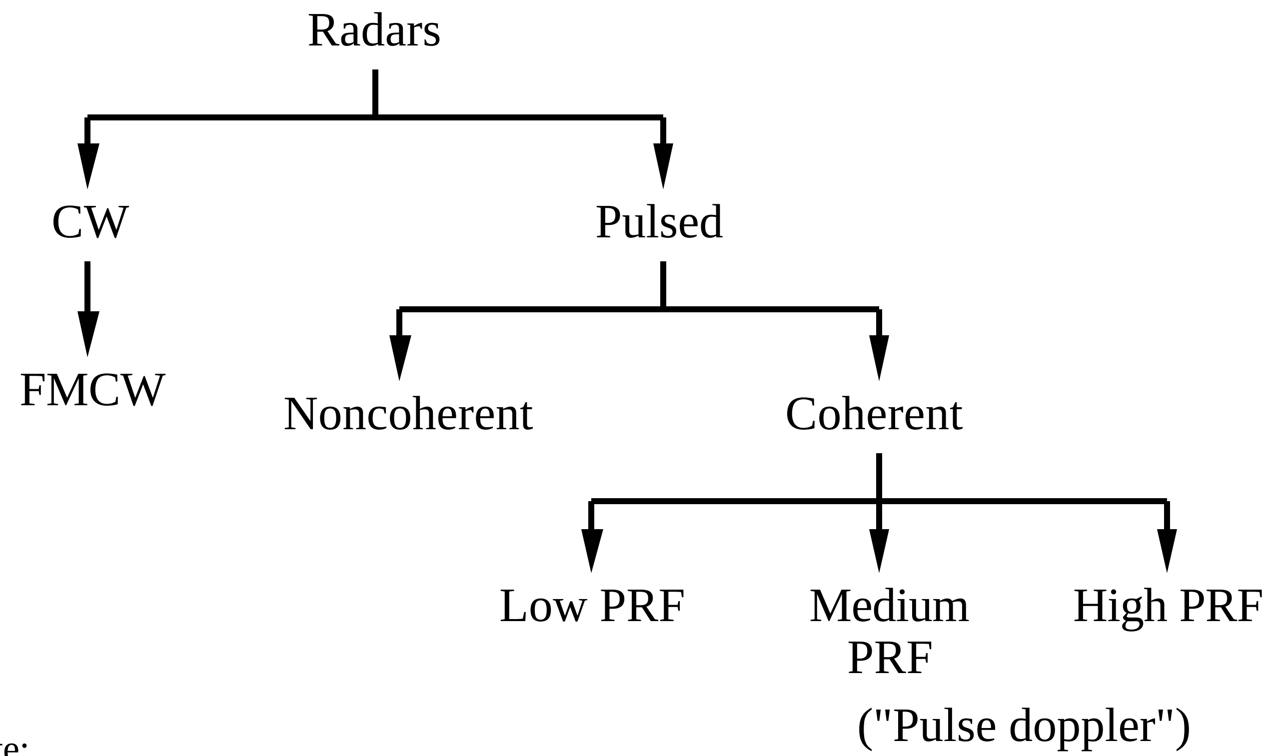
The complexity (cost & size) of the radar increases with the extent of the functions that the radar performs.

Radar Classification by Function



Many modern radars perform multiple functions ("multi-function radar")

Radar Classification by Waveform

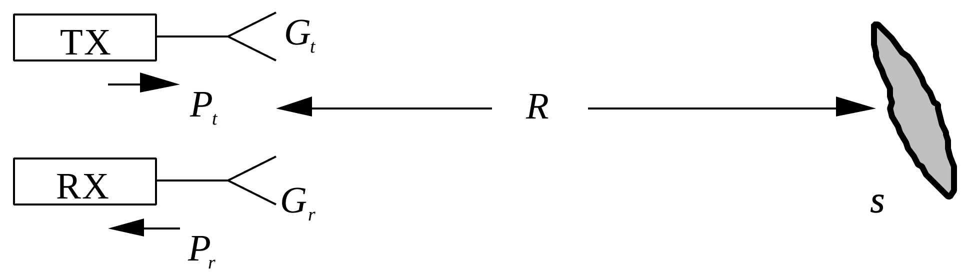


Note:

CW = continuous wave
FMCW = frequency modulated continuous wave
PRF = pulse repetition frequency

Basic Form of the Radar Range Equation (1)

"Quasi-monostatic" geometry



s = radar cross section (RCS) in square meters

P_t = transmitter power, watts

P_r = received power, watts

G_t = transmit antenna gain in the direction of the target (assumed to be the maximum)

G_r = receive antenna gain in the direction of the target (assumed to be the maximum)

$P_t G_t$ = effective radiated power (ERP)

From antenna theory: $G_r = \dfrac{4pA_{er}}{l^2}$

$A_{er} = A_p r$ = effective area of the receive antenna

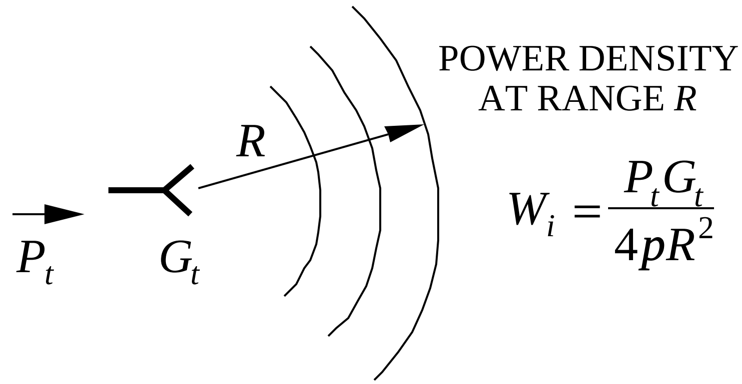
A_p = physical aperture area of the antenna

l = wavelength ($= c / f$)

r = antenna efficiency

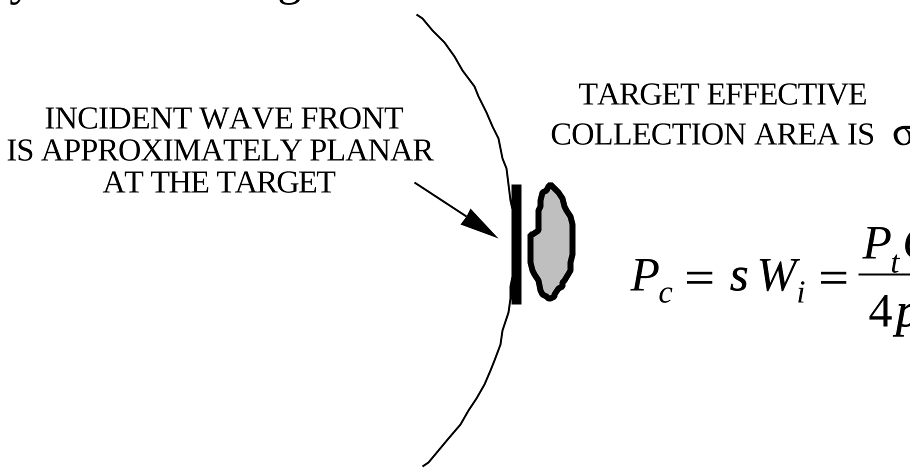
Basic Form of the Radar Range Equation (2)

Power density incident on the target



$$W_i = \frac{P_t G_t}{4\pi R^2} \quad (\mathrm{W/m^2})$$

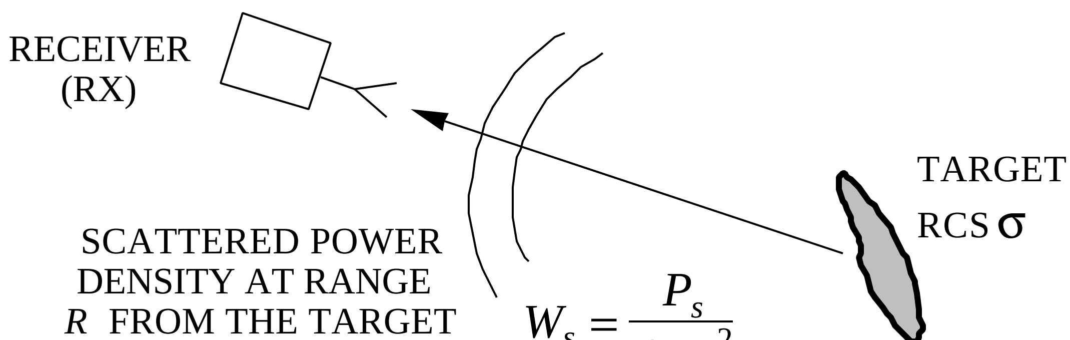
Power collected by the radar target



$$P_c = \sigma W_i = \frac{P_t G_t \sigma}{4\pi R^2}$$

Basic Form of the Radar Range Equation (3)

The RCS gives the fraction of incident power that is scattered back toward the radar. Therefore, $P_s = P_c$ and the scattered power density at the radar is obtained by dividing by $4\pi R^2$.



$$W_s = \frac{P_s}{4\pi R^2}$$

The target scattered power collected by the receiving antenna is $W_s A_{er}$. Thus the maximum target scattered power that is available to the radar is

$$P_r = \frac{P_t G_t \sigma A_{er}}{(4\pi R^2)^2} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

This is the classic form of the radar range equation (RRE).

Characteristics of the Radar Range Equation

$$P_r = \frac{P_t G_t s A_{er}}{(4pR^2)^2} = \frac{P_t G_t G_r s\, l^2}{(4p)^3 R^4}$$

For monostatic systems a single antenna is generally used to transmit and receive so that $G_t = G_r \equiv G$.

This form of the RRE is too crude to use as a design tool. Factors have been neglected that have a significant impact on radar performance:

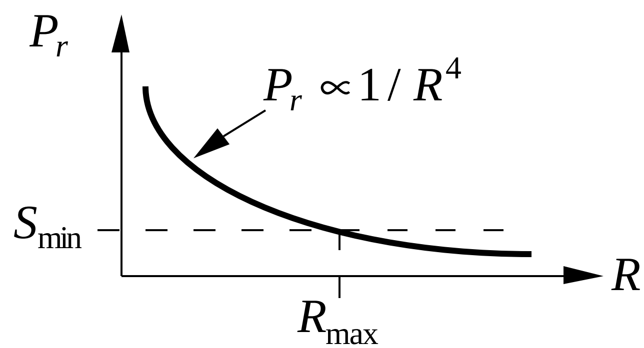
noise, system losses, propagation behavior, clutter, waveform limitations, etc.

We will discuss most of these in depth later in the course.

This form of the RRE does give some insight into the tradeoffs involved in radar design. The dominant feature of the RRE is the $1/R^4$ factor. Even for targets with relatively large RCS, high transmit powers must be used to overcome the $1/R^4$ when the range becomes large.

Maximum Detection Range

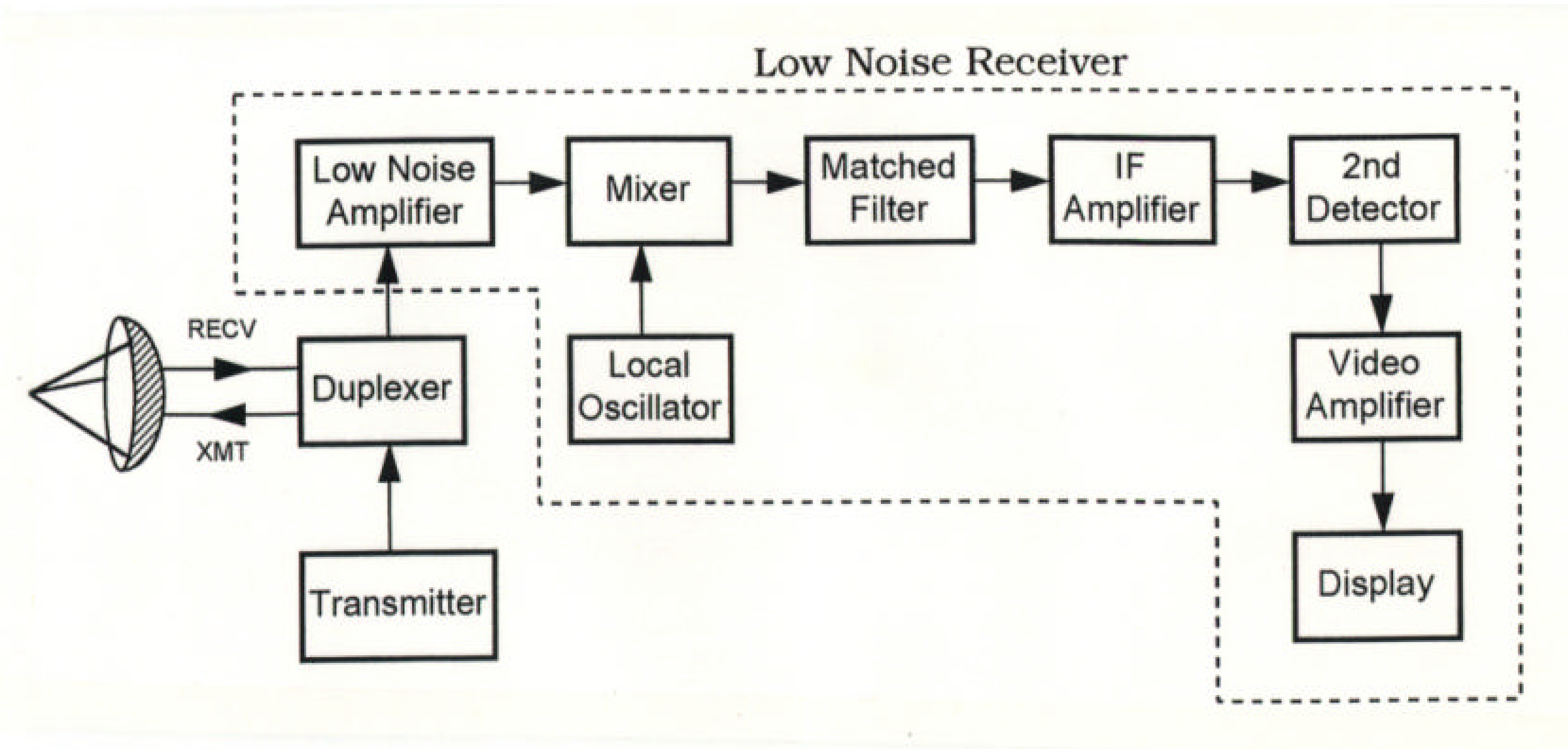
The minimum received power that the radar receiver can "sense" is referred to a the minimum detectable signal (MDS) and is denoted $S_{\min}$.



Given the MDS, the maximum detection range can be obtained:

$$P_r = S_{\min} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4} \Rightarrow R_{\max} = \left(\frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 S_{\min}} \right)^{1/4}$$

Generic Radar Block Diagram



This receiver is a superheterodyne receiver because of the intermediate frequency (IF) amplifier.

Brief Description of System Components

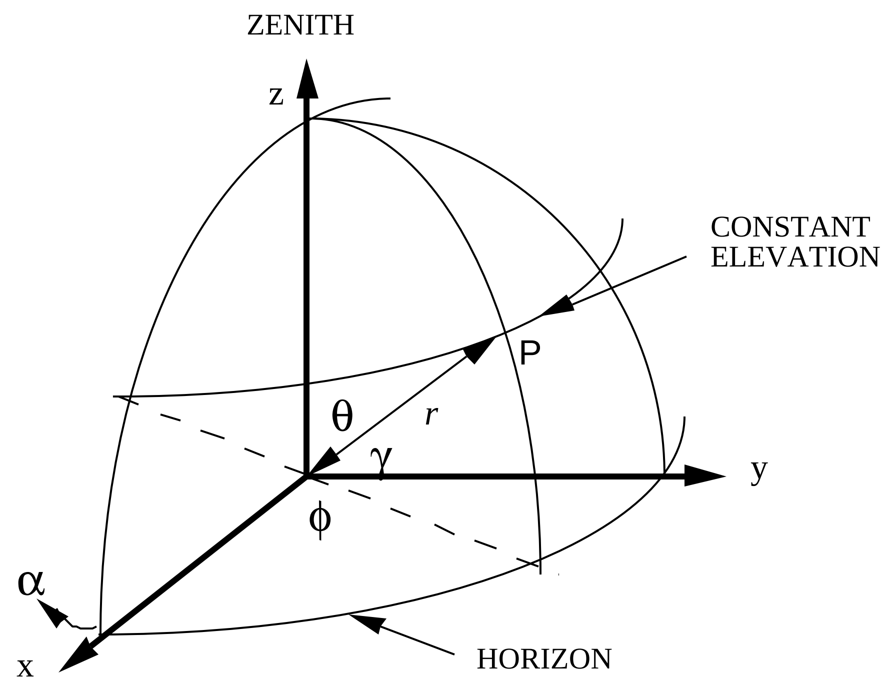
DUPLEXER	An antenna switch that allows the transmit and receive channels to share the antenna. Often it is a circulator. The duplexer must effectively isolate the transmit and receive channels.
TRANSMITTER	Generates and amplifies the microwave signal.
LOW NOISE AMPLIFIER (LNA)	Amplifies the weak received target echo without significantly increasing the noise level.
MIXER	Mixing (or heterodyning) is used to translate a signal to a higher frequency
MATCHED FILTER	Extracts the signal from the noise
IF AMPLIFIER	Further amplifies the intermediate frequency signal
DETECTOR	Translates the signal from IF to baseband (zero frequency)
VIDEO AMPLIFIER	Amplifies the baseband signal
DISPLAY	Visually presents the radar signal for interpretation by the operator

Coordinate Systems

Radar coordinate systems: spherical polar: (r, q, f)

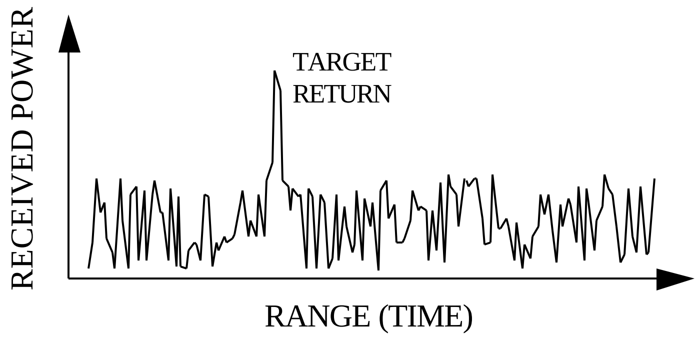
azimuth/elevation: (Az,El) or (a, g)

The radar is located at the origin of the coordinate system; the earth's surface lies in the *x*-*y* plane. Azimuth is generally measured clockwise from a reference (like a compass) but the spherical system azimuthal angle f is measured counterclockwise from the *x* axis. Therefore $a = 360 - f$ and $g = 90 - q$ degrees.

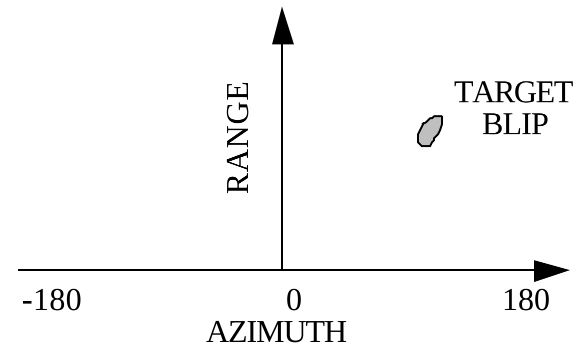


Radar Displays

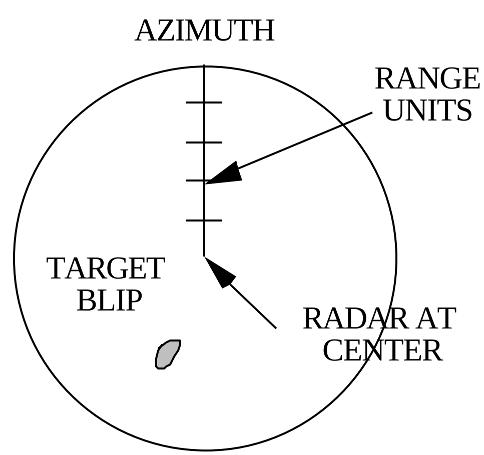
"A" DISPLAY



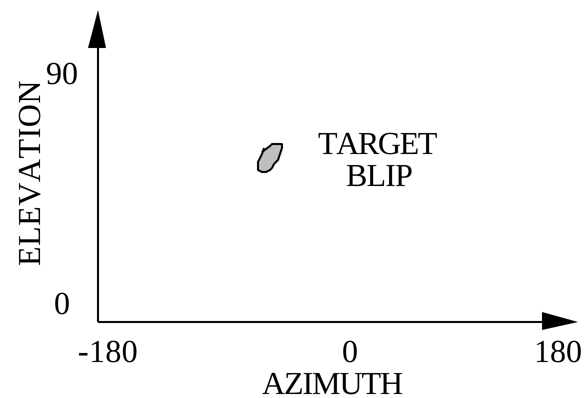
"B" DISPLAY



PLAN POSITION INDICATOR (PPI)



"C" DISPLAY

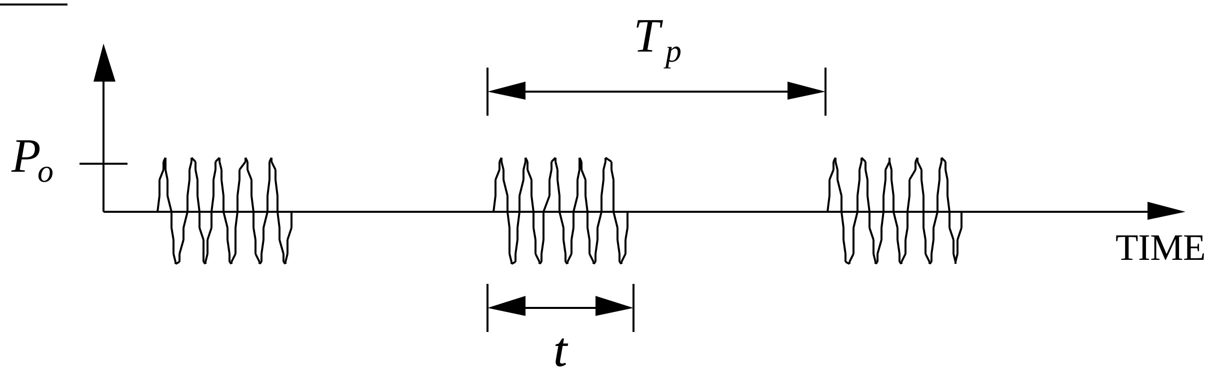


Pulsed Waveform

In practice pulses are continuously transmitted to:

1. cover search patterns,
2. track moving targets,
3. integrate (sum) several target returns to improve detection.

The pulse train is a common waveform

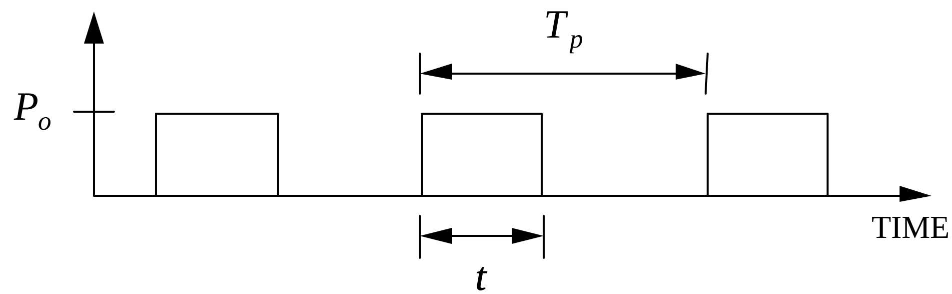


where: P_o = pulse amplitude (may be power or voltage)
t = pulse width (seconds)
T_p = pulse period (seconds)

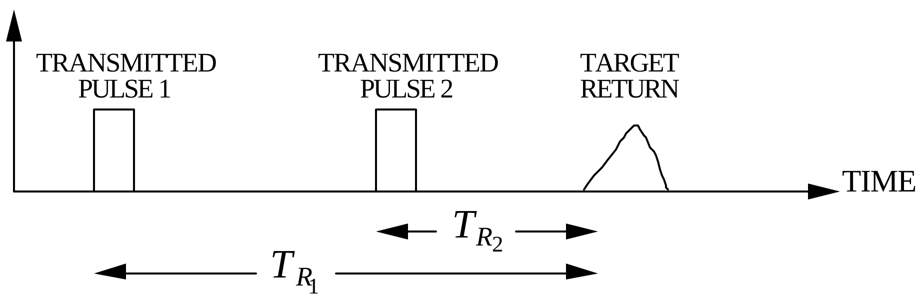
The pulse repetition frequency is defined as $\text{PRF} = f_p = \dfrac{1}{T_p}$

Range Ambiguities

For convenience we omit the sinusoidal carrier when drawing the pulse train



When multiple pulses are transmitted there is the possibility of a range ambiguity.

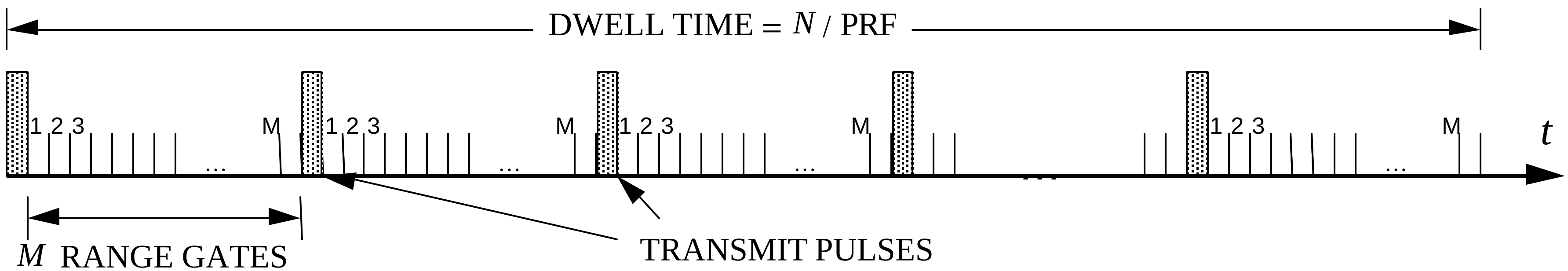


To determine the range unambiguously requires that $T_p \geq \frac{2R}{c}$. The unambiguous range is

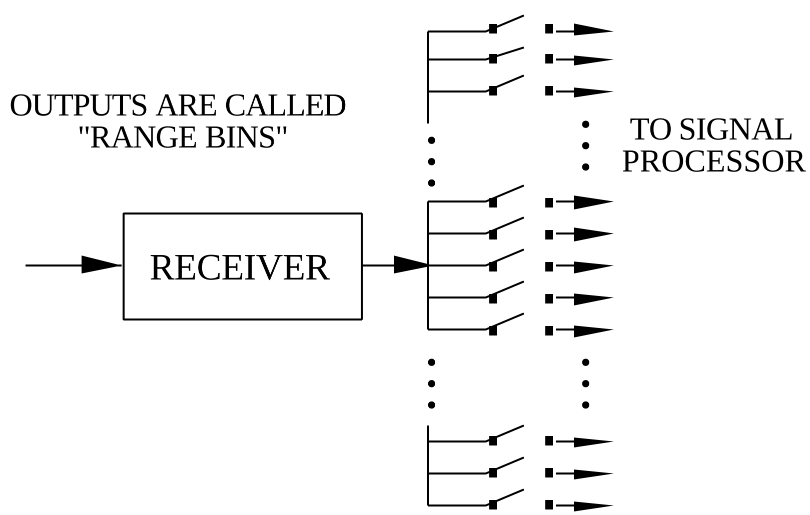
$$R_u = \frac{cT_p}{2} = \frac{c}{2f_p}$$ where f_p is the PRF.

Range Gates

Typical pulse train and range gates



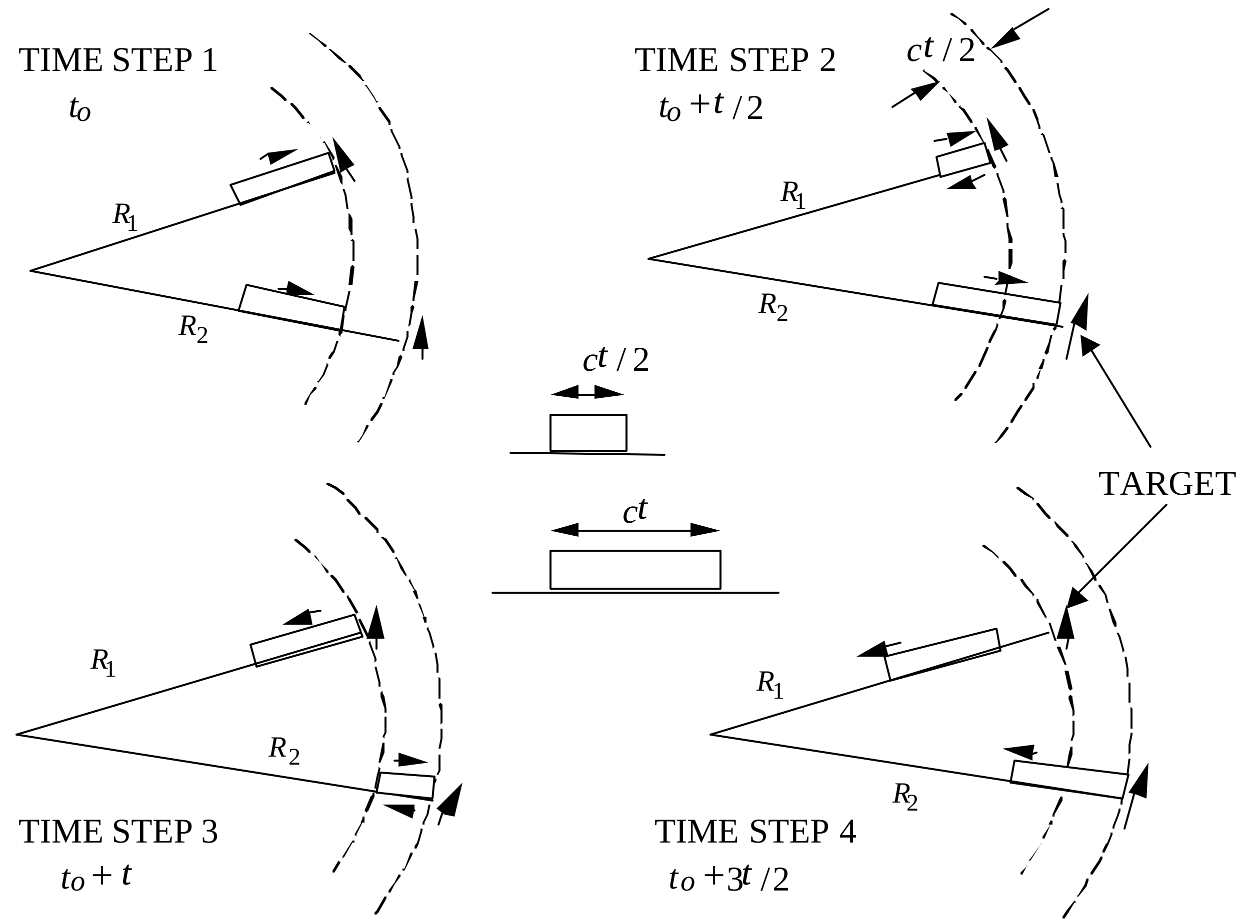
Analog implementation of range gates



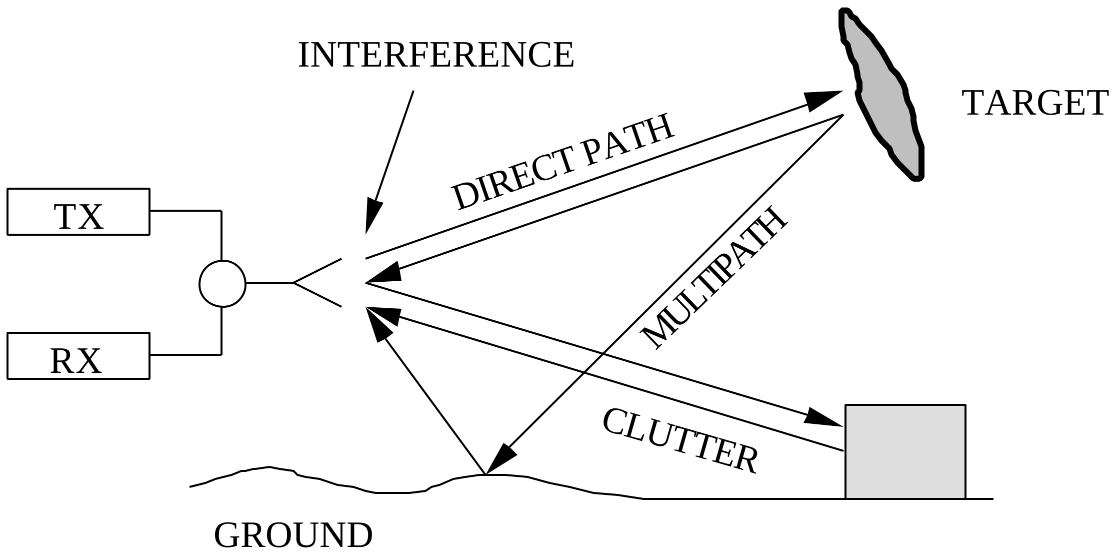
- Gates are opened and closed sequentially
- The time each gate is closed corresponds to a range increment
- Gates must cover the entire interpulse period or the ranges of interest
- For tracking a target a single gate can remain closed until the target leaves the bin

Range Bins and Range Resolution

Two targets are resolved if their returns do not overlap. The range resolution corresponding to a pulse width t is $\Delta R = R_2 - R_1 = ct/2$



Radar Operational Environment

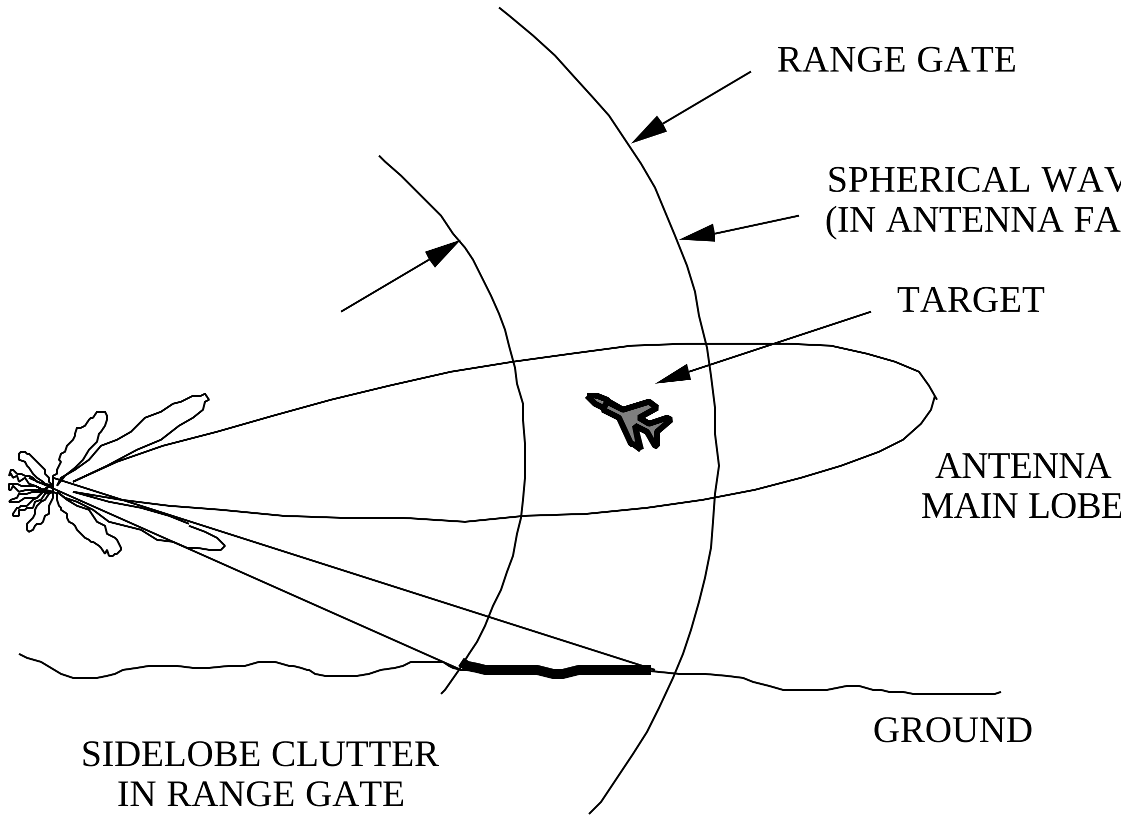


Radar return depends on:

1. target orientation (aspect angle) and distance (range)
2. target environment (other objects nearby; location relative to the earth's surface)
3. propagation characteristics of the path (rain, snow or foliage attenuation)
4. antenna characteristics (polarization, beamwidth, sidelobe level)
5. transmitter and receiver characteristics

Ground Clutter From Sidelobes

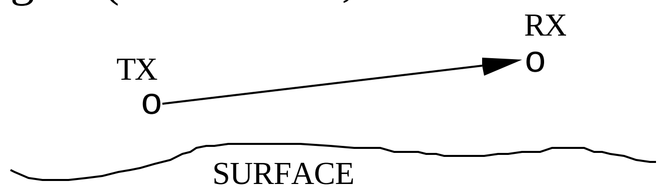
Sidelobe clutter competes with the mainbeam target return



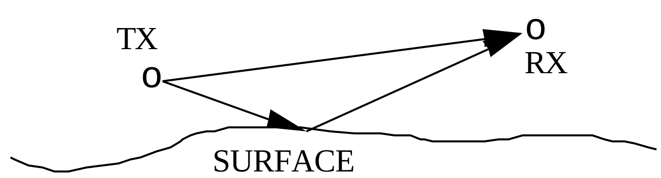
Survey of Propagation Mechanisms (1)

There are may propagation mechanisms by which signals can travel between the radar transmitter and receiver. Except for line-of-sight (LOS) paths, their effectiveness is generally a strong function of the frequency and radar/target geometry.

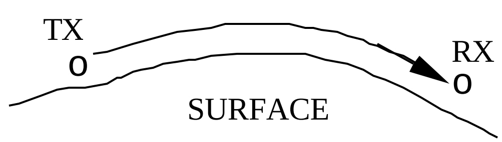
1. direct path or "line of sight" (most radars; SHF links from ground to satellites)



2. direct plus earth reflections or "multipath" (UHF broadcast; ground-to-air and air-to-air communications)

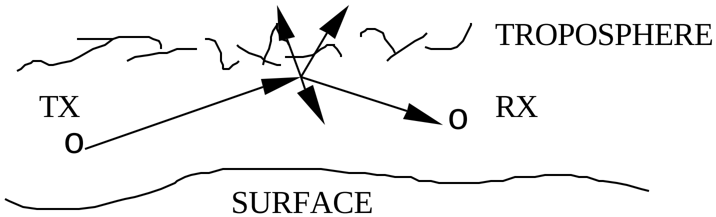


3. ground wave (AM broadcast; Loran C navigation at short ranges)

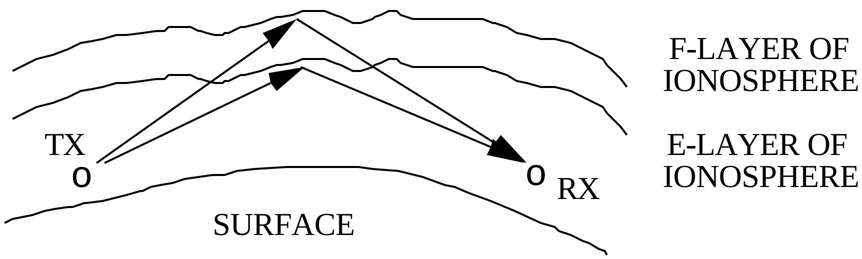


Survey of Propagation Mechanisms (2)

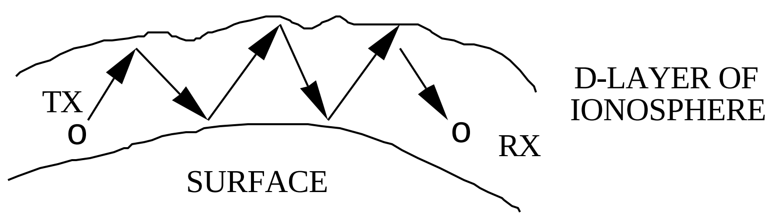
4. tropospheric paths or "troposcatter" (microwave links; over-the-horizon (OTH) radar and communications)



5. ionospheric hop (MF and HF broadcast and communications)



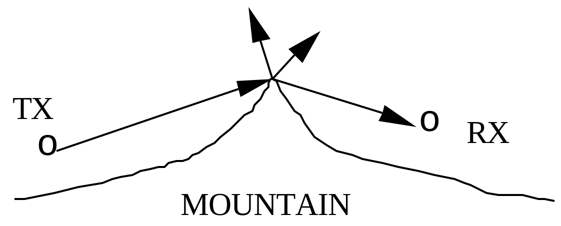
6. waveguide modes or "ionospheric ducting" (VLF and LF communications)



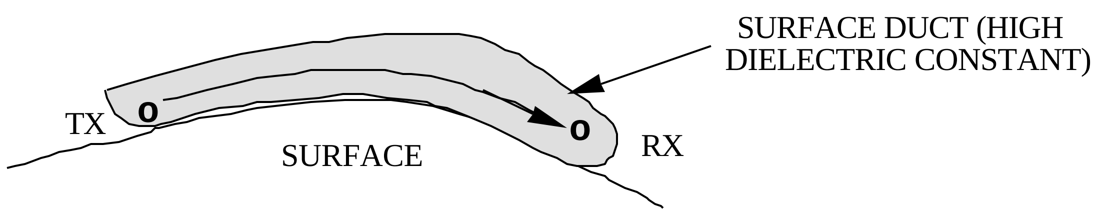
(Note: The distinction between waveguide modes and ionospheric hops is based more on the analysis approach used in the two frequency regimes rather than any physical difference.)

Survey of Propagation Mechanisms (3)

7. terrain diffraction



8. low altitude and surface ducts (radar frequencies)



9. Other less significant mechanisms: meteor scatter, whistlers

Radar System Design Tradeoffs

Choice of frequency affects:

size	high frequencies have smaller devices
transmit power	generally favors lower frequencies
antenna gain/HPWB	small high gain favors high frequencies
atmospheric attenuation	smaller loss a low frequencies
ambient noise	lowest in 1-10 GHz range
doppler shift	greater at high frequencies

Polarization affects:

clutter and ground reflections
RCS of the targets of interest
antenna deployment limitations

Waveform selection affects:

signal bandwidth (determined by pulse width)
PRF (sets the unambiguous range)
average transmitter power (determines maximum detection range)

Decibel Refresher

In general, a <u>dimensionless</u> quantity Q in decibels (denoted Q_{dB}) is defined by

$$Q_{\text{dB}} = 10\log_{10}(Q)$$

Q usually represents a ratio of powers, where the denominator is the reference, and $\log_{10}$ is simply written as log. Characters are added to the "dB" to denote the reference quantity, for example, dBm is decibels relative to a milliwatt. Therefore, if P is in watts:

$$P_{\text{dBW}} = 10\log(P/1) \text{ or } P_{\text{dBm}} = 10\log(P/0.001)$$

Antenna gain G (dimensionless) referenced to an isotropic source (an <u>isotropic</u> source radiates uniformly in all directions, and its gain is 1): $G_{\text{dB}} = 10\log(G)$

Note that:

1. Positive dB values > 1; negative dB values < 1
2. 10 dB represents an order of magnitude change in the quantity Q
3. When quantities are multiplied their dB values add. For example, the effective radiated power (ERP) can be computed directly from the dB quantities:

$$\text{ERP}_{\text{dBw}} = (PG)_{\text{dBw}} = P_{\text{dBw}} + G_{\text{dB}}$$

Note: The ERP is also referred to as the effective isotropic radiated power, EIRP.

Thermal Noise

Consider a receiver at the standard temperature, $T_o = 290$ degrees Kelvin (K). Over a range of frequencies of bandwidth B_n (Hz) the available noise power is

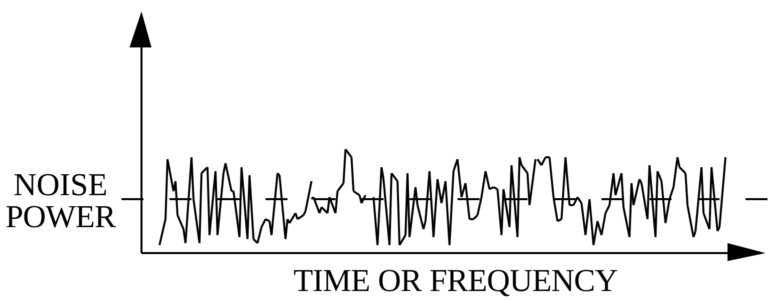
$$N_o = k T_o B_n$$

where $k = 1.38 \times 10^{-23}$ (Joules/K) is Boltzman's constant.

Other radar components will also contribute noise (antenna, mixer, cables, etc.). We define a system noise temperature T_s, in which case the available noise power is

$$N_o = k T_s B_n .$$

(We will address the problem of computing T_s later.) The quantity kT_s is the noise spectral density (W/Hz)

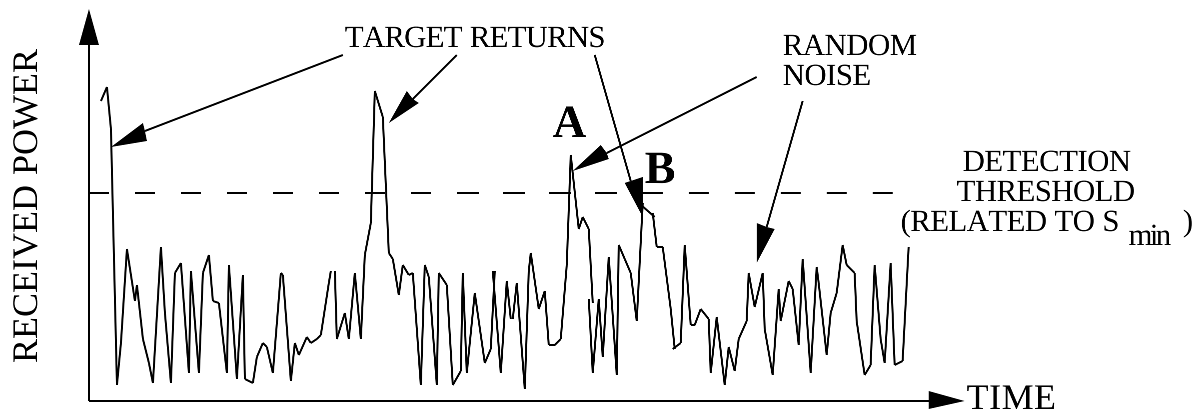


Noise in Radar Systems

In practice the received signal is "corrupted" (distorted from the ideal shape and amplitude):

1. Noise is introduced in the radar system components (antenna, receiver, etc.) and by the environment (interference sources, propagation path, etc.).
2. Signal dispersion occurs. Frequency components of the waveform are treated differently by the radar components and the environment.
3. Clutter return exits.

Typical return trace appears as follows:



Threshold detection is commonly used. If the return is greater than the detection threshold a target is declared. **A** is a false alarm: the noise is greater than the threshold level but there is no target. **B** is a miss: a target is present but the return is not detected.

Noise in Radar Systems

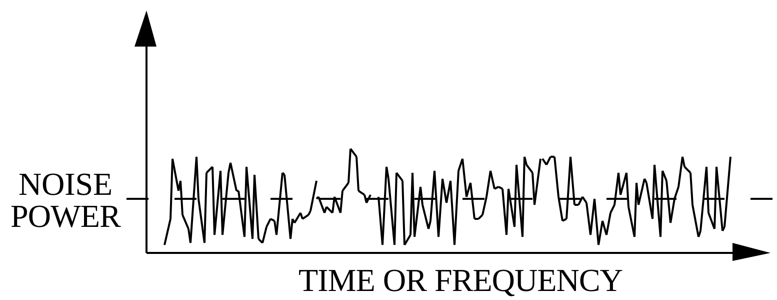
Conflicting requirements:

To avoid false alarms set the detection threshold higher
To avoid misses set the detection threshold lower

Noise is a random process and therefore we must use probability and statistics to assess its impact on detection and determine the "optimum" threshold level.

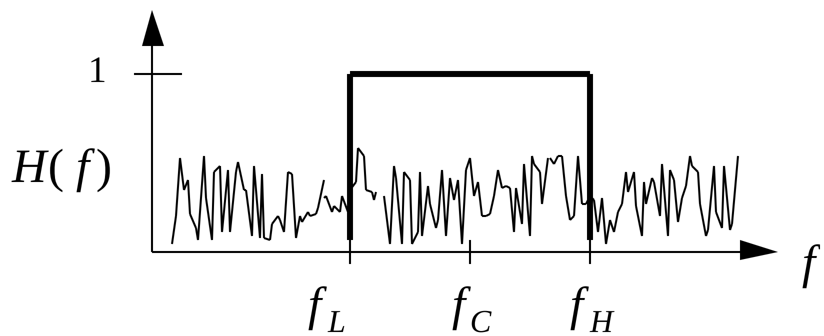
Thermal noise is generated by charged particles as they conduct. High temperatures result in greater thermal noise because of increased particle agitation.

Thermal noise exists at all frequencies. We will consider the noise to be constant with frequency ("white noise") and its statistics (average and variance) independent of time ("stationary").



Ideal Filter

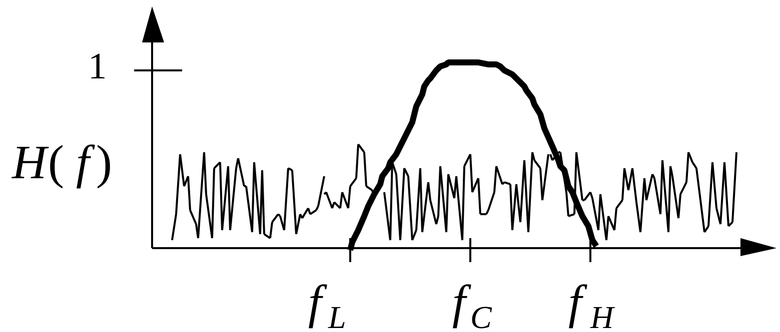
A filter is a device that passes signals with the desired frequencies and rejects all others. Below is shown the filter characteristic of an ideal bandpass filter. Filters are linear systems and the filter characteristic is the transfer function $H(f)$ in the frequency domain. (Recall that $H(f)$ is the Fourier transform of its impulse response, $h(t)$. For convenience $H(f)$ is normalized.)



The bandwidth of this ideal filter is $B = f_H - f_L$. The center frequency is given by $f_C = (f_H + f_L)/2$. Signals and noise in the pass band emerge from the filter unaffected. Therefore the noise power passed by this filter is $N_o = k\mathrm{T}_s B$. The noise bandwidth of an ideal filter is equal to the bandwidth of the filter: $B_n = B$.

Noise Bandwidth of an Arbitrary Filter

In practice $H(f)$ is not constant; in fact it may not even be symmetrical about the center frequency.



The noise bandwidth is defined as the bandwidth of an equivalent ideal filter with $H(f) = 1$:

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_C)|^2}$$

Furthermore, real filters are not strictly bandlimited (i.e., the characteristic is not zero outside of the passband). In this case we usually use the actual filter characteristic inside the 3dB (or sometimes 10 dB) points and zero at frequencies outside of these points.

Signal-to-Noise Ratio (*S/N*)

Considering the presence of noise, the important parameter for detection is the signal-to-noise ratio (S/N). We already have an expression for the signal returned from a target (P_r from the radar equation), and therefore the signal-to-noise ratio is

$$\text{SNR} = \frac{P_r}{N_o} = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4 k \text{T}_s B_n}$$

At this point we will consider only two noise sources:

1. background noise collected by the antenna (T_A)
2. total effect of all other system components (system effective noise temperature, T_e)

so that

$$\text{T}_s = \text{T}_\text{A} + \text{T}_e$$

Example: Police Radar

A police radar has the following parameters:

$$B_n = 1\,\text{kHz} \quad P_t = 100\,\text{mW} \quad D = 20\,\text{cm} \quad r = 0.6$$

$$f = 10.55\,\text{GHz} \quad \text{T}_s = 1000\,\text{K} \quad (\text{S/N})_{\text{min}} = 10\,\text{dB} \quad s = 0.2\,\text{m}^2$$

$$A_{er} = A_p r = p(D/2)^2 0.6 = 0.01884\,\text{m}^2, \quad l = c/f = 3\times 10^8 / 10.55\times 10^9 = 0.028\,\text{m}$$

$$G = \frac{4pA_{er}}{l^2} = \frac{4p(0.01884)}{0.028^2} = 292.6 = 24.66\,\text{dB}$$

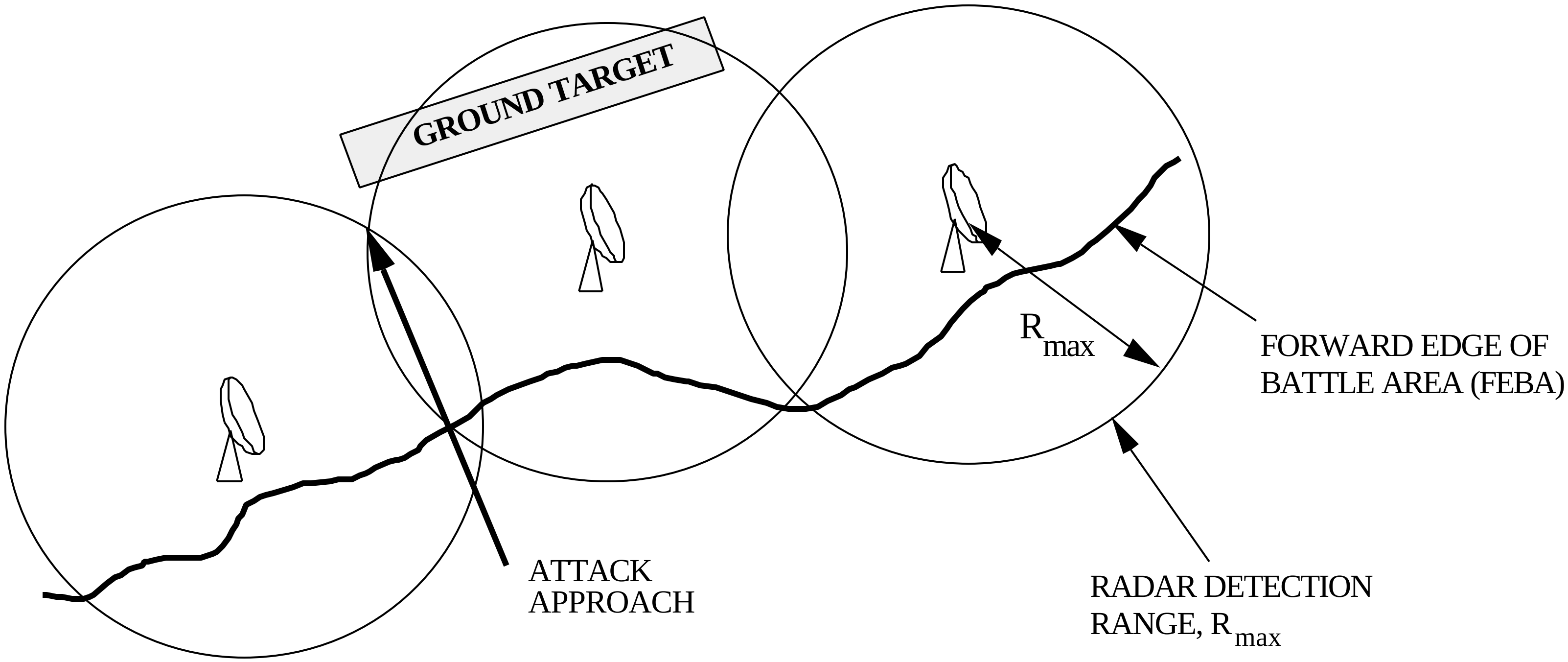
$$N_o = k\text{T}_s B_n = (1.38\times 10^{-23})(1000)(1000) = 1.38\times 10^{-17}$$

$$\text{SNR} = \frac{P_r}{N_o} = \frac{P_t G^2 s\, l^2}{(4p)^3 R^4 N_o} = 10\,\text{dB} = 10^{(10/10)} = 10$$

$$R^4 = \frac{P_t G^2 s\, l^2}{(4p)^3 10 N_o} = \frac{(0.1)(292.6)^2 (0.2)(0.028)^2}{(4p)^3 (1.38\times 10^{-16})} = 4.9\times 10^{12}$$

$$R = 1490\,\text{m} = 1.49\,\text{km} \approx 0.9\,\text{mi}$$

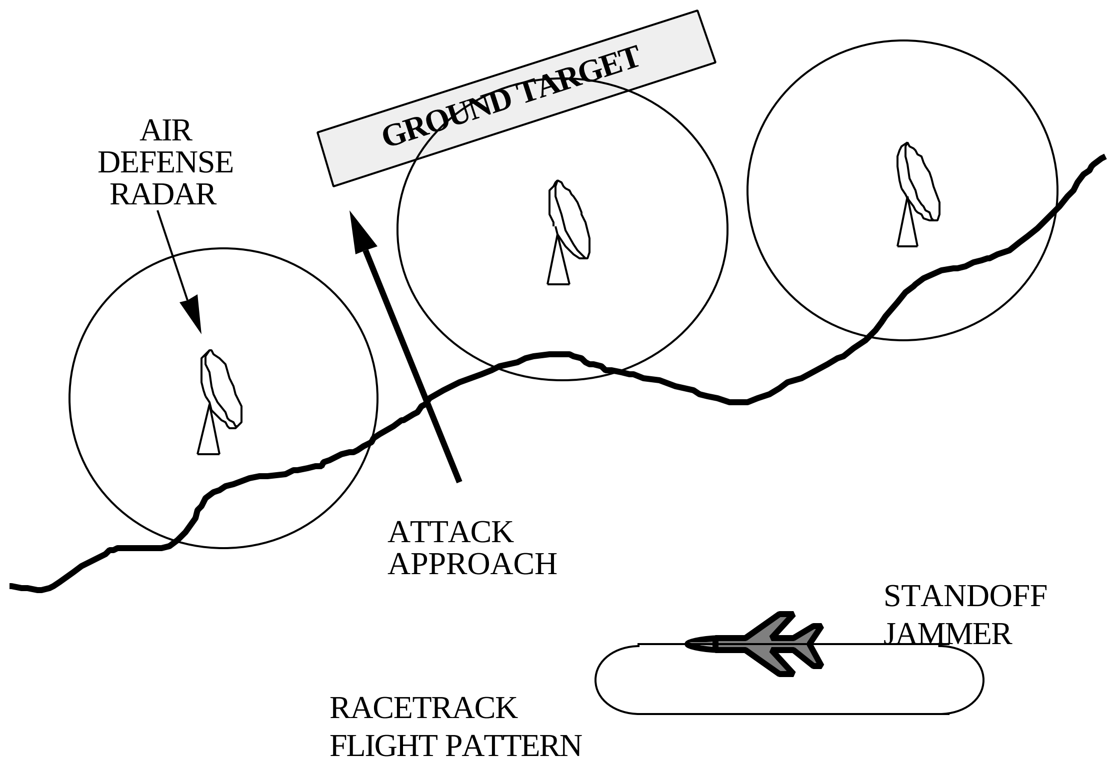
Attack Approach



A network of radars are arranged to provide continuous coverage of a ground target.

Conventional aircraft cannot penetrate the radar network without being detected.

Defeating Radar by Jamming

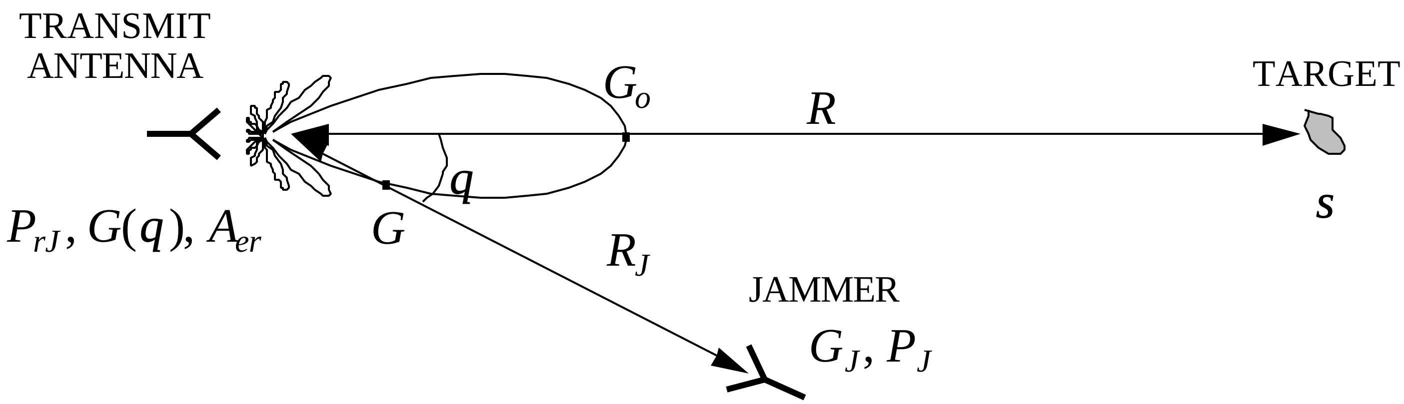


The barrage jammer floods the radar with noise and therefore decreases the SNR

The radar knows it's being jammed

Jammer Burnthrough Range (1)

Consider a <u>standoff jammer</u> operating against a radar that is tracking a target



The jammer power received by the radar is

$$P_{rJ} = W_i A_{er} = \left(\frac{P_J G_J}{4\pi R_J^2} \right) \left(\frac{\lambda^2 G(\theta)}{4\pi} \right) = \frac{P_J G_J \lambda^2 G(\theta)}{(4\pi R_J)^2}$$

Defining $G_o \equiv G(\theta = 0)$, the target return is

$$P_r = \frac{P_t G_o^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Jammer Burnthrough Range (2)

The signal-to-jam ratio is

$$\mathrm{SJR} = \frac{S}{J} = \frac{P_r}{P_{rJ}} = \left(\frac{P_t G_o}{P_J G_J}\right)\left(\frac{R_J^2}{R^4}\right)\left(\frac{\sigma}{4\pi}\right)\left(\frac{G_o}{G(\theta)}\right)$$

The <u>burnthrough range</u> for the jammer is the range at which its signal is equal to the target return (SJR=1).

Important points:

1. R_J^2 vs R^4 is a big advantage for the jammer.
2. G vs $G(\theta)$ is usually a big disadvantage for the jammer. Low sidelobe radar antennas reduce jammer effectiveness.
3. Given the geometry, the only parameter that the jammer has control of is the ERP ($P_J G_J$).
4. The radar knows it is being jammed. The jammer can be countered using waveform selection and signal processing techniques.

Noise Figure

Active devices such as amplifiers boost the signal but also add noise. For these devices the noise figure is used as a figure of merit:

$$F_n = \frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} = \frac{S_{\text{in}} / N_{\text{in}}}{S_{\text{out}} / N_{\text{out}}}$$

For an ideal network that does not add noise $F_n = 1$.

Solve for the input signal:

$$S_{\text{in}} = \frac{S_{\text{out}}}{N_{\text{out}}} F_n N_{\text{in}} = \left(\frac{S}{N}\right)_{\text{out}} F_n (k\text{T}_o B_n)$$

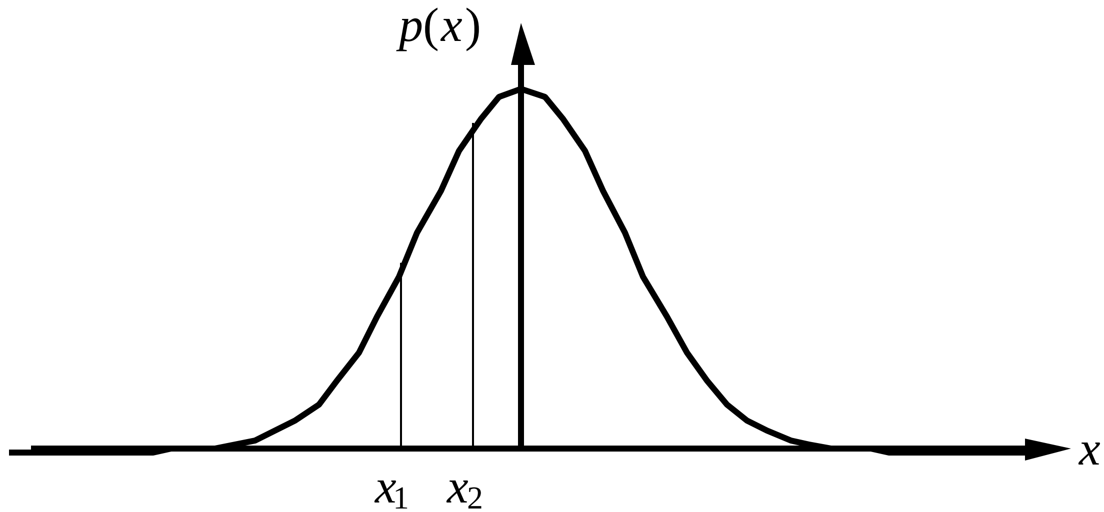
Let $S_{\text{in}} = S_{\text{min}}$ and find the maximum detection range

$$R_{\text{max}}^4 = \frac{P_t G_t A_{er} s}{(4p)^2 k\text{T}_o B_n F_n (S_{\text{out}} / N_{\text{out}})_{\text{min}}}$$

This equation assumes that the antenna temperature is T_o.

Probability & Statistics Refresher (1)

Probability density function (PDF) of a random variable x



Probability that x lies between x_1 and x_2:

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} p(x)\,dx$$

Since $p(x)$ includes all possible outcomes

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

Probability & Statistics Refresher (2)

The expected value (average, mean) is computed by

$$\langle x \rangle = \bar{x} = \int_{-\infty}^{\infty} x\, p(x)\, dx$$

In general, the expected value of any function of x, $g(x)$

$$\langle g(x) \rangle = \overline{g(x)} = \int_{-\infty}^{\infty} g(x)\, p(x)\, dx$$

Moments of the PDF: $\langle x \rangle = \bar{x}$ is the first moment, ..., $\langle x^m \rangle = \overline{x^m}$ is the mth moment

Central moments of the PDF: $\left\langle (x-\bar{x})^m \right\rangle$ mth central moment

The second central moment is the variance, $\overline{s^2} = \left\langle (x-\bar{x})^2 \right\rangle$

$$\overline{s^2} = \int_{-\infty}^{\infty} (x-\bar{x})^2 p(x)\, dx = \int_{-\infty}^{\infty} (x^2 - 2x\bar{x} + \bar{x}^2) p(x)\, dx$$

$$\overline{s^2} = \int_{-\infty}^{\infty} x^2 p(x)\, dx - 2\bar{x} \int_{-\infty}^{\infty} x\, p(x)\, dx + \bar{x}^2 \int_{-\infty}^{\infty} p(x)\, dx$$

with the final result: $\overline{s^2} = \left\langle (x-\bar{x})^2 \right\rangle = \overline{x^2} - \bar{x}^2$

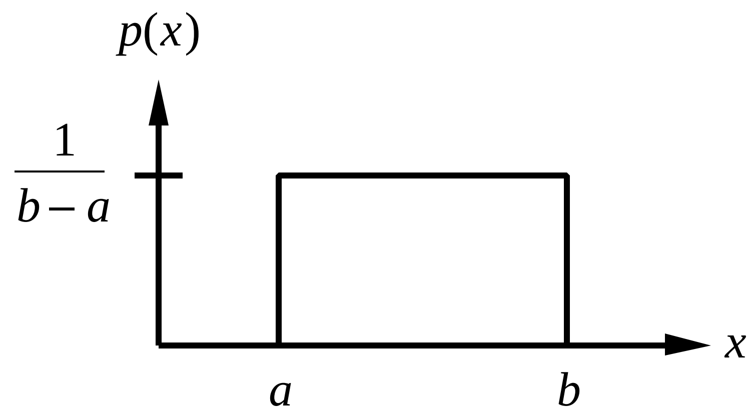
Physical significance: $\bar{x}$ is the mean value (dc); $\sqrt{\overline{s^2}}$ is the rms value

Probability & Statistics Refresher (3)

Special probability distributions we will encounter:

1. Uniform PDF

$$p(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{else} \end{cases}$$



Expected value:

$$\langle x \rangle = \bar{x} = \int_{-\infty}^{\infty} x\, p(x)\, dx = \frac{1}{2}\frac{b^2 - a^2}{b-a} = \frac{b+a}{2}$$

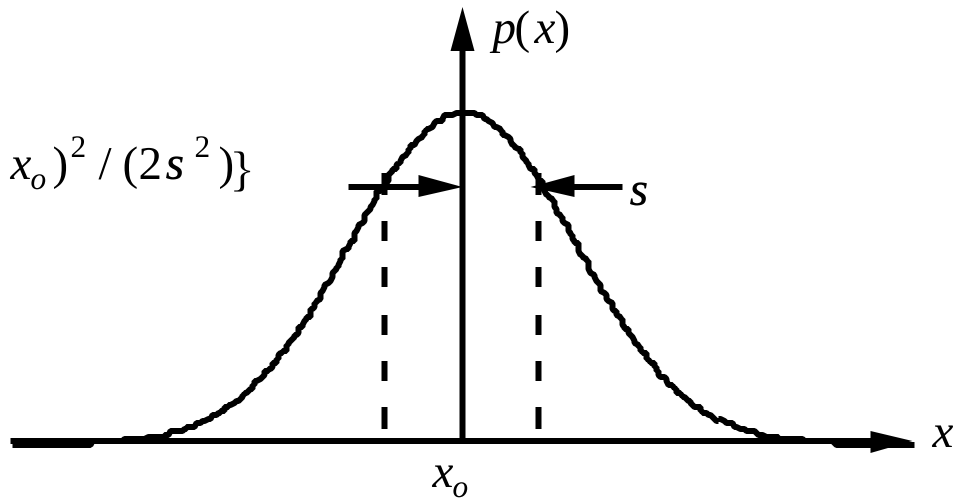
Variance:

$$\overline{s^2} = \left\langle (x-\bar{x})^2 \right\rangle = \int_a^b \left(\frac{(x-\bar{x})^2}{b-a} \right) dx = \frac{(b-a)^2}{12}$$

Probability & Statistics Refresher (4)

2. Gaussian PDF

$$p(x) = \frac{1}{\sqrt{2p\, s^2}} \exp\{-(x - x_o)^2 / (2s^2)\}$$



Expected value: $$\langle x \rangle = \bar{x} = \frac{1}{\sqrt{2p\, s^2}} \int_{-\infty}^{\infty} x \exp\left\{-(x - x_o)^2 / (2s^2)\right\} dx = x_o$$

Variance: $$\overline{x^2} = \frac{1}{\sqrt{2p\, s^2}} \int_{-\infty}^{\infty} x^2 \exp\left\{-(x - x_o)^2 / (2s^2)\right\} dx = x_o^2 + s^2$$

$$\overline{s^2} = \overline{x^2} - \bar{x}^2 = s^2$$

The standard normal distribution has $\bar{x} = 0$ and $\overline{s^2} = 1$.

Rayleigh Distribution (1)

Consider two independent gaussian distributed random variables x and y

$$p_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-x^2/(2\sigma^2)\right\} \text{ and } p_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-y^2/(2\sigma^2)\right\}$$

The joint PDF of two independent variables is the product of the PDFs:

$$p_{xy}(x,y) = \frac{1}{2\pi\sigma^2} \exp\left\{-(x^2+y^2)/(2\sigma^2)\right\}$$

If x and y represent noise on the real and imaginary parts of a complex signal, we are interested in the PDF of the magnitude, $r^2 = x^2 + y^2$. Transform to polar coordinates (r, ϕ)

$$\int_{-\infty}^{\infty}\int p_{xy}(x,y)\, dx\, dy = \int_0^{\infty}\int_0^{2\pi} p_{r\phi}(r,\phi)\, r\, dr\, d\phi$$

In polar form the PDF is independent of ϕ

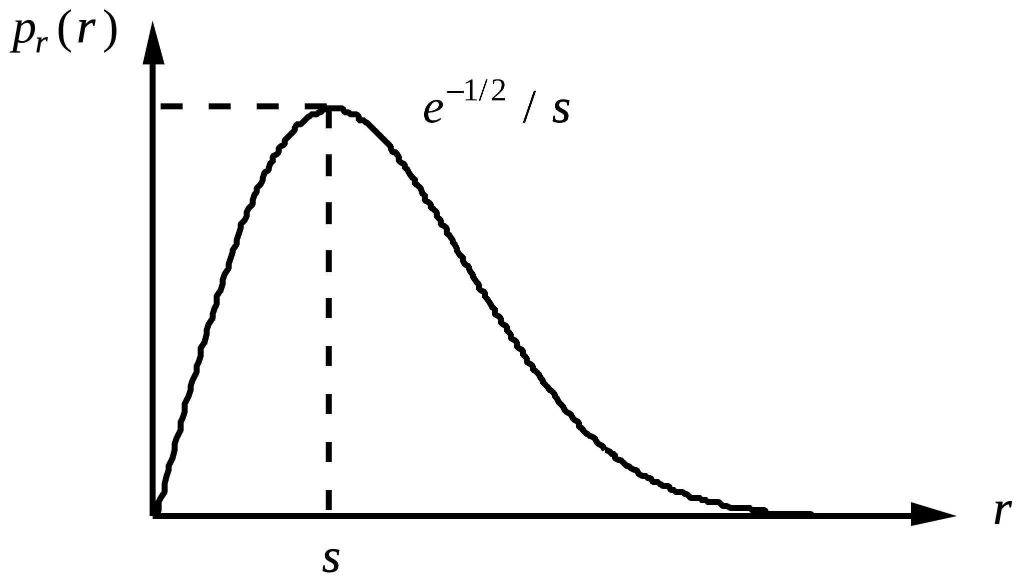
$$p_{r\phi}(r,\phi) = \frac{1}{2\pi\sigma^2} \exp\left\{-r^2/2\sigma^2\right\}$$

Therefore, the ϕ integration simply gives a factor of 2π.

Rayleigh Distribution (2)

The Rayleigh PDF is:

$$p_r(r) = \frac{r}{s^2} \exp\left\{-r^2/(2s^2)\right\}$$



s is the "mode" or most probable value

$\bar{r} = \sqrt{\frac{p}{2}}\, s$ is the expected value of r

$\overline{r^2} = 2s^2$ (noise power) and the variance is $\left(2 - \frac{p}{2}\right) s^2$

Central Limit Theorem

The central limit theorem states that the probability density function of the sum of N independent identically distributed random variables is asymptotically normal. If $x = x_1 + x_2 + \cdots + x_N$, where the x_i have mean $\bar{x}$ and variance s then

$$\lim_{N \to \infty} P\left(a \le \frac{x - N\bar{x}}{s\sqrt{N}} \le b\right) = \frac{1}{\sqrt{2p}} \int_a^b e^{-u^2/2} du$$

Samples from any distribution will appear normally distributed if we take "enough" samples. Usually 10 samples are sufficient.

For our purposes, the central limit theorem usually permits us to model most random processes as gaussian.

Example: N uniformly distributed random variables between the limits a and b.
The central limit theorem states that the joint PDF is gaussian with

mean: $\bar{x} = N\frac{b+a}{2}$

variance: $s^2 = \frac{N}{12}\left(\frac{b+a}{2}\right)^2$

Transformation of Variables

Given that a random variable has a PDF of $p_x(x)$ we can find the PDF of any function of x, say $g(x)$. Let $a = g(x)$ and the inverse relationship denoted by $x = \hat{g}(a)$. Then

$$p_a(a) = p_x(x)\left|\frac{dx}{da}\right| = p_x(\hat{g}(a))\left|\frac{d\hat{g}(a)}{da}\right|$$

Example: A random signal passed through a square law detector is squared (i.e., the output is proportional to x^2). Thus let $a = x^2 \equiv g(x)$ or

$$x = \sqrt{a} \equiv \hat{g}(a) \text{ and } \frac{d\,\hat{g}(a)}{da} = \frac{d\,(a)^{1/2}}{da} = \frac{1}{2\sqrt{a}}$$

Therefore,

$$p_a(a) = p_x(\sqrt{a})\frac{1}{2\sqrt{a}}$$

Let

$$p_x(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{else} \end{cases}$$

with the final result:

$$p_a(a) = \frac{1}{2\sqrt{a}}\, e^{-\sqrt{a}} \text{ for } a > 0$$

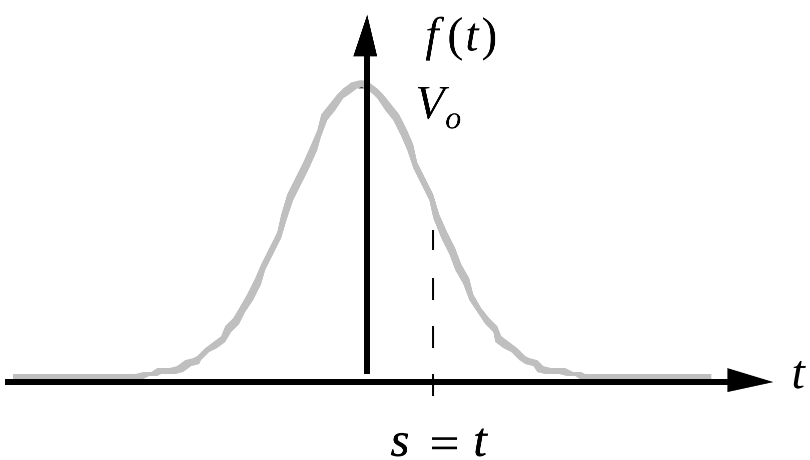
Fourier Transform Refresher (1)

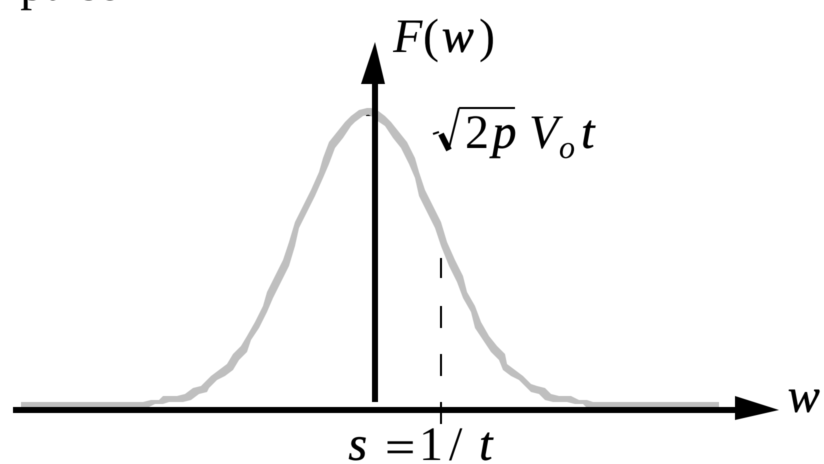
We will be using the Fourier transform and the inverse Fourier transform to move between time and frequency representations of a signal. A Fourier transform pair $f(t)$ and $F(w)$ (where $w = 2p\, f$) are related by

$$f(t) = \frac{1}{2p} \int_{-\infty}^{\infty} F(w) e^{jwt} dw \leftrightarrow F(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt} dt$$

Some transform pairs we will be using:

gaussian pulse





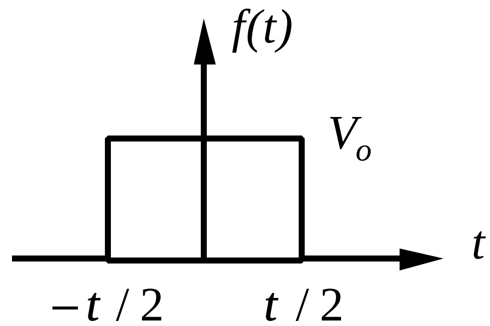
$$f(t) = V_o e^{-t^2/(2t)^2}$$

$$F(w) = \sqrt{2p}\, V_o t \; e^{-t^2 w^2/2}$$

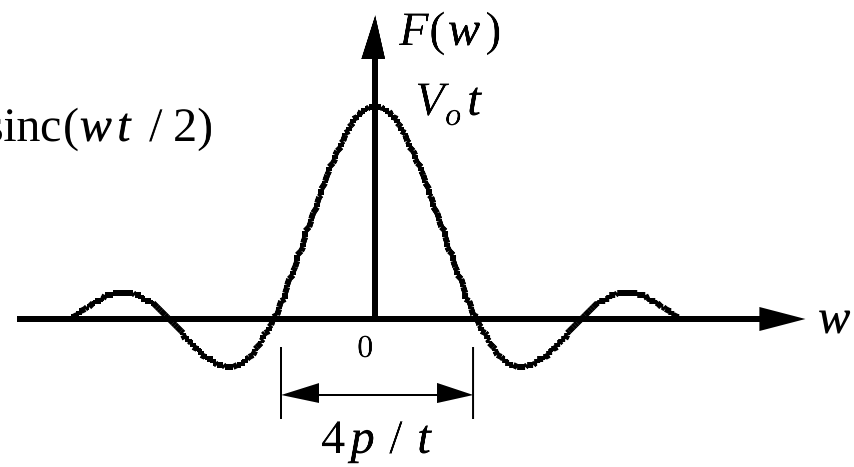
Fourier Transform Refresher (2)

rectangular pulse

$$f(t) = \begin{cases} V_o, & |t| < t/2 \\ 0, & \text{else} \end{cases}$$



$$F(w) = V_o t \frac{\sin(wt/2)}{wt/2} \equiv V_o t \,\mathrm{sinc}(wt/2)$$



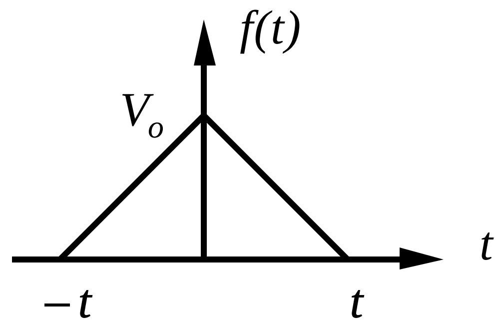
Bandwidth between first nulls for this signal:

$$2(2p B_1) = 4p/t \Rightarrow B_1 = 1/t$$

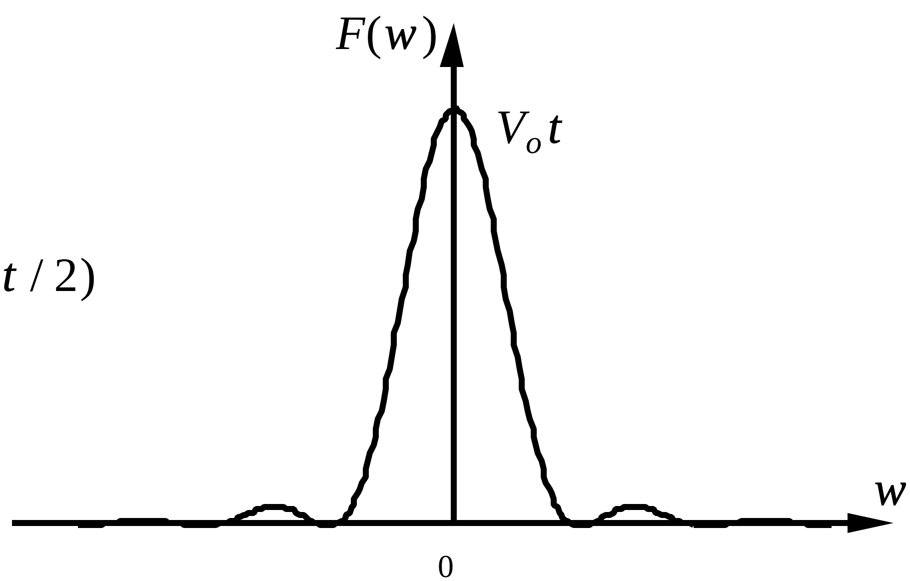
Fourier Transform Refresher (3)

triangular pulse

$$f(t) = \begin{cases} V_o(1 - |t|/t), & |t| < t \\ 0, & \text{else} \end{cases}$$



$$F(w) = V_o t \, \text{sinc}^2(wt/2)$$



Fourier Transform Refresher (4)

Important theorems and properties of the Fourier transform:

1. symmetry between the time and frequency domains

$$\text{if } f(t) \leftrightarrow F(w), \text{ then } F(t) \leftrightarrow 2p\ f(-w)$$

2. time scaling

$$f(at) \leftrightarrow 1/\,|\,a\,|\,F(w\,/\,a)$$

3. time shifting

$$f(t - t_o) \leftrightarrow F(w)\,e^{-jwt_o}$$

4. frequency shifting

$$f(t)e^{jw_o t} \leftrightarrow F(w - w_o)$$

5. time differentiation

$$\frac{d^n f(t)}{dt^n} \leftrightarrow (jw)^n F(w)$$

6. frequency differentiation

$$(-jt)^n f(t) \leftrightarrow \frac{d^n F(w)}{dw^n}$$

Fourier Transform Refresher (5)

7. conjugate functions

$$f^*(t) \leftrightarrow F^*(-w)$$

8. time convolution

$$f_1(t) \leftrightarrow F_1(w)$$

$$f_2(t) \leftrightarrow F_2(w)$$

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(t) f_2(t - t)\, dt \leftrightarrow F_1(w) F_2(w)$$

9. frequency convolution

$$f_1(t) f_2(t) \leftrightarrow F_1(w) * F_2(w)$$

10. Parseval's formula

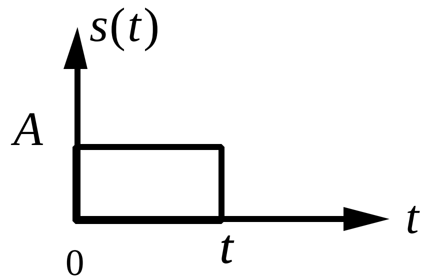
$$F(w) = A(w) e^{j\Phi(w)}, \quad |F(w)| = A(w)$$

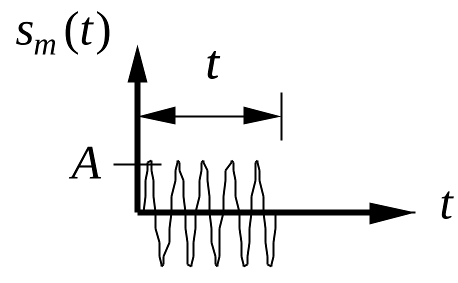
$$\int_{-\infty}^{\infty} |\, f(t)\,|^2\, dt = \frac{1}{2p} \int_{-\infty}^{\infty} A(w)^2\, dw$$

Modulation of a Carrier (1)

A carrier is modulated by a pulse to shift frequency components higher. (High frequency transmission lines and antennas are more compact and efficient than low frequency ones.) A sinusoidal carrier modulated by the waveform $s(t)$ is given by

$$s_m(t) = s(t)\cos(w_c t) = \frac{s(t)}{2}\left[e^{jw_c t} + e^{-jw_c t}\right]$$





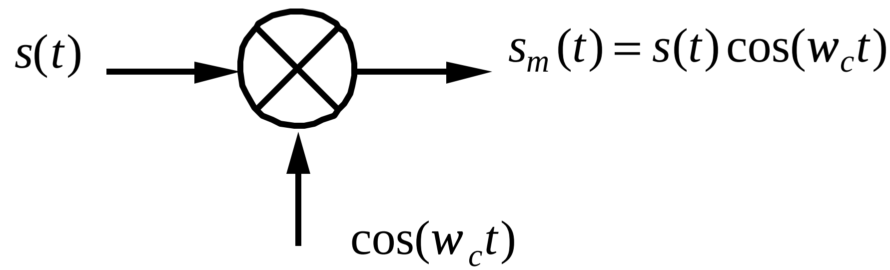
The Fourier transform of the modulated wave is easily determined using the shifting theorem

$$F_m(w) = \frac{1}{2}\left[F(w + w_c) + F(w - w_c)\right]$$

where $s(t) \leftrightarrow F(w)$ and $s_m(t) \leftrightarrow F_m(w)$. Thus, in the case of a pulse, $F(w)$ is a sinc function, and it has been shifted to the carrier frequency.

Modulation of a Carrier (2)

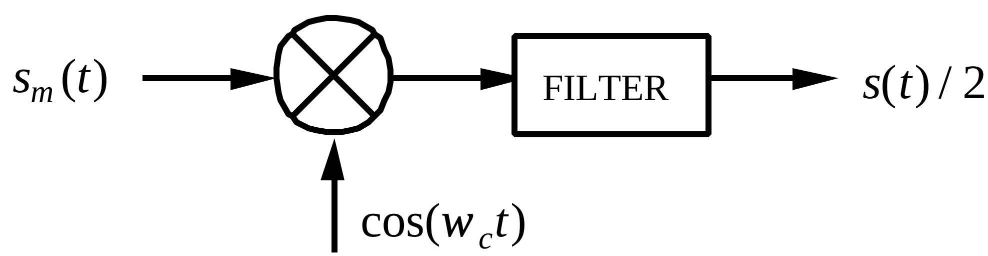
The frequency conversion, or shifting, is achieved using a modulator (mixer) which essentially multiplies two time functions



To recover $s(t)$ from $s_m(t)$ we demodulate. This can be done by multiplying again by $\cos(w_c t)$

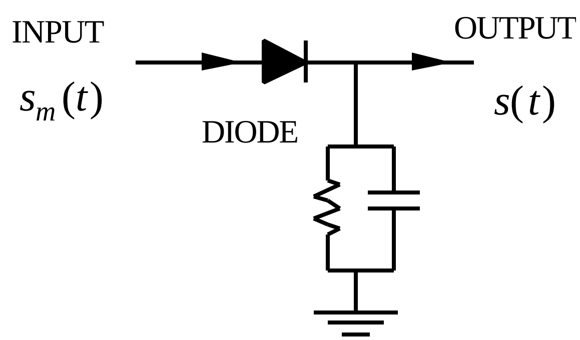
$$s_m(t)\cos(w_c t) = s(t)\cos^2(w_c t) = \frac{s(t)}{2}\left[1 + \cos(2w_c t)\right] = \frac{s(t)}{2} + \frac{s(t)}{4}\left[e^{j2w_c t} + e^{-j2w_c t}\right]$$

$s(t)/2$ is the desired baseband signal (centered at zero frequency). The other terms are rejected using filters.



Modulation of a Carrier (3)

We can save work by realizing that $s(t)$ is simply the envelope of $s_m(t)$. Therefore we only need an envelope detector:



In a real system both signal and noise will be present at the input of the detector

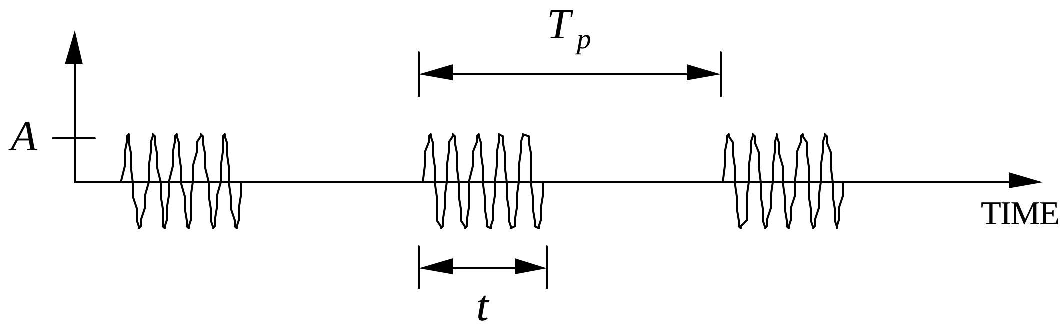
$$s_{\text{in}} = s_m(t) + n(t)$$

The noise is assumed to be gaussian white noise, that is, constant noise power as a function of frequency. Furthermore, the statistics of the noise (mean and variance) are independent of time. (This is a property of a stationary process.)

An important question that needs to be addressed is: how is noise affected by the demodulation and detection process? (Or, what is the PDF of the noise out of the detector?)

Fourier Transform of a Pulse Train (1)

A <u>coherent</u> pulse train is shown below:



Coherent implies that the pulses are periodic sections of the same parent sinusoid. The finite length pulse train can be expressed as the product of three time functions:

1. infinite pulse train which can be expanded in a Fourier series

$$f_1(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n w_o t)$$

where $w_o = \frac{2p}{T_p} = 2p\ f_p$ and

$$a_0 = \frac{1}{T_p} \int_{-t/2}^{t/2} (1)\, dt = \frac{t}{T_p}$$

$$a_n = \frac{2}{T_p} \int_{-t/2}^{t/2} \cos(n w_o t)\, dt = \frac{2t}{T_p} \operatorname{sinc}(n w_o T_p / 2)$$

Fourier Transform of a Pulse Train (2)

2. rectangular window of length $N_p T_p$ that turns on N_p pulses.

$$f_2(t) = \begin{cases} 1, & |t| \le N_p T_p / 2 \\ 0, & \text{else} \end{cases}$$

3. infinite duration sinusoid $f_3(t) = A\cos(w_c t)$ where w_c is the carrier frequency.

Thus the time waveform is:

$$f(t) = f_1(t) f_2(t) f_3(t) = \frac{At}{T_p}\left\{1 + 2\sum_{n=1}^{\infty}\cos(nw_o t)\operatorname{sinc}(nw_o t/2)\right\}\cos(w_c t)$$

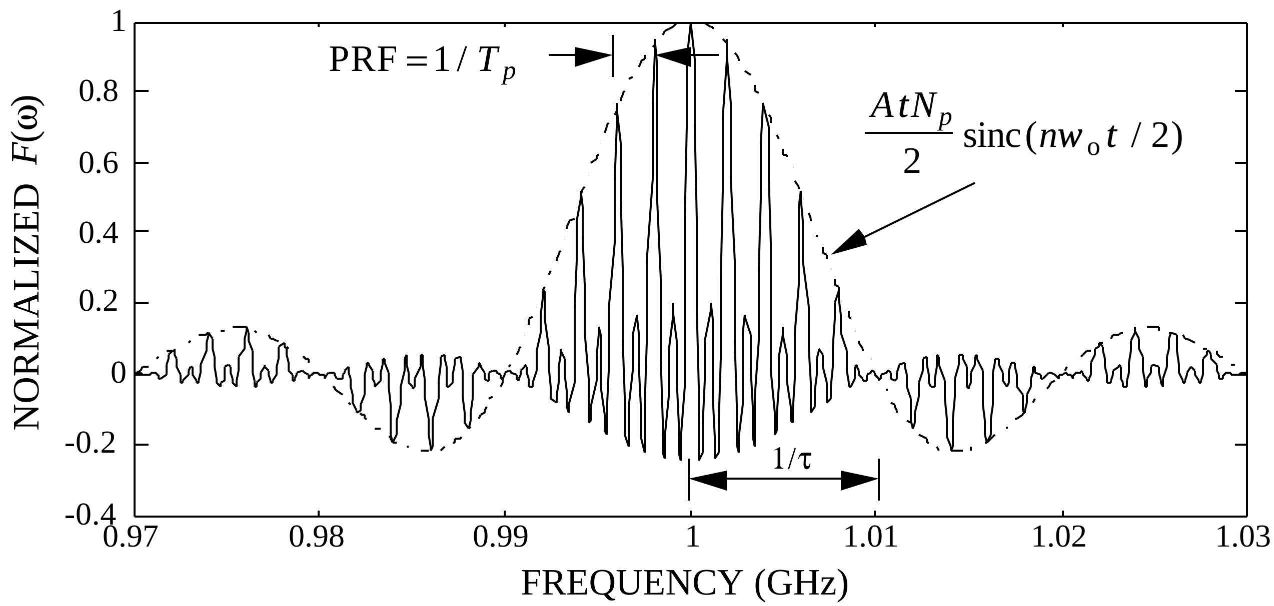
for $|t| \le N_p T_p / 2$. Now we must take the Fourier transform of $f(t)$. The result is:

$$F(w) = \frac{AtN_p}{T_p}\left\{\operatorname{sinc}\left((w+w_c)\frac{N_p T_p}{2}\right) + \sum_{n=1}^{\infty}\operatorname{sinc}\left(nw_o\frac{t}{2}\right)\left[\operatorname{sinc}\left((w+w_c+nw_o)\frac{N_p T_p}{2}\right) + \operatorname{sinc}\left((w+w_c-nw_o)\frac{N_p T_p}{2}\right)\right]\right.$$
$$\left. + \operatorname{sinc}\left((w-w_c)\frac{N_p T_p}{2}\right) + \sum_{n=1}^{\infty}\operatorname{sinc}\left(nw_o\frac{t}{2}\right)\left[\operatorname{sinc}\left((w-w_c+nw_o)\frac{N_p T_p}{2}\right) + \operatorname{sinc}\left((w-w_c-nw_o)\frac{N_p T_p}{2}\right)\right]\right\}$$

Fourier Transform of a Pulse Train (3)

A plot of the positive frequency portion of the spectrum for the following values:

$$N_p = 5, \quad f_c = 1 \text{ GHz}, \quad t = 0.1 \times 10^{-6} \text{ sec}, \quad T_p = 5t \text{ sec.}$$



- The envelope is determined by the pulse width; first nulls at $w_c \pm 2p / t$.
- The "spikes" are located at $w_c \pm n w_o$; the width between the first nulls of each spike is $4p / (N_p T_p)$.
- The number of spikes is determined by the number of pulses.

Response of Networks (1)

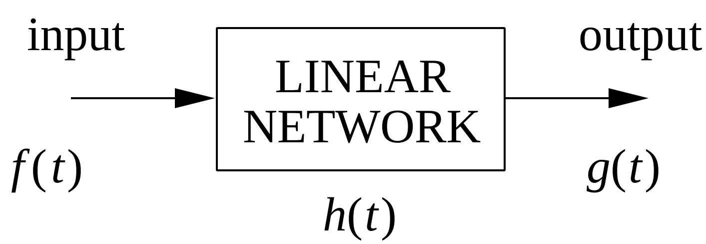
Consider a network that is:

linear, time invariant, stable (bounded output), causal

input: $f(t) \leftrightarrow F(w)$; output: $g(t) \leftrightarrow G(w)$; impulse response: $h(t)$;

transfer function (frequency response): $H(w) = A(w) e^{j\Phi(w)}$

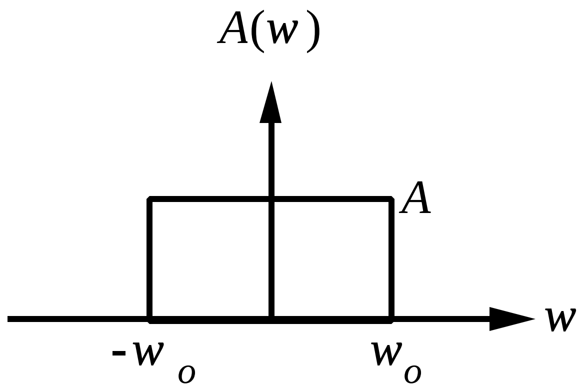
$$g(t) = \frac{1}{2p} \int_{-\infty}^{\infty} H(w) F(w) e^{jwt} dw \leftrightarrow G(w) = H(w) F(w)$$

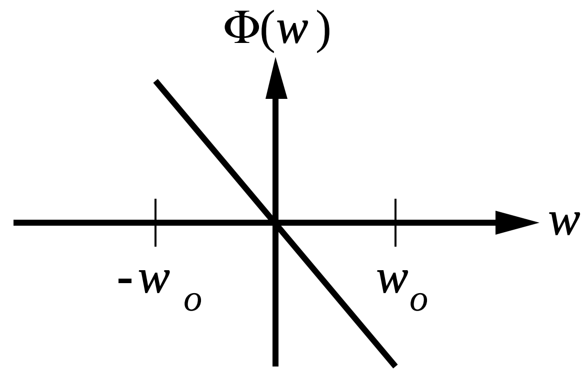


As an example, consider an ideal linear filter (constant amplitude and linear phase).

Response of Networks (2)

Linear filter with cutoff frequency $w_o = 2pB$. What is the output if *f(t)* is a pulse?





Now $F(w) = V_o t \, \mathrm{sinc}(wt/2)$ and $G(w) = V_o t \, \mathrm{sinc}(wt/2) A e^{j\Phi(w)}$. Let $\Phi(w) = -t_o w$:

$$g(t) = \frac{AV_o t}{2p} \int_{-w_o}^{w_o} \mathrm{sinc}\left(\frac{wt}{2}\right) e^{jw(t-t_o)} dw$$

$$g(t) = \frac{AV_o t}{2p} \, 2 \int_{0}^{w_o} \mathrm{sinc}\left(\frac{wt}{2}\right) \cos[w(t-t_o)] \, dw$$

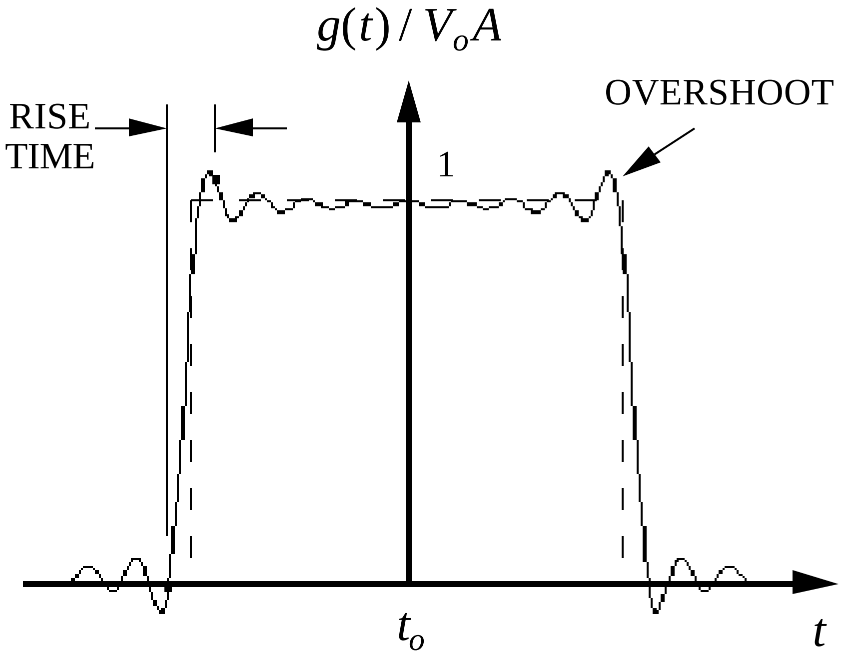
Use trig identity (cos*A* cos*B* =...) and substitution of variables to reduce the integrals to "sine integral" form, which is tabulated.

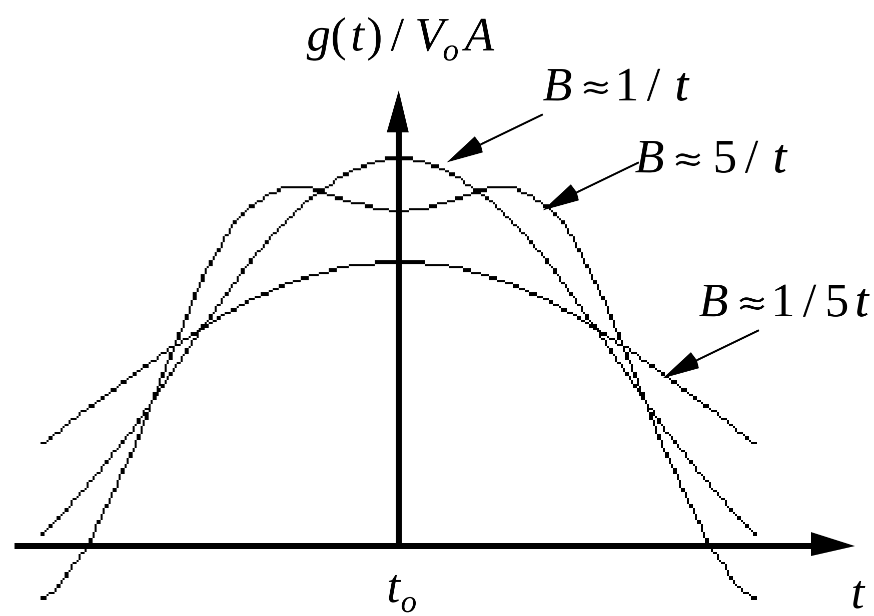
Response of Networks (3)

Final form of the output signal:

$$g(t) = \frac{AV_o}{p} \left\{ \mathrm{Si}\,[w_o(t - t_o + t\ /2)] + \mathrm{Si}\,[w_o(t - t_o - t\ /2)] \right\}$$

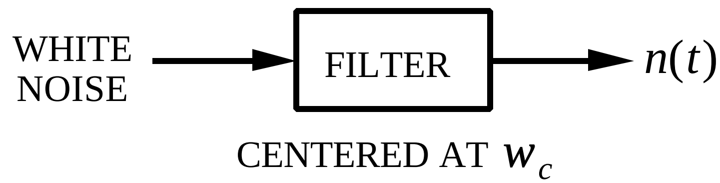
where $\mathrm{Si}(B) = \int_0^B \mathrm{sinc}(a)\,da$. Rule of thumb: $B \approx 1/t$ for good pulse fidelity.





Signals and Noise Through Networks (1)

Consider white noise through an envelope detector and filter

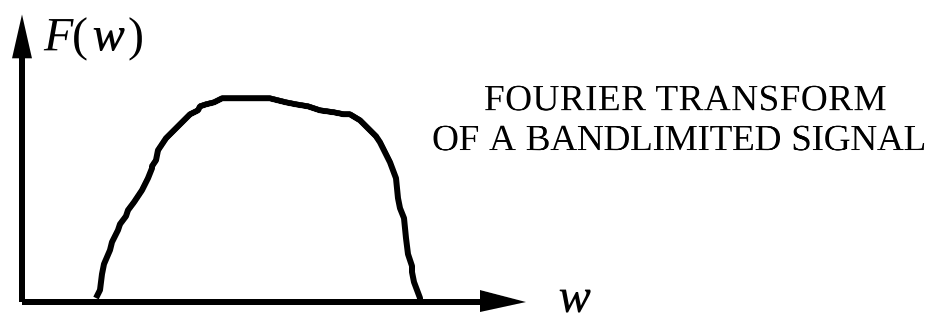


The noise at the output is a complex random variable

$$n(t) = r(t)e^{j(w_c t + q(t))} = x(t)\cos(w_c t) + y(t)\sin(w_c t)$$

The right-hand side is a rectangular form that holds for a narrowband process. The cosine term is the in-phase (I) term and the sine term the quadrature (Q) term.

Assume that the Fourier transform of $s(t)$ is <u>bandlimited</u>, that is, its Fourier transform is zero except for a finite number of frequencies



Signals and Noise Through Networks (2)

If the filter characteristic has the same bandwidth as $s(t)$ and is shifted to the frequency w_c then the carrier modulated signal $s_m(t)$ will pass unaffected. The signal plus noise at the output will be

$$s_{\text{out}}(t) = \underbrace{[s(t) + x(t)]}_{\equiv x',\ \text{in-phase term}} \cos(w_c t) + \underbrace{y(t)\sin(w_c t)}_{\text{quadrature term}}$$

If x and y are normally distributed with zero mean and variance s^2, their joint PDF is the product

$$p_{xy}(x,y) = \frac{e^{-(x^2+y^2)/(2s)}}{2ps^2}$$

When the signal is added to the noise, the random variable x' is shifted

$$p_{x'y}(x', y) = \frac{e^{-[(\overbrace{x+s}^{x'})^2+y^2]/(2s)}}{2ps^2}$$

Now transform to polar coordinates: $x' = r\cos q$ and $y = r\sin q$ and use a theorem from probability theory

$$p_{rq}(r,q)\,dr\,dq = p_{x'y}(x',y)\,dx'\,dy$$

Signals and Noise Through Networks (3)

With some math we find that

$$p_{rq}(r,q) = \frac{1}{2p\,s^{2}} e^{-s^{2}/(2s^{2})} r \exp\left\{-(r^{2} - 2rs\cos q)/(2s^{2})\right\}$$

At the output of the detector we are only dealing with $p_r(r)$, the phase gets integrated out. Thus we end up with the following expression for the PDF of the signal plus noise

$$p_r(r) = \frac{r}{2p\,s^{2}} e^{-(s+r)^{2}/(2s^{2})} \underbrace{\int_{0}^{2p} e^{-rs\cos q/s^{2}}\,dq}_{\equiv 2p\, I_o(rs/s^{2})}$$

where $I_o(\cdot)$ is the modified Bessel function of the first kind (a tabulated function). Final form of the PDF is

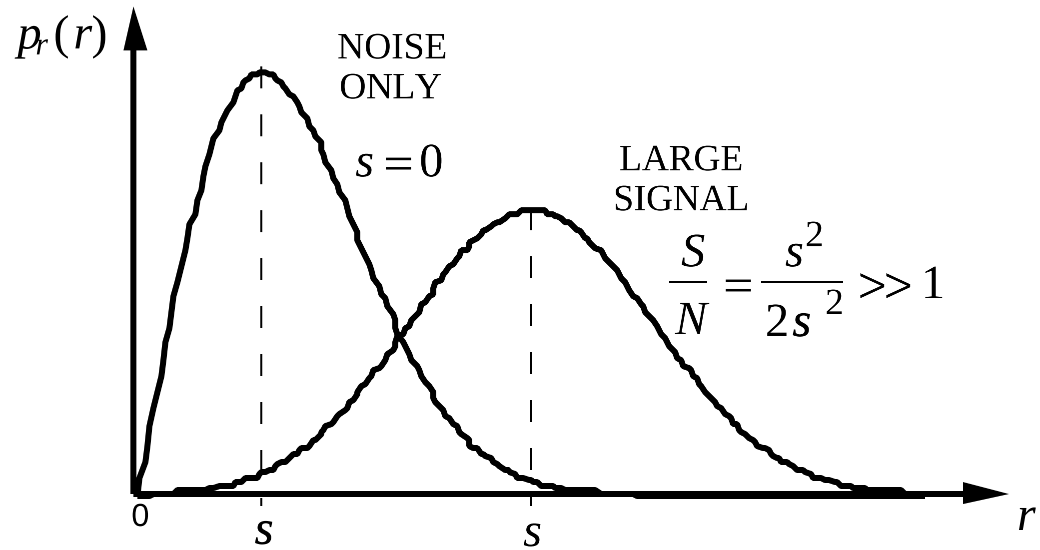
$$p_r(r) = \frac{r}{s^{2}} e^{-(s+r)^{2}/(2s^{2})} I_o(rs/s^{2})$$

This is a Rician PDF. Note that for noise only present $s = 0 \Rightarrow I_o(0) = 1$ and the Rician PDF reduces to a Rayleigh PDF.

(Note that Skolnik has different notation: $s \to A,\quad r \to R,\quad s^{2} \to y_{o}$)

Rician Distribution

Some examples of the Rician distribution:



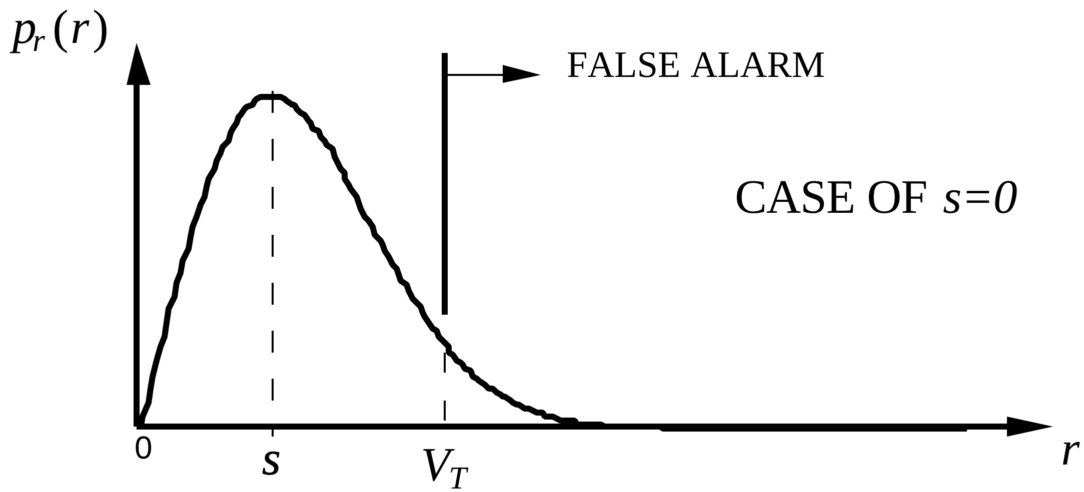
For $s = 0$ the Rician distribution becomes a Rayleigh distribution.

For $\frac{s^2}{2s^2} >> 1$ the Rician distribution approaches a normal distribution.

Probability of False Alarm (1)

For detection, a threshold is set. There are two cases to consider: $s = 0$ and $s \neq 0$.

1. $s = 0$: If the signal exceeds the threshold a target is declared even though there is none present

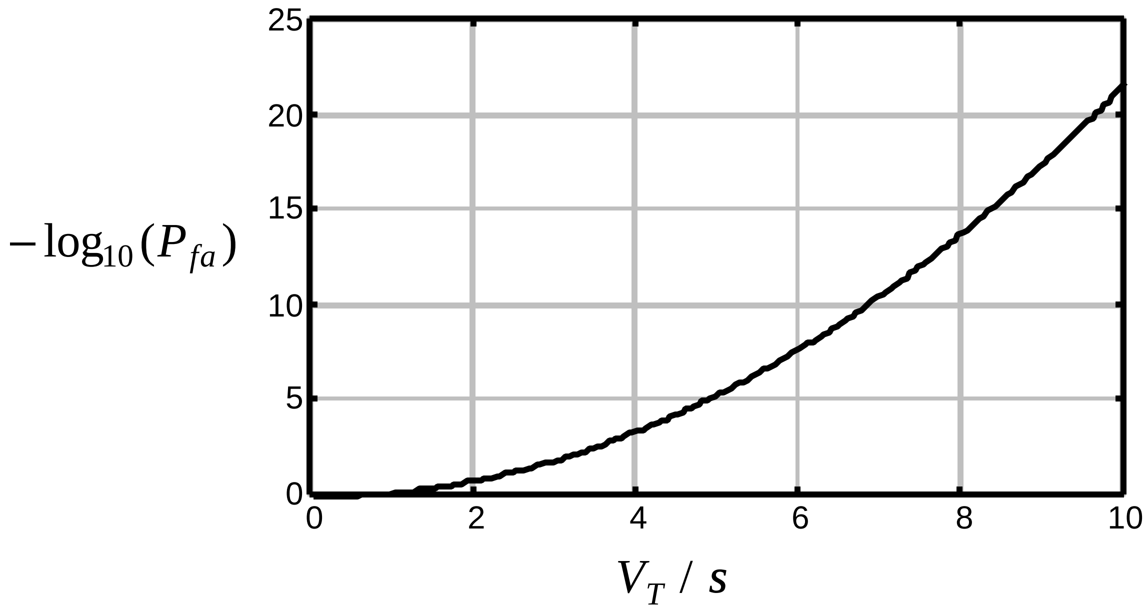


The probability of a false alarm is the area under the PDF to the right of V_T

$$P_{fa} = \int_{V_T}^{\infty} p_r(r) dr = \int_{V_T}^{\infty} \frac{r}{s^2} e^{-r^2/(2s^2)} dr = e^{-V_T^2/(2s^2)}$$

Probability of False Alarm (2)

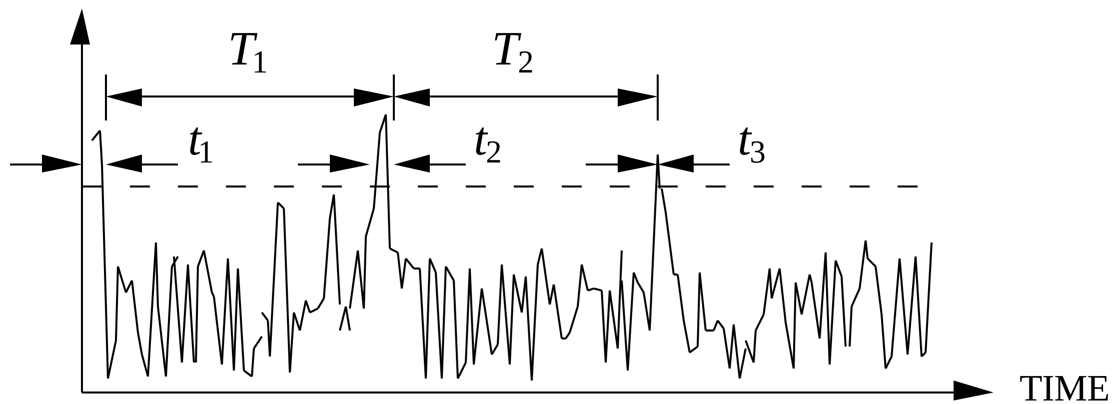
Probability of a false alarm vs. threshold level (this is essentially Figure 2.5 in Skolnik)



Probability of a false alarm can also be expressed as the fraction of time that the threshold is crossed divided by the total observation time:

$$P_{fa} = \frac{\text{time that the threshold has been crossed}}{\text{time that the threshold could have been crossed}}$$

Probability of False Alarm (3)



or, referring to the figure (see Figure 2.4)

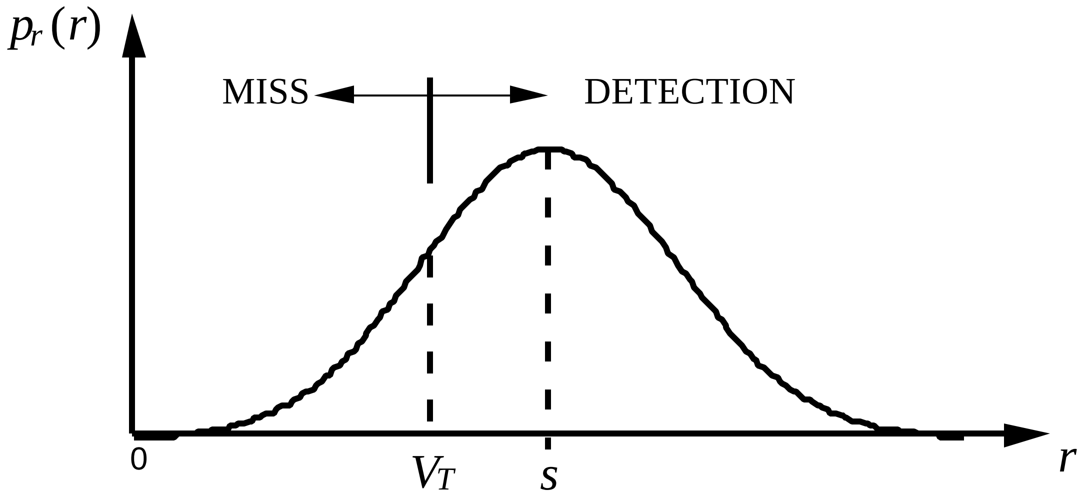
$$P_{fa} = \frac{\sum_n t_n}{\sum_n T_n} = \frac{\langle t_n \rangle}{\langle T_n \rangle} \approx \frac{1/B_n}{T_{fa}}$$

where T_{fa} is the false alarm time, a quantity of more practical interest than P_{fa}. Finally,

$$P_{fa} = \frac{1}{T_{fa} B_n} = e^{-V_T^2/(2s^2)}$$

Probability of Detection (1)

2. $s \neq 0$: There is a target present. The probability of detecting the target is given by the area under the PDF to the right of V_T



$$P_d = \int_{V_T}^{\infty} p_r(r)\,dr = \int_{V_T}^{\infty} \frac{r}{s^2} e^{-(s^2 - r^2)/(2s^2)} I_o(rs/s^2)\,dr$$

The probability of a miss is given by the area under the PDF to the left of V_T, or since the total area under the curve is 1,

$$P_m = 1 - P_d$$

Probability of Detection (2)

For a large SNR = $\frac{s^2}{2s^2} >> 1$ and a large argument approximation for the modified Bessel function can be used in the expression for the PDF: $I_o(x) \approx e^x / (2px)$. The Rician PDF is approximately gaussian

$$p_r(r) = \frac{1}{2ps^2} e^{-(s-r)^2/(2s^2)}$$

Use the standard error function notation

$$\mathrm{erf}(x) = \frac{1}{\sqrt{p}} \int_0^z e^{-u^2} du$$

which is tabulated in handbooks. The probability of detection becomes

$$P_d = \frac{1}{2}\left\{1 - \mathrm{erf}\left(\frac{V_T}{\sqrt{2s^2}} - \sqrt{\mathrm{SNR}}\right)\right\}, \quad \mathrm{SNR} >> 1$$

Recall that

$$P_{fa} = e^{-V_T^2/(2s^2)}$$

Probability of Detection (3)

Eliminate V_T and solve for the SNR

$$\text{SNR} \approx A + 0.12AB + 1.7B$$

where $A = \ln(0.62 / P_{fa})$ and $B = \ln[P_d / (1 - P_d)]$. This is referred to as Albertsheim's approximation, and is good for the range $10^{-7} \le P_{fa} \le 10^{-3}$ and $0.1 \le P_d \le 0.9$

Note: The SNR is not in dB. This equation gives the same results as Figure 2.6

Design Process:

1. choose an acceptable P_{fa} (10^{-3} to 10^{-12})
2. find V_T for the chosen P_{fa}
3. choose an acceptable P_d (0.5 to 0.99)
4. for the chosen P_d and V_T find the SNR

Probability of Detection (4)

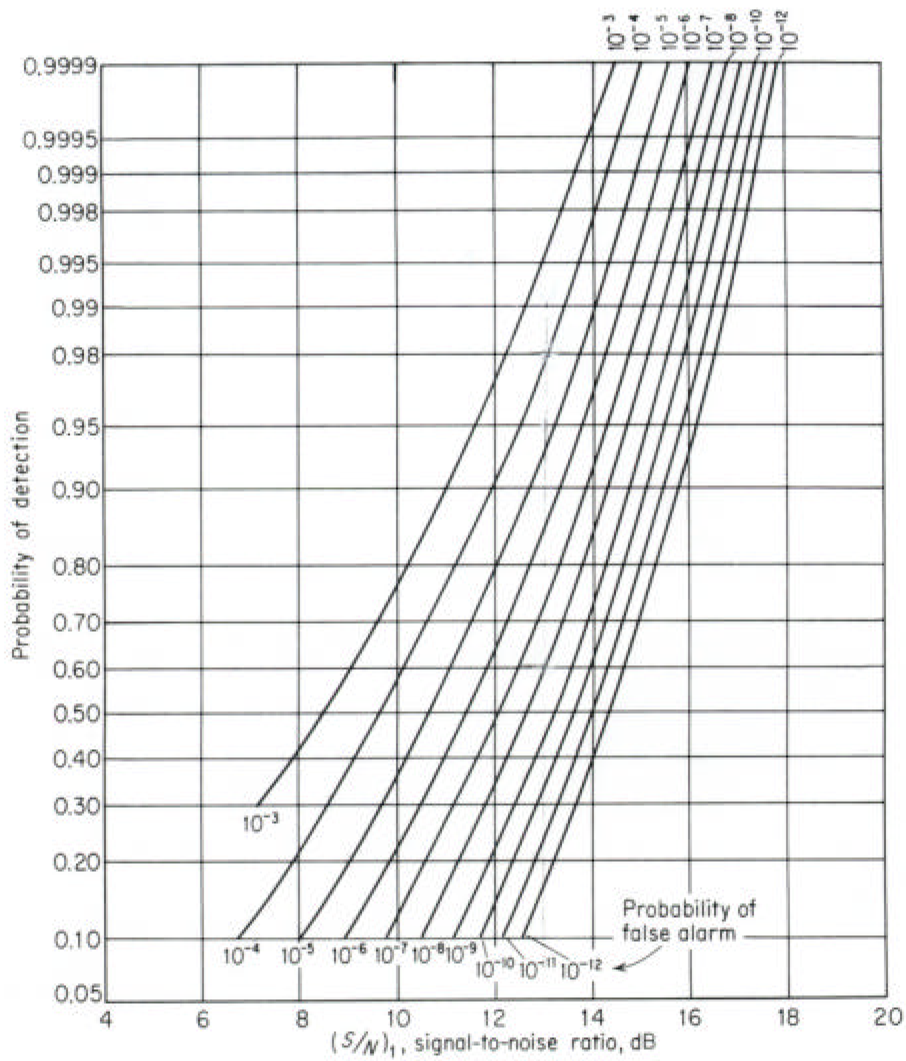
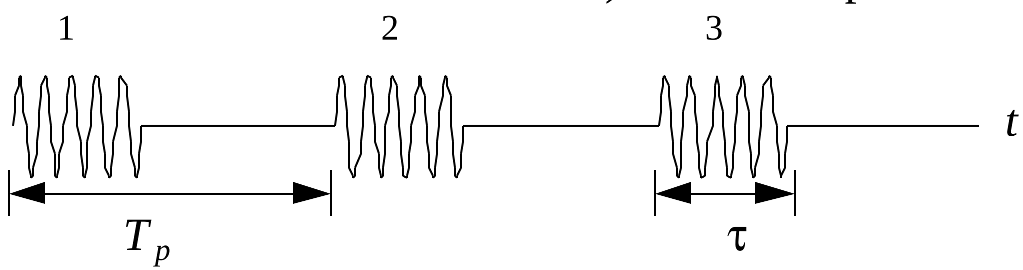


Figure 2.6 in Skolnik

SNR Improvement Using Integration

The SNR can be increased by integrating (summing) the returns from several pulses. Integration can be coherent or noncoherent .

1. Coherent integration (predetection integration): performed before the envelope detector (phase information must be available). Coherent pulses must be transmitted.



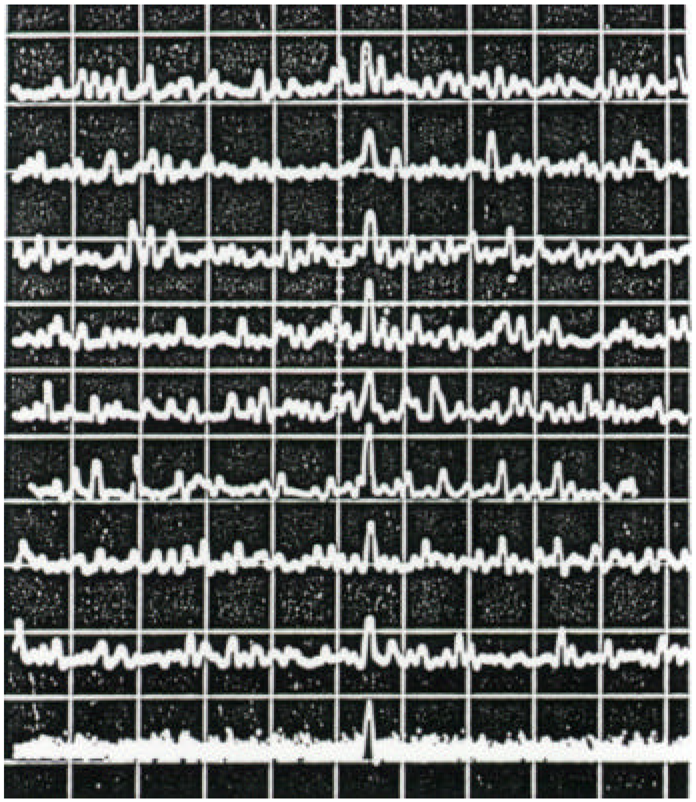
Returns from pulses 1 and 2 are delayed in the receiver so that they contiguous (i.e., they touch) and then summed coherently. The result is essentially a pulse length n times greater than that of a single pulse when n pulses are used. The noise bandwidth is $B_n \approx 1/(nt)$ compared to $B_n \approx 1/t$ for a single pulse. Therefore the noise has been reduced by a factor n and

$$\text{SNR} \propto \frac{n(P_r)_1}{B_n}$$

where $(P_r)_1$ is the received power for a single pulse. The improvement in SNR by coherently integrating n pulses is n. This is also referred to as a perfect integrator.

Illustration of Coherent Integration

For coherent integration to be effective the propagation medium and target cannot randomize the phases of the pulses. The last trace shows the integrated signal.



SNR Improvement Using Integration

2. Noncoherent integration (postdetection integration): performed after the envelope detector. The magnitudes of the returns from all pulses are added.

Procedure:

- N samples (pulses) out of the detector are summed
- the PDF of each sample is Ricean
- the joint PDF of the N samples is obtained from a convolution of Ricean PDFs
- once the joint PDF is known, set V_T and integrate to find expressions for P_{fa} and P_d

Characteristics:

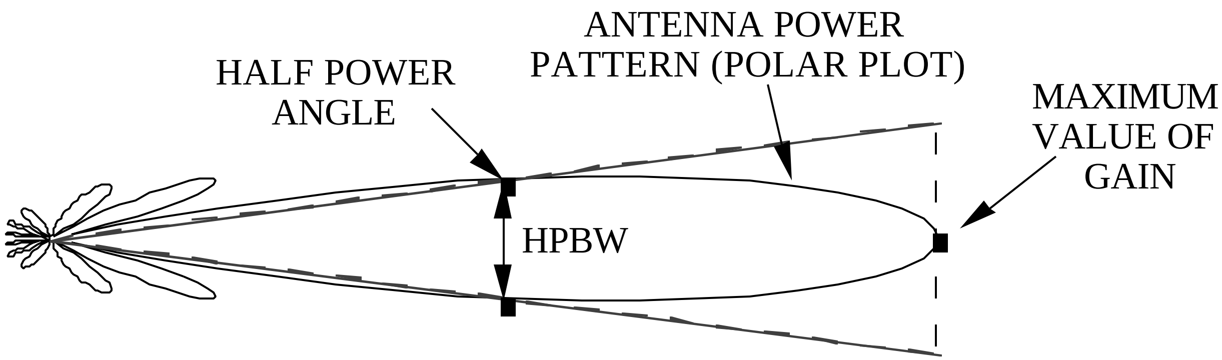
- noise never sums to zero as it can in the coherent case
- does not improve signal-to-clutter ratio
- only used in non-coherent radars (most modern radars are coherent)

Improvement:

$$\mathrm{SNR} \propto \frac{n_{\mathrm{eff}} (P_r)_1}{B_n}$$

where the effective number of pulses is $n_{\mathrm{eff}} \approx n$ for small n and $n_{\mathrm{eff}} \approx \sqrt{n}$ for large n

Approximate Antenna Model



For systems analysis an approximation of the actual antenna pattern is sufficient. We ignore the beam shape and represent the antenna pattern by

$$G = \begin{cases} G_o\,, & \text{within HPBW } (= q_B\,) \\ 0, & \text{outside of HPBW} \end{cases}$$

where G_o is the maximum antenna gain. Thus the sidelobes are neglected and the gain inside of the half power beamwidths is constant and equal to the maximum value.

Number of Pulses Available

The antenna beam moves through space and only illuminates the target for short periods of time. Use the approximate antenna model

$$G(q) = \begin{cases} G_o, |q| < q_H \; (= q_B / 2) \\ 0, \quad \text{else} \end{cases}$$

where G_o is the maximum gain and q_H the half power angle and q_B the half power beamwidth (HPBW). If the aperture has a diameter D and uniform illumination, then $q_B \approx l / D$. The beam scan rate is w_s in revolutions per minute or $\frac{dq_s}{dt} = \dot{q}_s$ in degrees per second. (The conversion is $\frac{dq_s}{dt} = 6w_s$.) The time that the target is in the beam (<u>dwell time</u>[1] or <u>look</u>) is

$$t_{\text{ot}} = q_B / \dot{q}_s$$

and the number of pulses that will hit the target in this time is

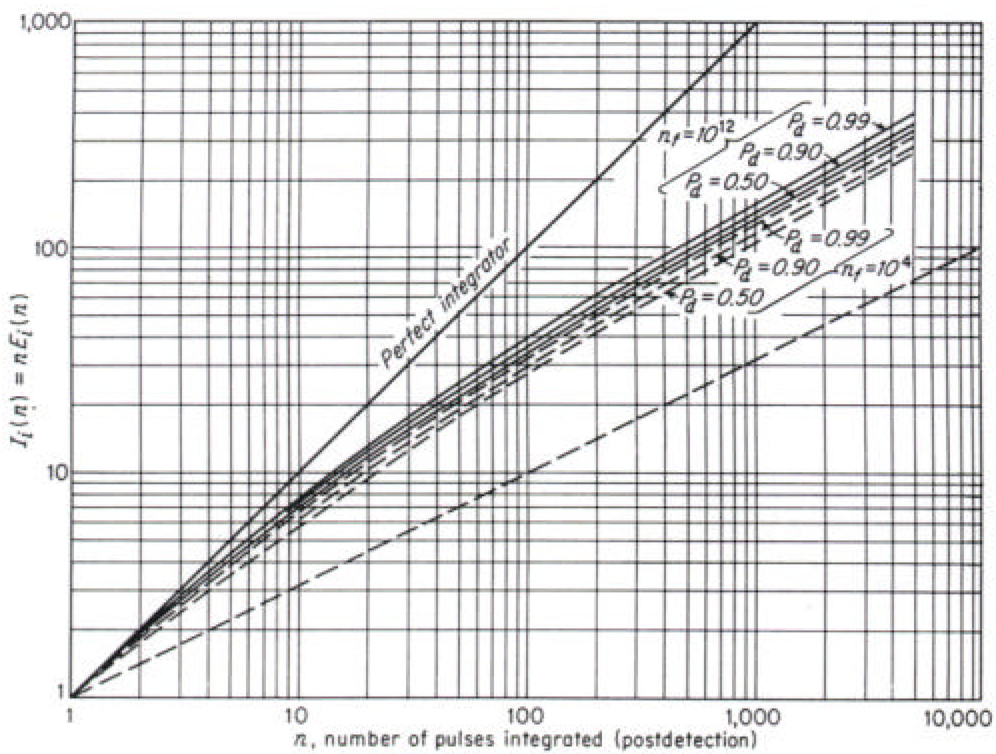
$$n_B = t_{\text{ot}} f_p$$

---

[1] The term dwell time does not have a standardized definition. It can also mean the time that a pulse train is hitting the target, or data collection time. By this definition, if multiple PRFs are used while the target is in the beam, then there can be multiple dwells per look.

Integration Improvement Factor

The <u>integration efficiency</u> is defined as $E_i(n) = \dfrac{\mathrm{SNR}_1}{n\,(\mathrm{SNR}_n)}$ where SNR_1 is the signal-to-noise ratio for one pulse and SNR_n is that to obtain the same P_d as SNR_1 when integrating n pulses. The improvement in signal-to-noise ratio when n pulses are integrated is the <u>integration improvement factor</u>: $I_i(n) = n\,E_i(n)$



Skolnik Figure 2.7 (a)

- for a square law detector
- <u>false alarm number</u>

$$n_{fa} = 1/P_{fa} = T_{fa}B_n$$

RRE for Pulse Integration

To summarize:

Coherent (predetection) integration: $E_i(n) = 1$ and $I_i(n) = n$

$$\mathrm{SNR}_n = \frac{1}{n}\mathrm{SNR}_1$$

Noncoherent (postdetection) integration: $I_i(n) < n$

In the development of the RRE we used the single pulse SNR; that is

$$(\mathrm{SNR_{out}})_{\min} = \mathrm{SNR}_1$$

For n pulses integrated

$$(\mathrm{SNR_{out}})_{\min} = \mathrm{SNR}_n = \frac{\mathrm{SNR}_1}{nE_i(n)}$$

This quantity should be used in the RRE.

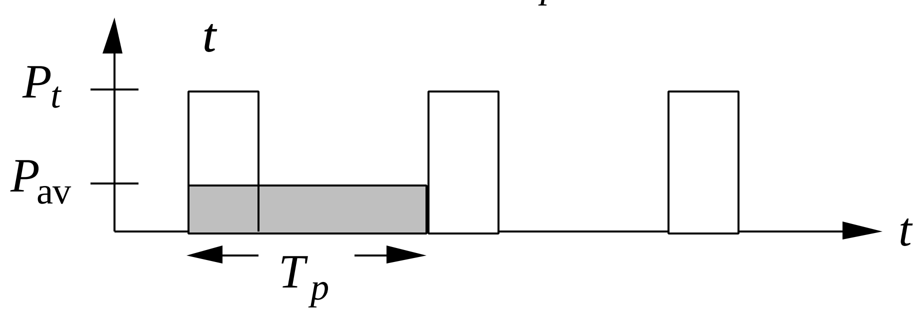
RRE for Pulse Integration

Integrating pulses increases the detection range of a radar by increasing the signal-to-noise ratio

$$R_{\max}^4 = \frac{P_t G_t A_{er} \sigma \, n \, E_i(n)}{(4\pi)^2 k \, T_s B_n (\mathrm{SNR})_1}$$

where $(\mathrm{SNR})_1$ is the signal-to-noise ratio for single pulse detection.

In the RRE, P_t is the peak pulse envelope power. The duty cycle is the fraction of the interpulse period that the pulse is on $(= \tau / T_p)$



P_{av} is the average power: computed as if the energy in the pulse $(= P_t \tau)$ were spread over the entire interpulse period T_p: $P_{\mathrm{av}} = P_t \tau / T_p$. Using the average power gives a form of the RRE that is waveform independent.

RRE for Pulse Integration

Note that ordinarily P_t is the time-averaged power (one half the maximum instantaneous) when working with a pulse train waveform

$$R_{\max}^4 = \frac{P_t G_t A_{er} s\, n E_i(n)}{(4p)^2 k \mathrm{T}_s B_n \mathrm{SNR}_1}$$

<u>Design process using Figures 2.6 and 2.7</u>:

1. choose an acceptable P_{fa} from $P_{fa} = 1/(T_{fa} B_n)$
2. choose an acceptable P_d $(0.5 \text{ to } 0.99)$ and with P_{fa} find SNR_1 from Figure 2.6
3. for the chosen P_d, and <u>false alarm number</u> $n_{fa} = 1/P_{fa} = T_{fa} B_n$ find the integration improvement factor, $I_i(n)$, from Figure 2.7(a)

<u>Design process using Albertsheim's approximation</u>:

The SNR per pulse when n pulses are integrated noncoherently is approximately

$$\mathrm{SNR}_n, \mathrm{dB} \approx -5 \log n + \left(6.2 + 4.54/\sqrt{n + 0.44}\right) \log(A + 0.12AB + 1.7B)$$

where $A = \ln(0.62/P_{fa})$ and $B = \ln[P_d/(1 - P_d)]$, $10^{-7} \le P_{fa} \le 10^{-3}$, and $0.1 \le P_d \le 0.9$

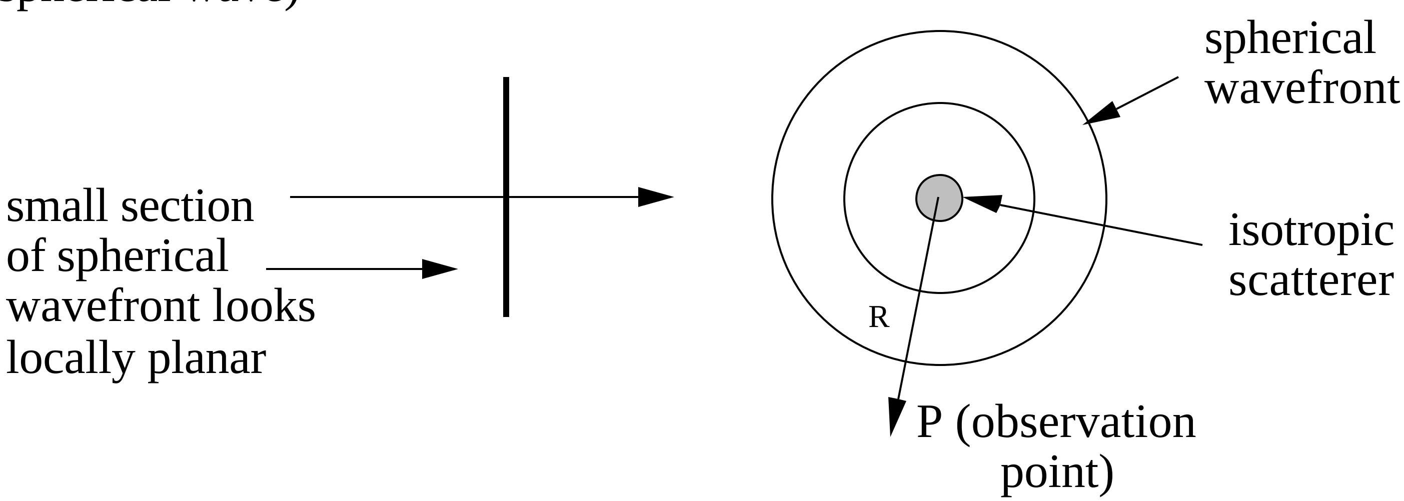
Radar Cross Section (1)

Definition of radar cross section (RCS)

$$\sigma = \frac{\text{power reflected toward source per unit solid angle}}{\text{incident power density}/4\pi} = \lim_{R \to \infty} 4\pi R^2 \frac{|\vec{W}_s|}{|\vec{W}_i|}$$

$\vec{W}_i$ = power density incident on the target (Poynting vector)
$\vec{W}_s$ = scattered power density from target returned to the radar

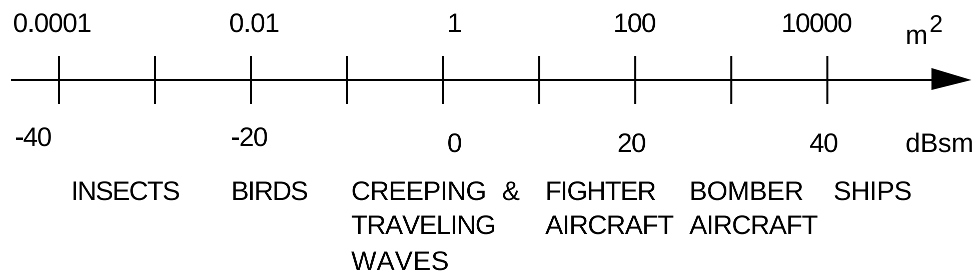
Expressed in decibels relative to a square meter (dBsm): $\sigma_{\text{dBsm}} = 10\log_{10}(\sigma)$.
RCS is used to describe a target's scattering properties just as gain (or directivity) is used for an antenna. An <u>isotropic scatterer</u> will scatter equally in all directions (i.e., a spherical wave)



Radar Cross Section (2)

RCS is a function of:
1. wave properties (polarization and frequency)
2. aspect angle (viewing angle)

Typical values of RCS:



Consider an arbitrary target with a "characteristic dimension," L. The RCS has three distinct frequency regions as illustrated by the RCS of a sphere:

1. low frequencies (Rayleigh region): $kL << 1$

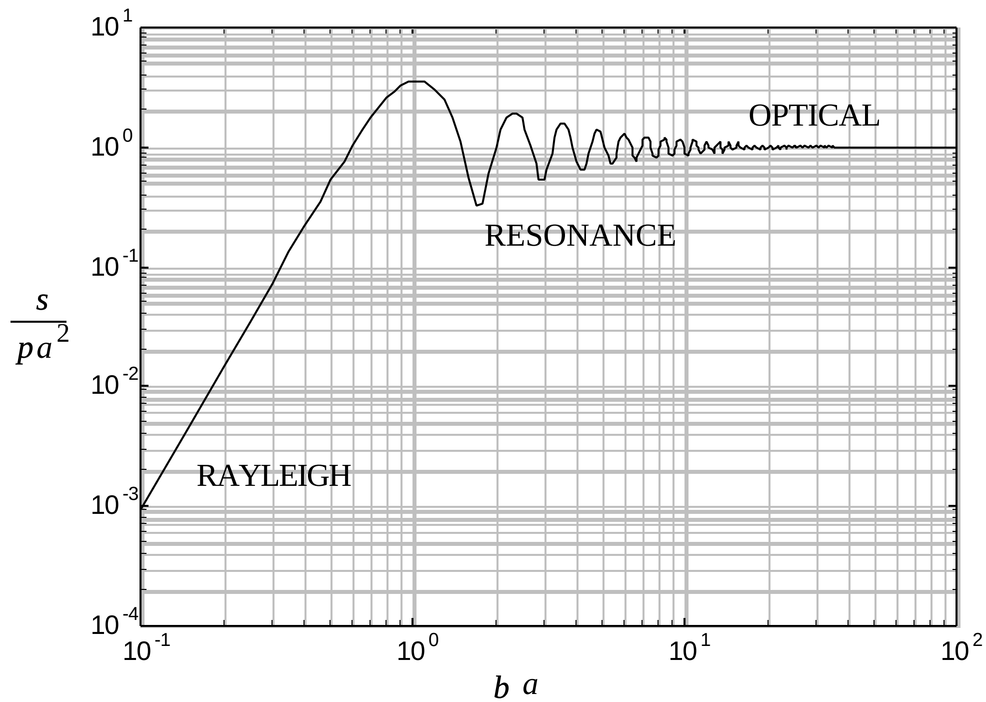
   $$s \propto 1/l^4,\ s \text{ vs } kL \text{ is smooth, } s \propto (\text{volume})^2$$

2. resonance region (Mie region): $kL \approx 1$, s vs kL oscillates

3. high frequencies (optical region): $kL >> 1$, s vs kL is smooth and may be independent of l

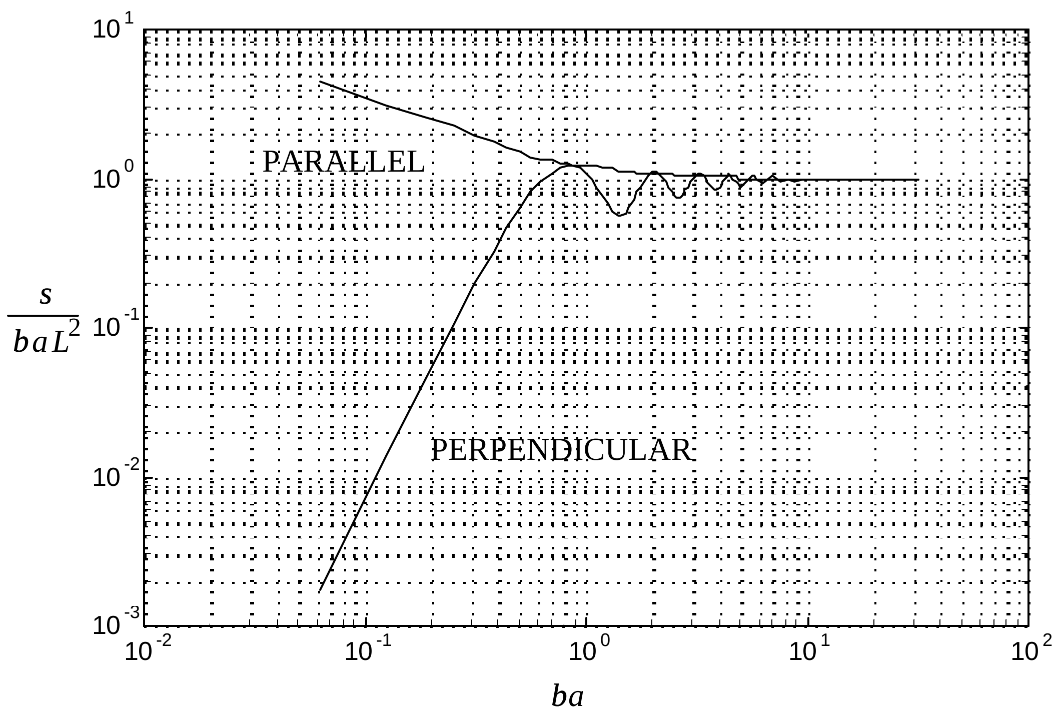
Radar Cross Section of a Sphere

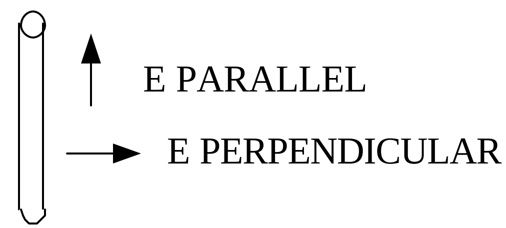
Monostatic RCS of a sphere, $b = 2p\ /\ l\ (= k)$, $a = \text{radius}$, illustrates the three frequency regions: (1) Rayleigh, (2) Mie, and (3) optical



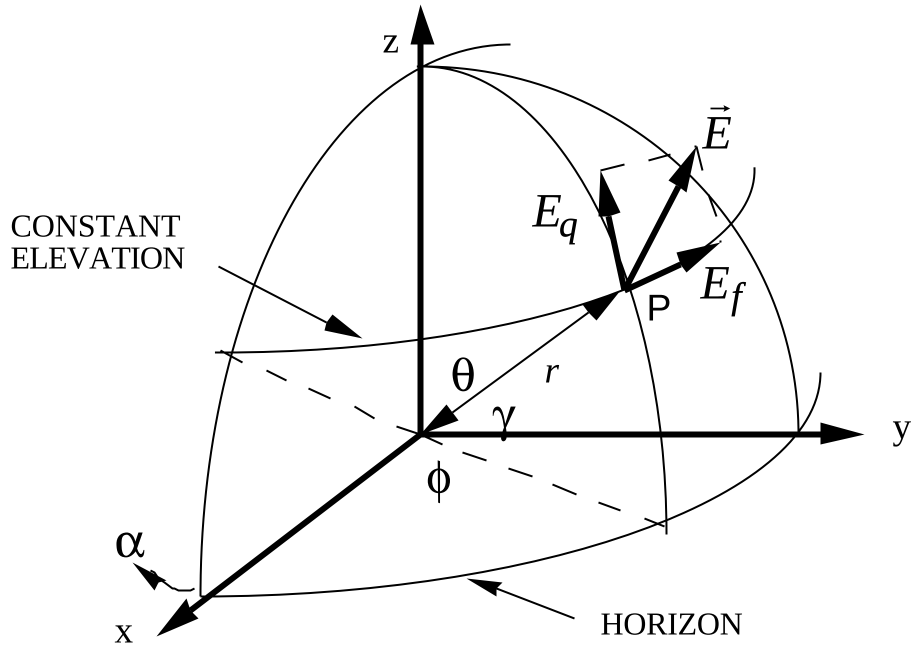
Radar Cross Section of a Cylinder

Monostatic RCS of a cylinder, $b = 2p / l \ (= k)$, $a =$ radius, illustrates dependence on polarization. L is the length.





Target Scattering Matrix (1)



Arbitrary wave polarization can be decomposed into spherical components.

$$\vec{E}_i = E_{iq}\hat{q} + E_{if}\hat{f}$$

Similarly for the scattered field

$$\vec{E}_s = E_{sq}\hat{q} + E_{sf}\hat{f}$$

Target Scattering Matrix (2)

Define the RCS for combinations of incident and scattered wave polarizations

$$s_{pq} = \lim_{R\to\infty} 4pR^2 \frac{\left|\vec{E}_{sp}\right|^2}{\left|\vec{E}_{iq}\right|^2}$$

where $p,q = q$ or f. The index p denotes the polarization of the scattered wave and q the polarization of the incident wave. In general, a scattering matrix can be defined that relates the incident and scattered fields

$$\begin{bmatrix} E_{sq} \\ E_{sf} \end{bmatrix} = \frac{1}{\sqrt{4pR^2}} \begin{bmatrix} s_{qq} & s_{qf} \\ s_{fq} & s_{ff} \end{bmatrix} \begin{bmatrix} E_{iq} \\ E_{if} \end{bmatrix}$$

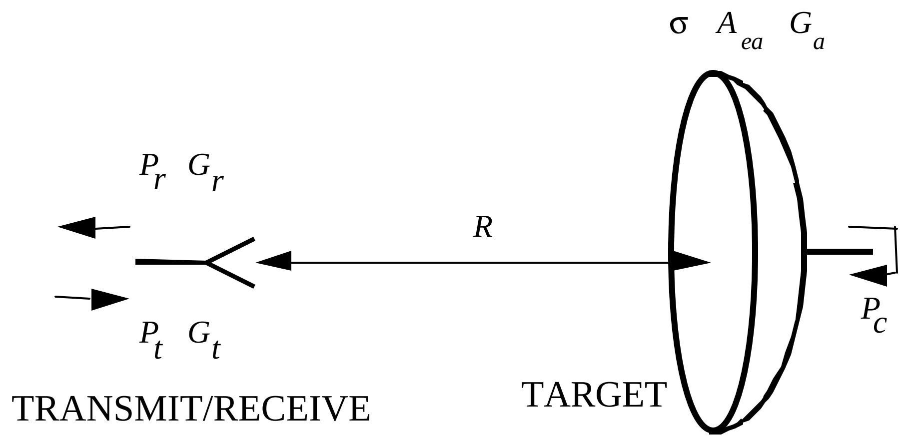
where

$$s_{pq} = \sqrt{s_{pq}}\, e^{jy_{pq}}, \quad y_{pq} = \tan^{-1}\left\{ \frac{\mathrm{Im}(E_{sp}/E_{iq})}{\mathrm{Re}(E_{sp}/E_{iq}} \right\}$$

Copolarized RCS: $p = q$ cross polarized: $p \neq q$

Example: Antenna as a Radar Target

Antenna at range R



Received power is

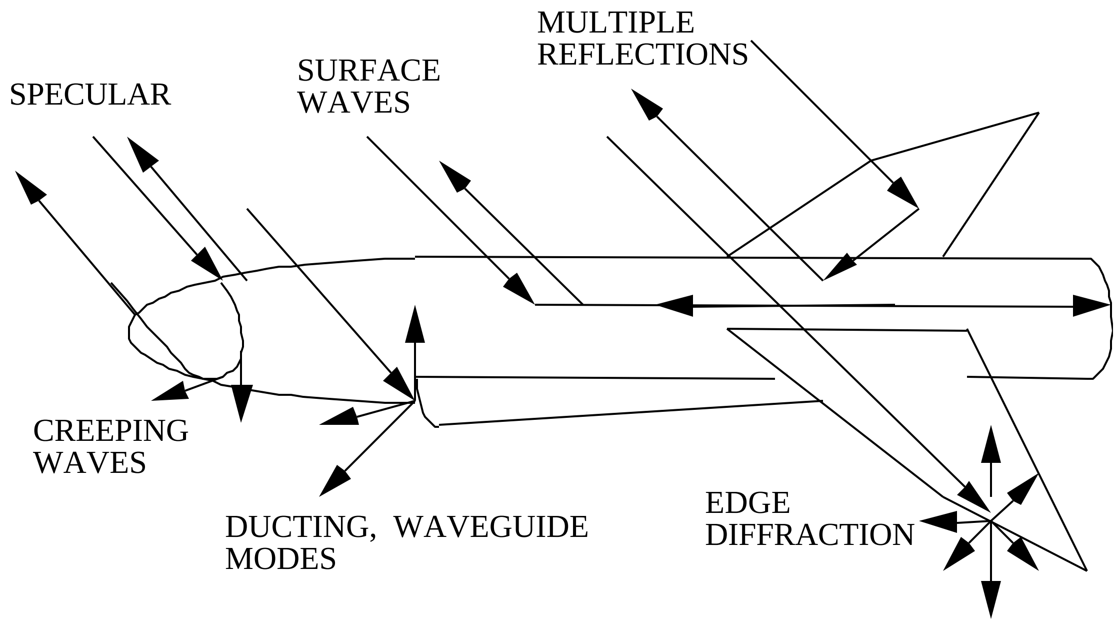
$$P_r = \underbrace{\left(\frac{P_t G_t}{4pR^2}\right)(A_{ea})\left(\frac{4pA_{ea}}{l^2}\right)}_{P_s}\left(\frac{1}{4pR^2}\right)(A_{er})$$

Compare this result with the original form of the radar equation and find that

$$A_{ea}^2 = \frac{l^2 s}{4p} \Rightarrow s = \frac{4pA_{ea}^2}{l^2} \approx \frac{4pA_p^2}{l^2}$$

Important result -- applies to any large relatively flat scattering area.

Scattering Mechanisms

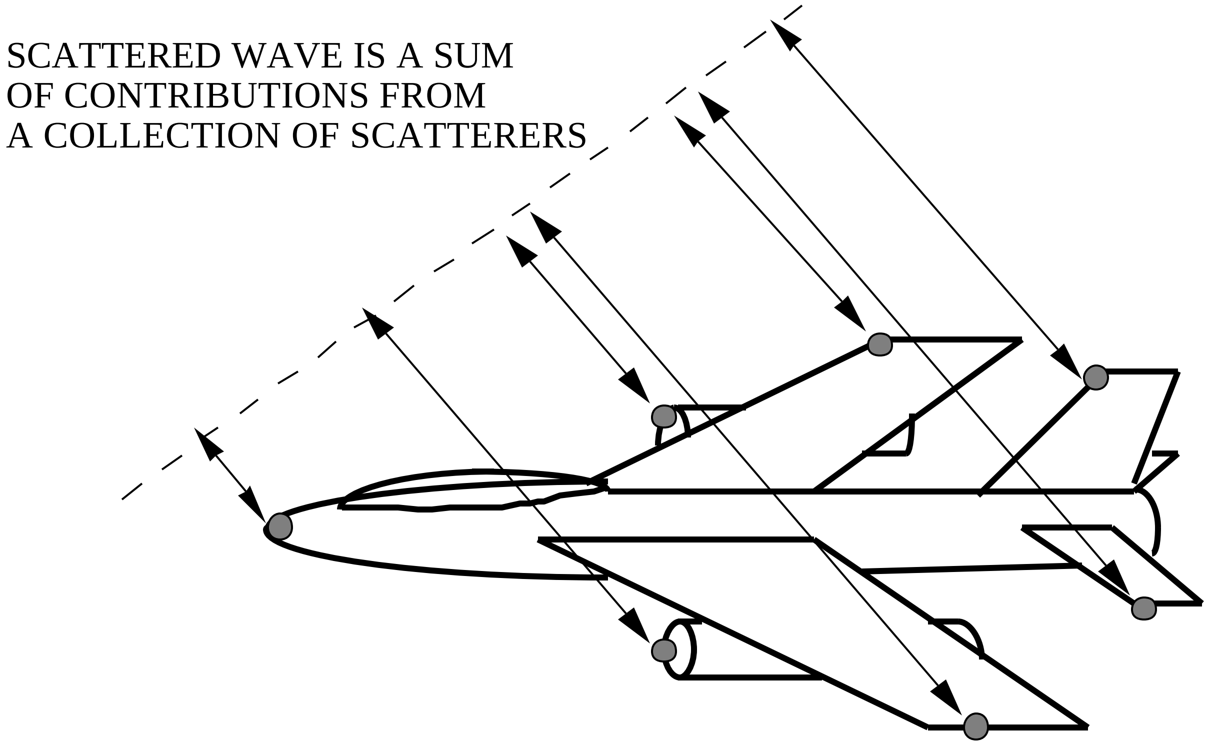


Scattering mechanisms are used to describe wave behavior. Especially important for standard radar targets (planes, ships, etc.) at radar frequencies:

specular = "mirror like" reflections that satisfy Snell's law
surface waves = the body acts like a transmission line guiding waves along its surface
diffraction = scattered waves that originate at abrupt discontinuities (e.g., edges)

Scattering Sources for a Complex Target

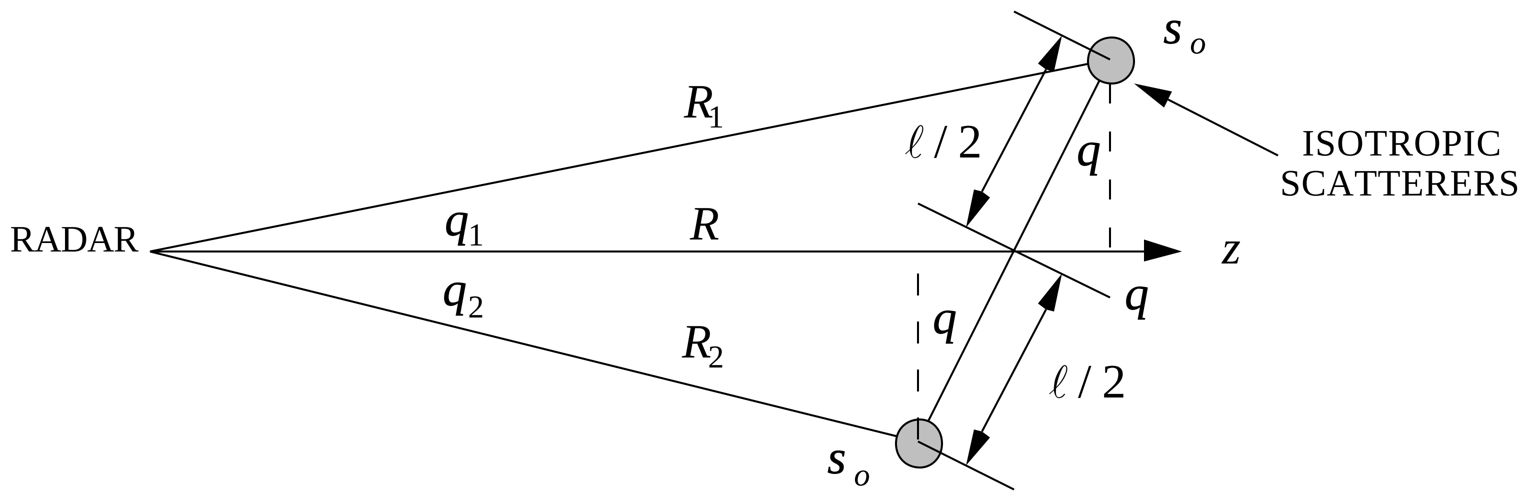
Typical for a target in the optical region (i.e., target large compared to wavelength)



In some directions all scattering sources may add in phase and result in a large RCS.
In other directions some sources may cancel other sources resulting in a very low RCS.

Two Sphere RCS (1)

Consider the RCS obtained from two isotropic scatterers (approximated by spheres).



Law of cosines:

$$R_1 = \sqrt{R^2 + (\ell/2)^2 - 2R(\ell/2)\cos(q + p/2)} = R\sqrt{1 + (\ell/2R)^2 + 2(\ell/2R)\sin q}$$

$$R_2 = \sqrt{R^2 + (\ell/2)^2 - 2R(\ell/2)\cos(q - p/2)} = R\sqrt{1 + (\ell/2R)^2 - 2(\ell/2R)\sin q}$$

Let $a = \ell \sin q / R$ and note that

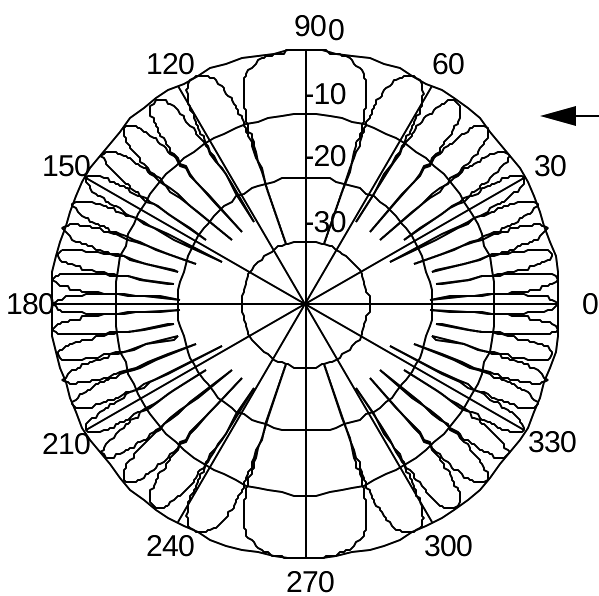
$$(1 \pm a)^{1/2} = 1 \pm \frac{1}{2} a \mp \underbrace{\frac{3}{8} a^2 \pm \cdots}_{\text{NEGLECT SINCE } a \ll 1}$$

Two Sphere RCS (2)

Keeping the first two terms in each case leads to the approximate expressions $R_1 \approx R + (\ell/2)\sin q$ and $R_2 \approx R - (\ell/2)\sin q$. Total received field for two spheres is:

$$P_{\text{tot}} \propto \left(\frac{\sqrt{s_o}}{R_1^2} e^{-j2kR_1} + \frac{\sqrt{s_o}}{R_2^2} e^{-j2kR_2} \right)^2 = \frac{s_o}{R^4} \underbrace{\left(e^{-jk\ell \sin q} + e^{+jk\ell \sin q} \right)^2}_{=4\cos^2(k\ell \sin q)}$$

where $k = 2p / l$. The "effective RCS" of the two spheres is $s_{\text{eff}} = 4 s_o \cos^2(k\ell \sin q)$. This can easily be extended to N spheres.

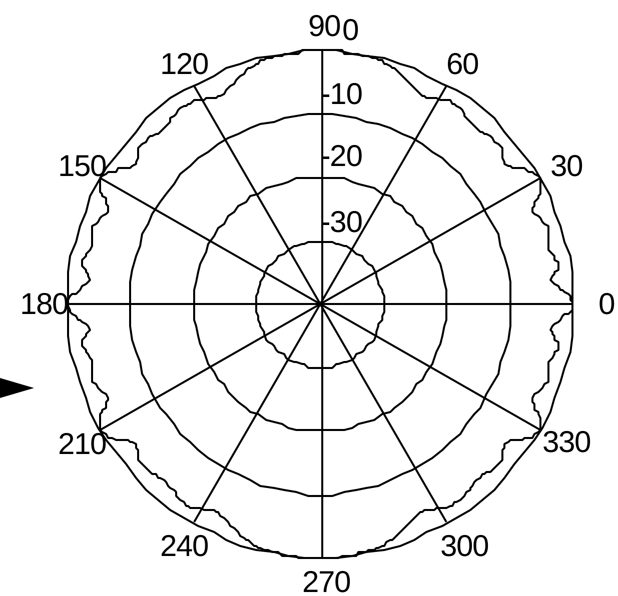


NORMALIZED PLOT OF $s_{\text{eff}} / s_{\text{o}}$ FOR 2 SPHERES SPACED $10\,l$

$s_{on} = (1,1)$

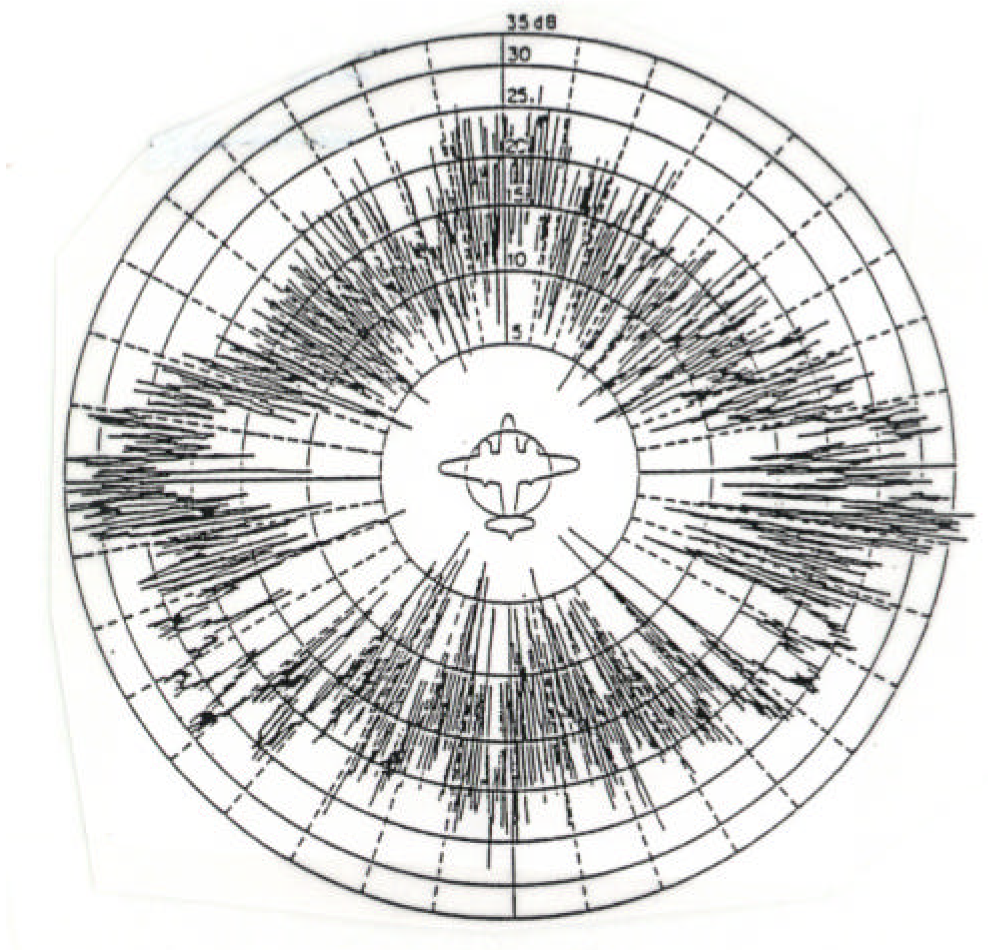
NORMALIZED PLOT OF $s_{\text{eff}} / s_{\text{o}}$ FOR 7 SPHERES SPACED $2\,l$

$s_{on} = (1,1,1,10,1,1,1)$



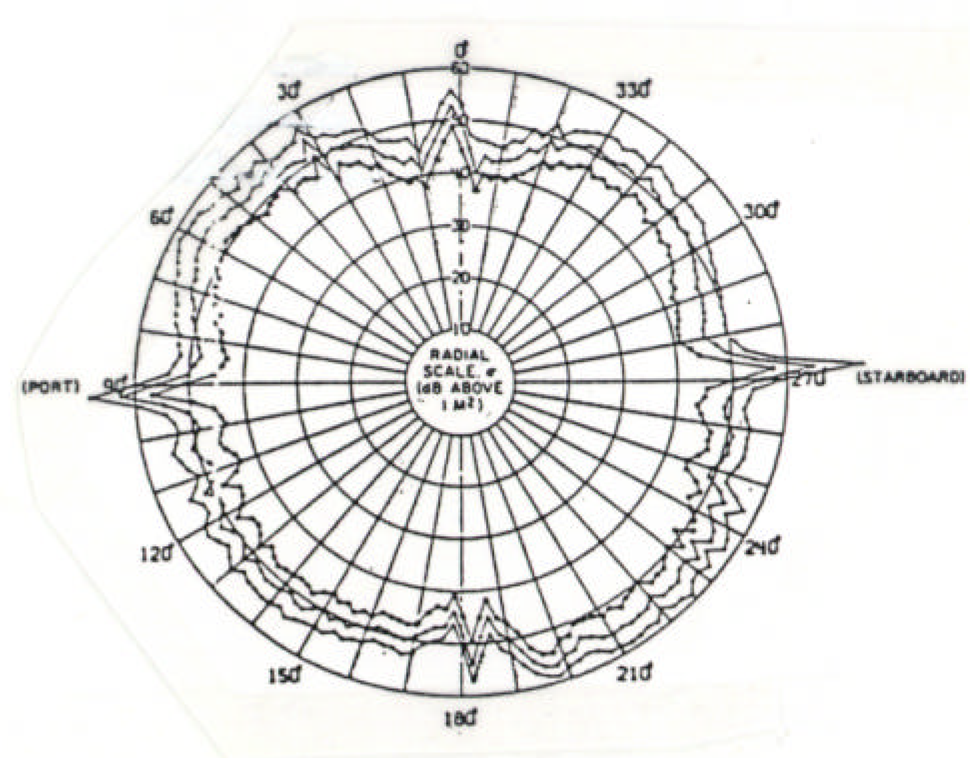
RCS of a Two Engine Bomber

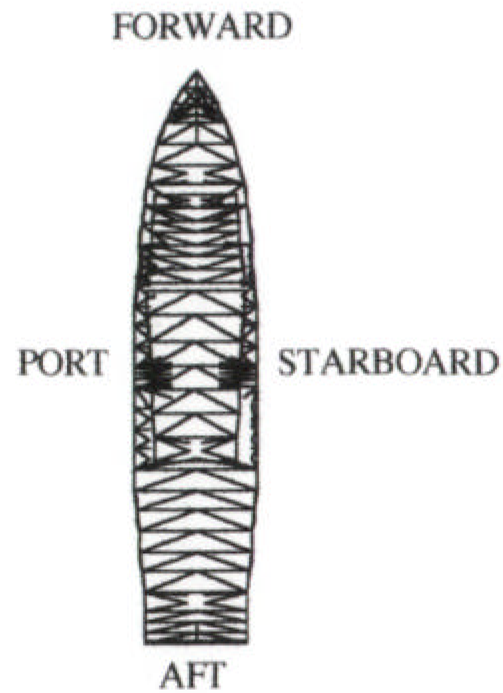
S-Band (3000 MHz) Horizontal Polarization Maximum RCS = 40 dBsm



RCS of a Naval Auxiliary Ship

S-Band (2800 MHz) Horizontal Polarization Maximum RCS = 70 dBsm





(Curves correspond to 20th, 50th and 80th percentiles)

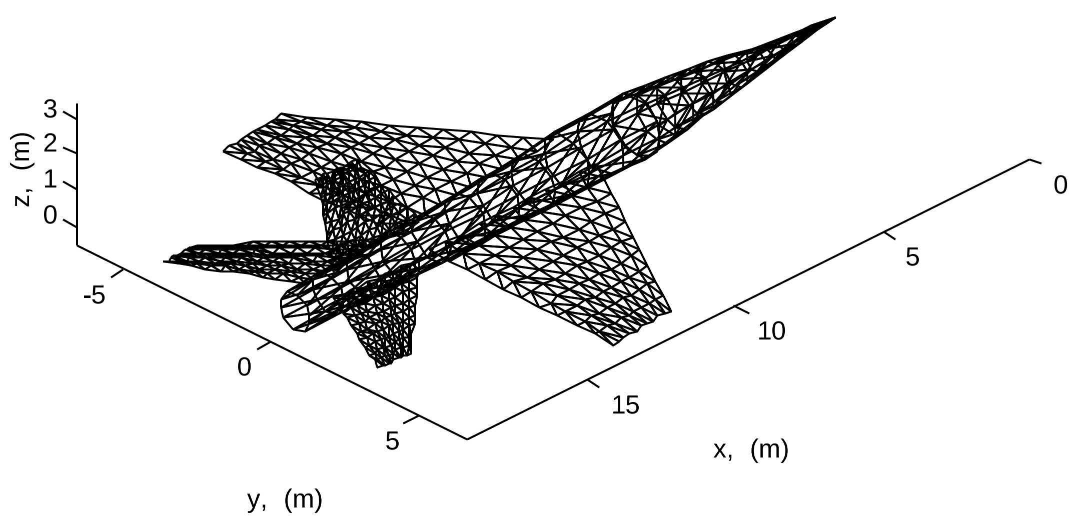
RCS of a Geometrical Components Jet

Frequency = 1 GHz
Bistatic and monostatic azimuth patterns
Bistatic advantages:

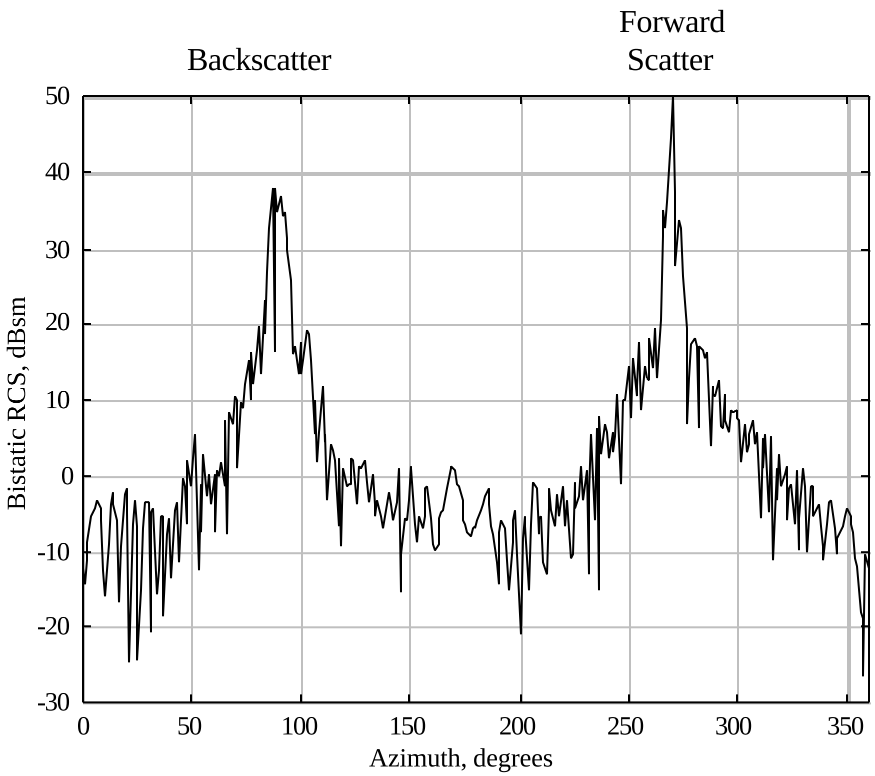
- always a large RCS in the forward direction ($\phi = \phi_i + 180^\circ$)
- forward scatter can be larger than backscatter
- lobes are wider (in angular extent)

Bistatic disadvantage:

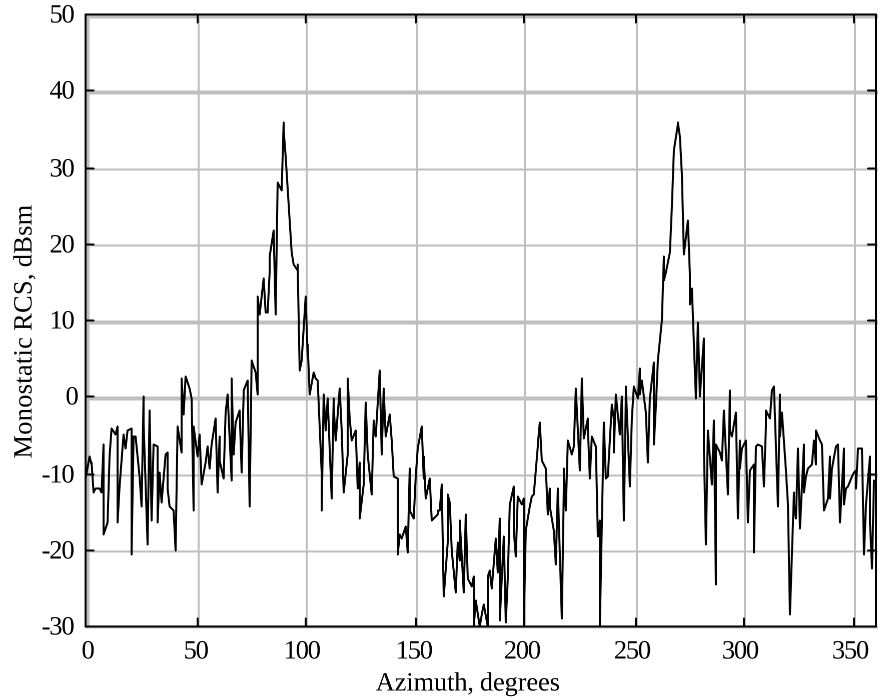
- restricted radar transmit/receive geometry



Geometrical Components Jet



Bistatic, $f_i = 90^\circ$



Monostatic

Fluctuating Targets

The target return appears to vary with time due to sources other than a change in range:

1. meteorological conditions and path variations
2. radar system instabilities (platform motion and equipment instabilities)
3. target aspect changes

For systems analysis purposes we only need to know the "gross" behavior of a target, not the detailed physics behind the scattering. Let the s be a random variable with a probability density function (PDF) that depends on the factors above. Two PDFs are commonly used:

1. $p(s) = \frac{1}{\bar{s}} e^{-s/\bar{s}}$ (this is a negative exponential PDF)

These are Rayleigh targets which consist of many independent scattering elements of which no single one (or few) predominate.

2. $p(s) = \frac{4s}{\bar{s}^2} e^{-2s/\bar{s}}$

These targets have one main scattering element that dominates, together with smaller independent scattering sources.

Swerling Types

Using PDFs #1 and #2 we define four Swerling target types:

Type I: PDF #1 with slow fluctuations (scan-to-scan)
Type II: PDF #1 with rapid fluctuations (pulse-to-pulse)
Type III: PDF #2 with slow fluctuations (scan-to-scan)
Type IV: PDF #2 with rapid fluctuations (pulse-to-pulse)

When the target scattering characteristics are unknown, Type I is usually assumed.

Now we modify our design procedure (same as in Skolnik's 2nd edition)

1. Find the SNR for a given P_{fa} and P_d as before
2. Get a correction factor from Figure 2.23 in Skolnik (reproduced on the next page)
3. Get $I_i(n)$ from Figure 2.24 (2nd edition in Skolnik if more than one pulse is used
4. Use $\bar{s}$ in the radar equation for RCS

Note: There are many charts available to estimate the SNR from integrating *n* pulses for fluctuating targets (e.g., charts by Swerling, Blake, Kantor and Marcum). Although the details of the processes are different, they all involve modifying the SNR for a single pulse by the appropriate fluctuation loss and estimating the integration improvement factor.

Correction & Improvement Factors (1)

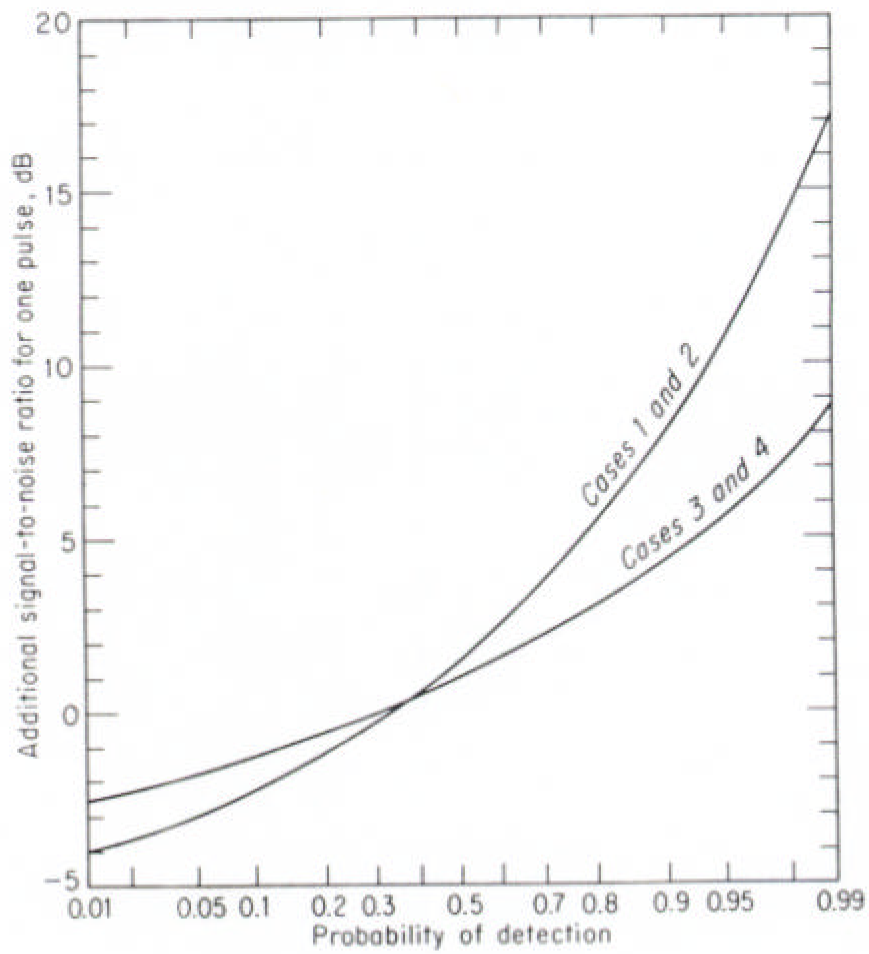


Figure 2.23 in Skolnik

Correction & Improvement Factors (2)

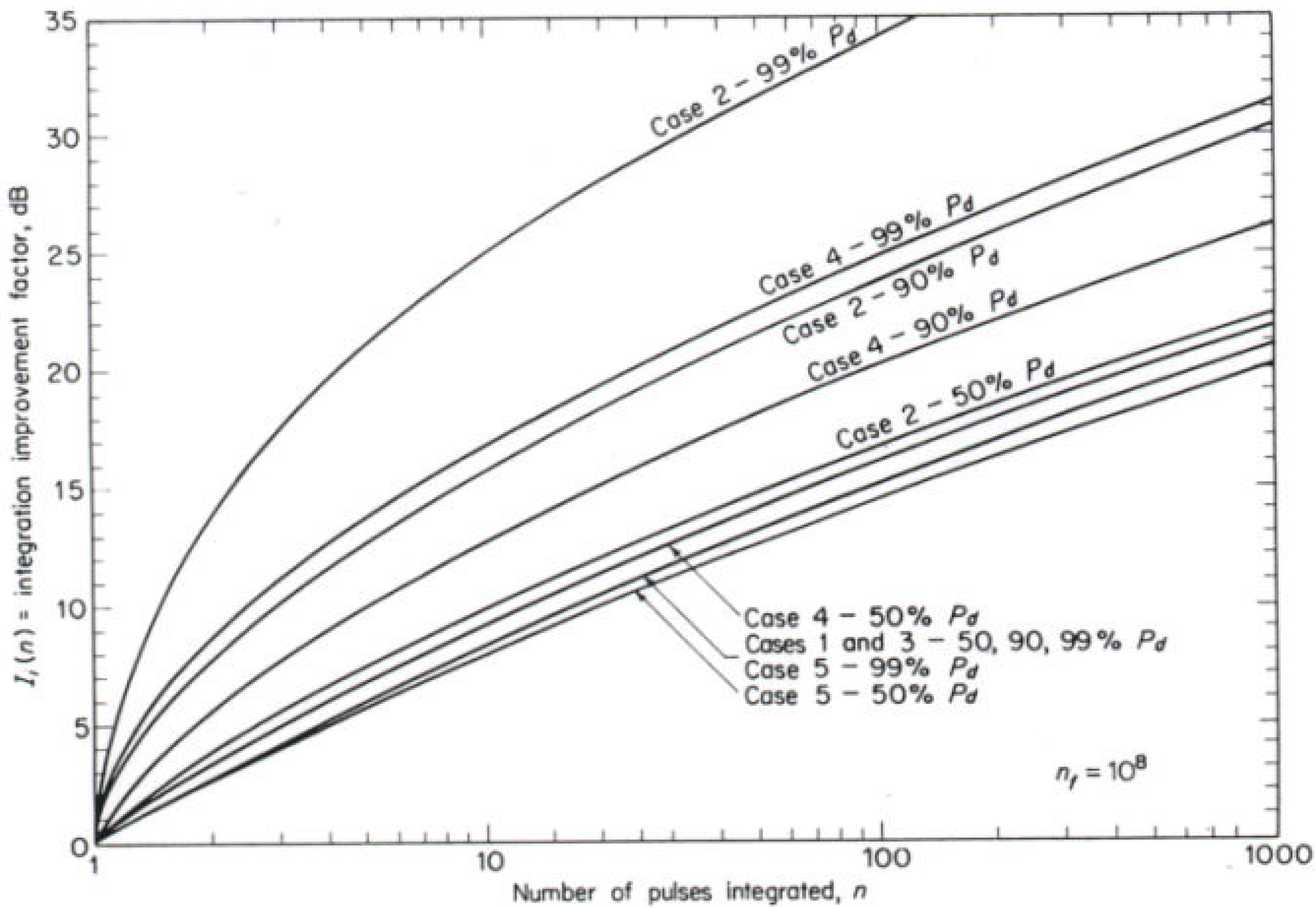


Figure 2.24 in Skolnik (2nd edition)

Detection Range for Fluctuating Targets

The maximum detection range for a fluctuating target is given by

$$R_{\max}^{4} = \frac{P_{\mathrm{av}} G_t A_{er} \bar{\sigma}\, n\, E_i(n)}{(4\pi)^2 k \mathrm{T}_s B_n \tau\ f_p \mathrm{SNR}_1}$$

where $I_i(n) = n\, E_i(n)$ and

$$\mathrm{SNR}_1 = (\mathrm{SNR}_1 \text{ for } P_d \text{ and } P_{fa} \text{ from Figure 2.6}) \times \\ (\text{correction factor from Figure 2.23})$$

(Note: if the quantities are in dB then they are added not multiplied.)

In general, the effect of fluctuations is to require higher SNRs for high probability of detection and lower values for low probability of detection, than those with non-fluctuating targets.

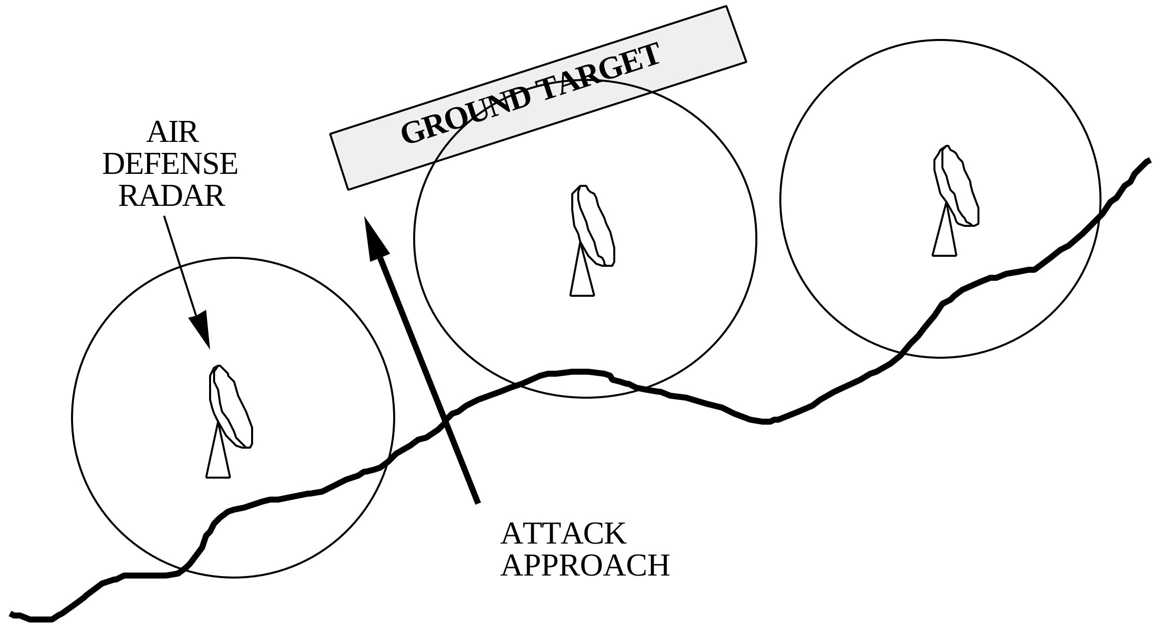
Example

A target's RCS is described by a single predominant scatterer whose echo fluctuates from pulse-to-pulse (Type IV). Find the SNR required if $P_{fa} = 10^{-10}$, n = 15 and P_d = 0.95.

Method 1: (as described in Skolnik 2nd edition)
1. From Figure 2.6: $\text{SNR}_1 = 15.5 \text{ dB} = 35.5$
2. From Figure 2.23, the correction factor (fluctuation loss, L_f) for the Type IV target and P_d = 0.95 is about 5.5 dB. Thus, for one pulse, $\text{SNR}_{\min} = 15.5 \text{ dB} + 5.5 \text{ dB} = 21 \text{ dB}$
3. From Figure 2.24 $I_i(n) \approx 15 \text{ dB}$. For n pulses $\text{SNR}_n = \text{SNR}_1 / I_i(n)$, or in dB
$\text{SNR}_n = \text{SNR}_1 - I_i(n) = 21\text{dB} - 15\text{dB} = 6\text{dB}$

Method 2: (as described in Skolnik 3nd edition, generally less accurate than Method 1)
1. Follow steps 1 and 2 from above
2. Adjust the fluctuation loss according to $L_f(n_e) = (L_f)^{1/n_e}$ where n_e is defined on page 69. (For Swerling Types I and III $n_e = 1$; for Types II and IV $n_e = n$.) Working in dB
$L_f(15) = 5.5/15 = 0.37$ dB
3. Use Figure 2.7 to get the integration improvement factor, $I_i \approx 10 \text{ dB}$
4. $\text{SNR}_n = \text{SNR}_1 + L_f(n_e) - I_i(n) = 15.5\text{dB} + 0.37\text{dB} - 10\text{dB} = 5.87\text{dB}$

Defeating Radar by Low Observability



Detection range depends on RCS, $R_{\max} \propto \sqrt[4]{s}$, and therefore RCS reduction can be used to open holes in a radar network.

Want to reduce RCS with a particular threat in mind:
clutter environment, frequency band, polarization, aspect, radar waveform, etc.

There are cost and performance limitations to RCS reduction

Method of RCS Reduction and Control

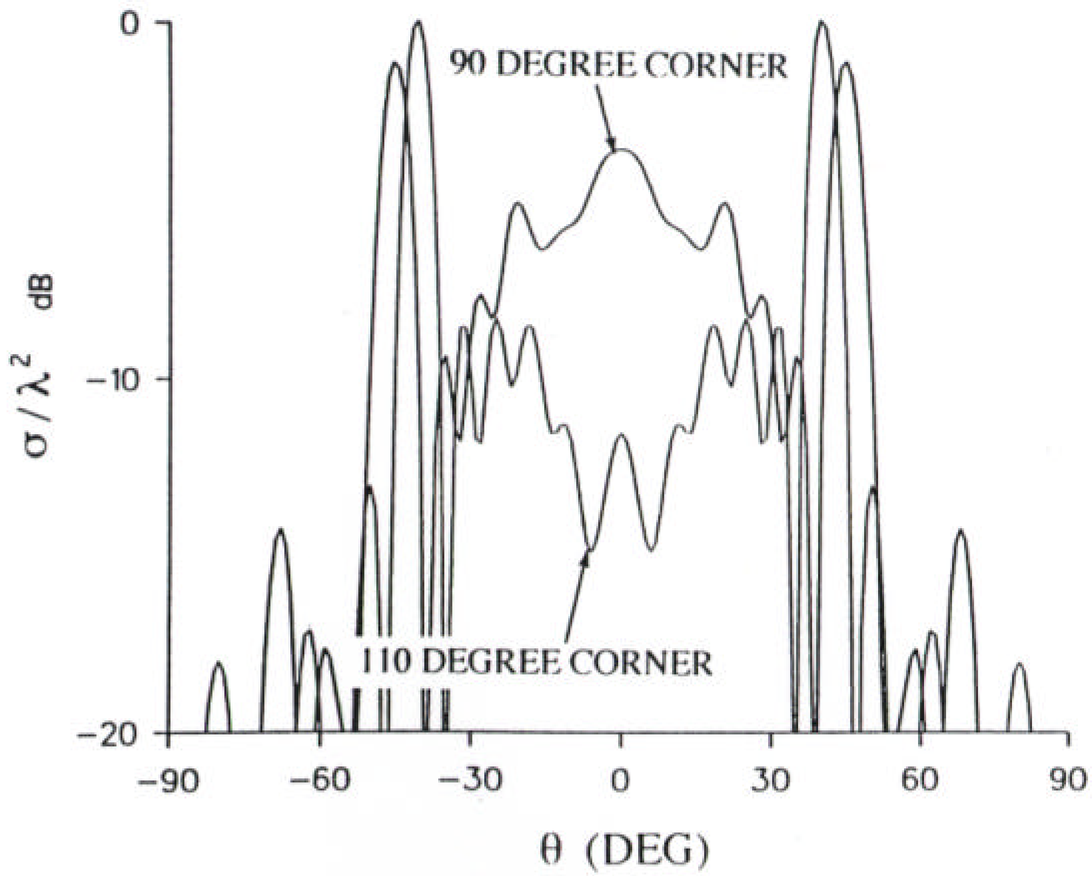
Four approaches:

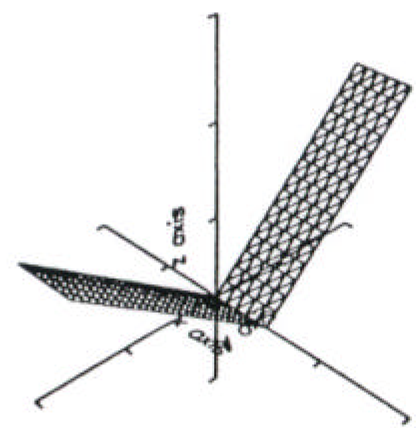
1. geometrical shaping - Direct the specular or traveling waves to low-priority directions. This is a high-frequency technique.
2. radar absorbing material (RAM) - Direct waves into absorbing material where it is attenuated. Absorbers tend to be heavy and thick.
3. passive cancellation - A second scattering source is introduced to cancel with scattering sources on the "bare" target. Effective at low frequencies for small targets.
4. active cancellation - Devices on the target either modify the radar wave and retransmits it (semi-active) or, generates and transmits its own signal. In either case the signal radiated from the target is adjusted to cancel the target's skin return. Requires expensive hardware and computational resources on the target.

Except for shaping, these methods are narrowband reduction techniques. Wideband radar is an effective way to defeat narrowband reduction methods.

Corner Reflector Reduction by Shaping

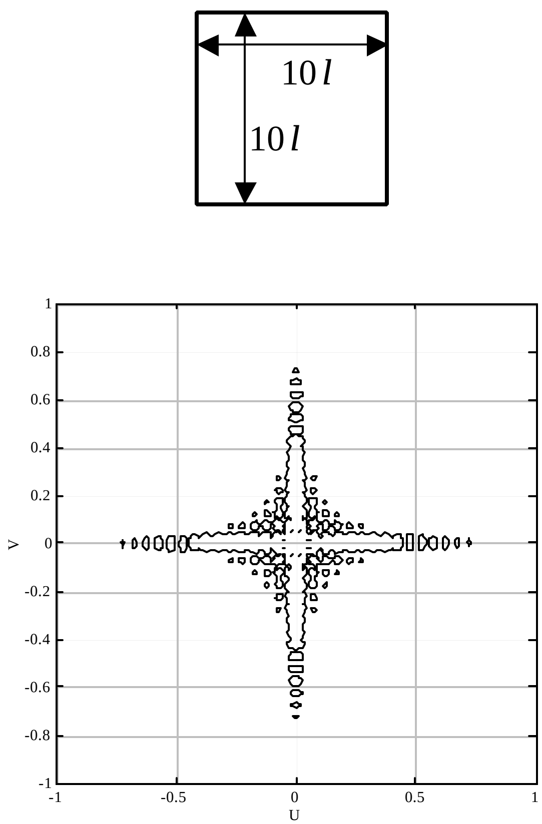
A 90 degree corner reflector has high RCS in the angular sector between the plates due to multiple reflections. Dihedrals are avoided in low observable designs (e.g., aircraft tail surfaces are canted).

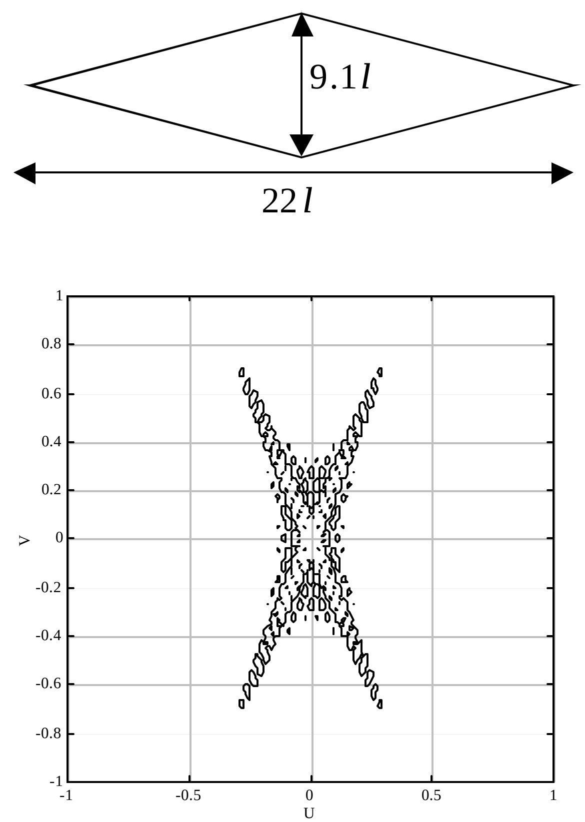




RCS of Shaped Plates

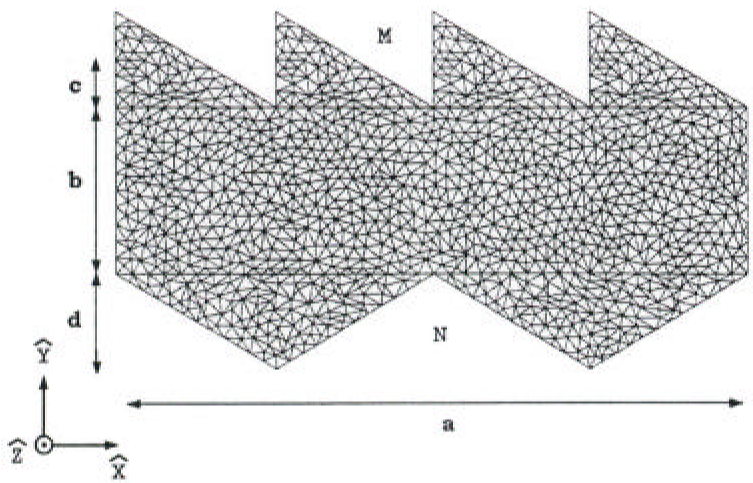
RCS contours of square and diamond shaped plates. Both have an area of 100 m^2.

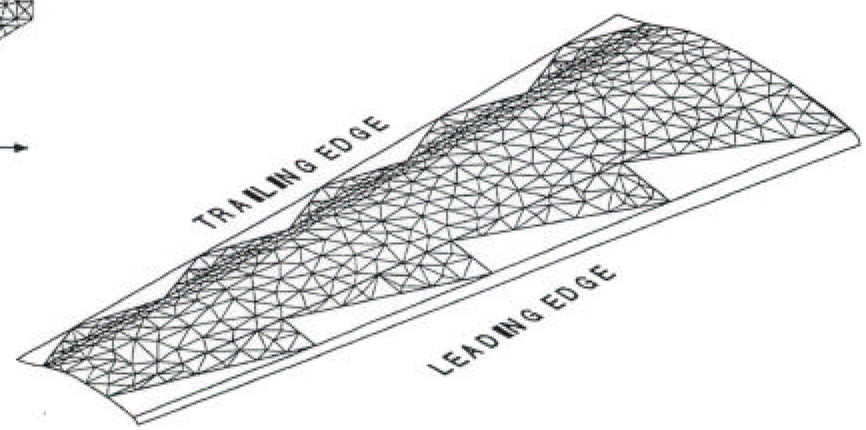




Application of Serrations to Reduce Edge Scattering

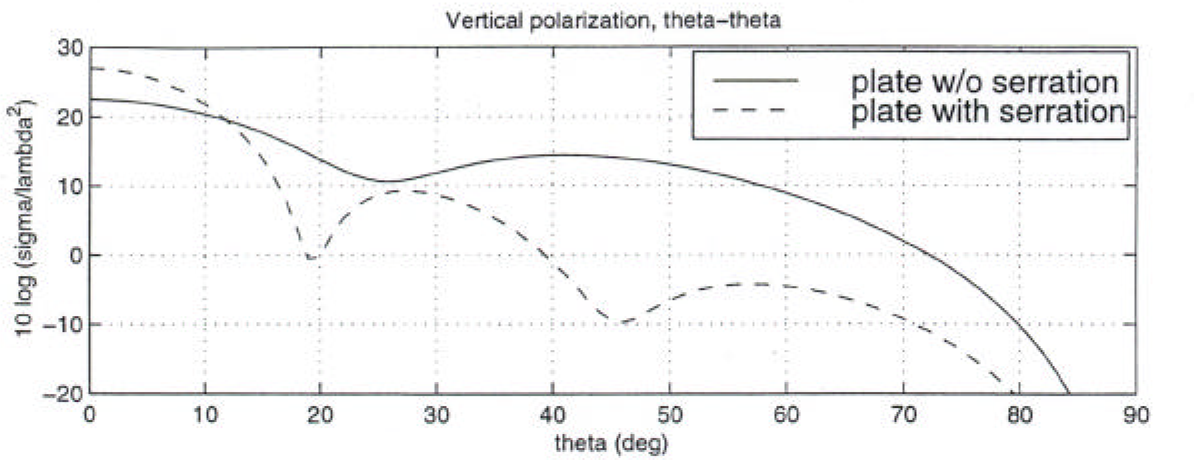
GENERAL PLATE APPLICATION

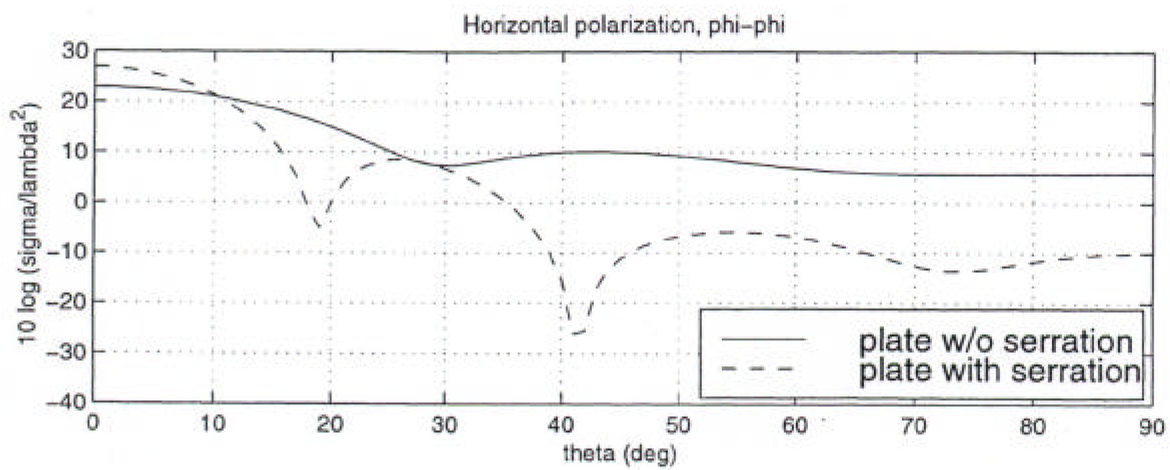




APPLICATION TO A WING

Application of Serrations to Reduce Edge Scattering

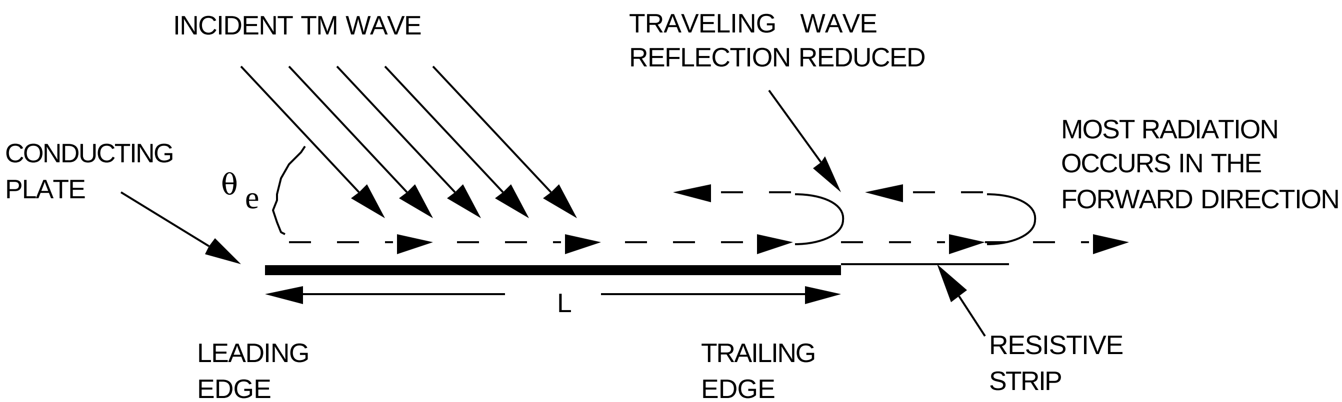




(RESULTS FOR $a = 4\,l,\quad b = l,\quad c = 0.577\,l,\quad M = 4,\quad N = 2$)

Traveling Waves

A traveling wave is a very loosely bound surface wave that occurs for gently curved or flat conducting surfaces. The surface acts as a transmission line; it "captures" the incident wave and guides it until a discontinuity is reached. The surface wave is then reflected, and radiation occurs as the wave returns to the leading edge of the surface.

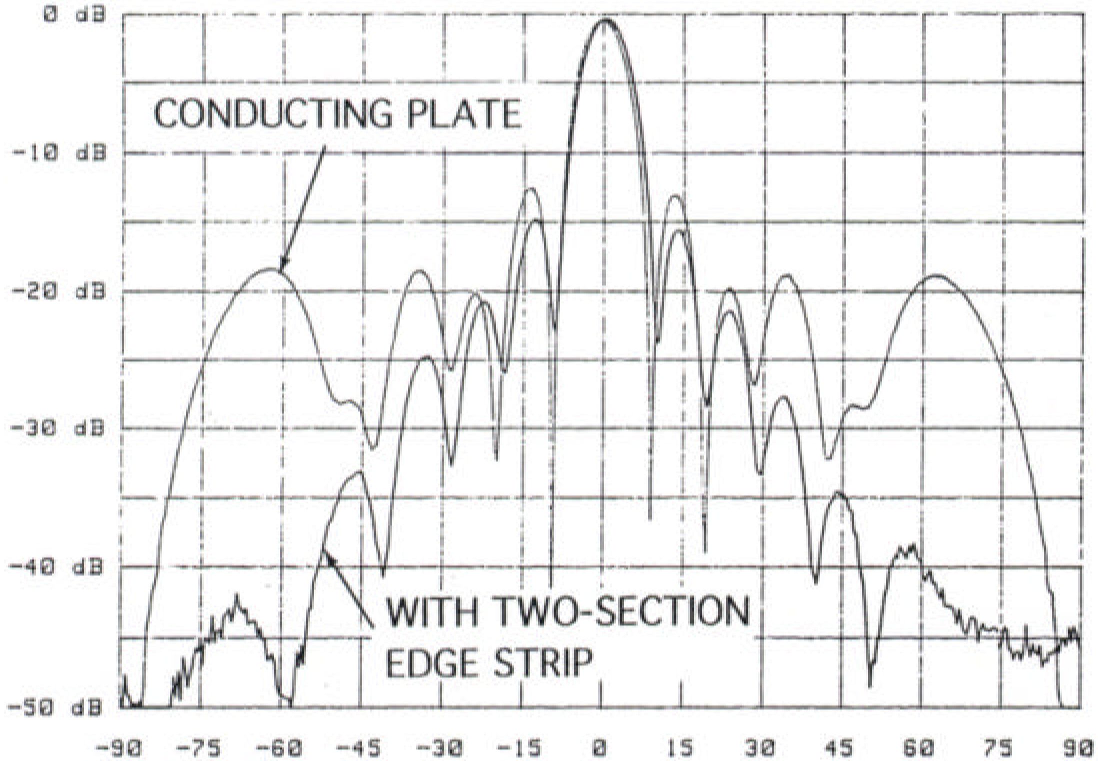


For a conducting surface the electric field must have a component perpendicular to the leading and trailing edges for a traveling wave to be excited. This is referred to as transverse magnetic (TM) polarization because the magnetic field is transverse to the plane of incidence. (Recal that the plane of incidence is defined by the wave propagation vector and the surface normal. Therefore, TM is the same as parallel polarization.) Transverse electric (TE) polarization has the electric field transverse to the plane of incidence. (TE is the same as perpendicular polarization.)

Trailing Edge Resistive Strips

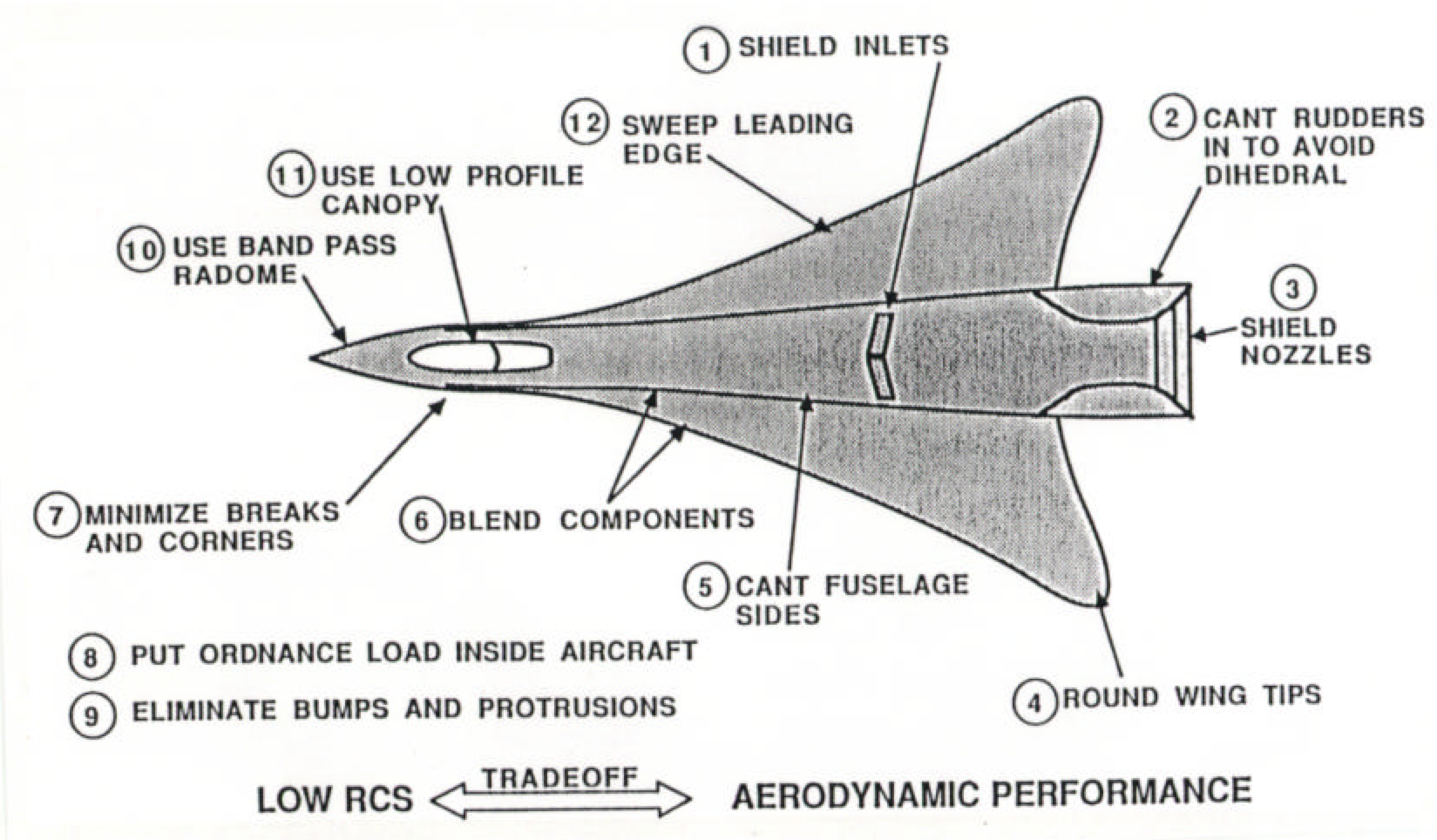
Quarter wave resistive strips can be used to eliminate traveling wave reflections at trailing edges

Normalized RCS of a plate with and without edge strips



Angle from plate normal, degrees

Application of Reduction Methods



From Prof. A. E. Fuhs

Low Observable Platforms: F-117

(USAF Photo)

Low Observable Platforms: B-2

(USAF Photo)

Low Observable Platforms: Sea Shadow

(USN Photo)