
Chapter 1

Radar Fundamentals

곽 영 길 교수
한국항공대학교

Contents

- RADAR Classification
- Range and Resolution
- Doppler Frequency
- Coherence
- RADAR Equations
 - . LPRF / HPRF Radar
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- RADAR Losses

RADAR - Electronic Eye

명칭 : RADAR : RAdio Detection And Ranging

정보 : - Range (거리), Angle (각도) : 위치정보

- Velocity (속도) : 도플러 정보

- Image (영상) : 고해상도 식별 정보

특징 :

전천후 고감도 전자 눈 (Electronic Eye)

민 군 겸용기술 (Dual Use Technology)

통합 산업기술 (Integrated System Technology)

- Electronics + Mechanics + Applications (전기,전자,통신,기계)
- 전자파(RF)+반도체+통신+신호처리+제어+컴퓨터+기계 구조
- High Value-Added Industrial Technology

고도의 통합기술 → 고부가가치 산업기술

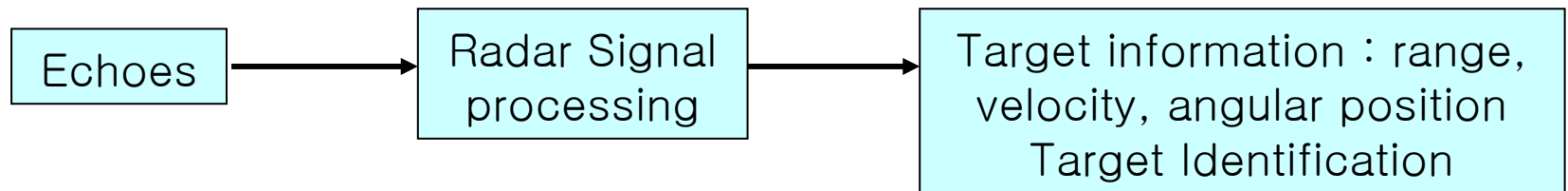
레이다 분류 – 기술과 용도

- RANGE : SHORT, MIDEUM, LONG RANGE
- FUNCTION : SURVEILLANCE, TRACKING
- INFORMATION : 1D, 2D, 3D, 4D, IMAGE(SAR)
- FREQUENCY : HF, UHF, L, S, C, X, Ku, Millimeter
- PROCESSING : MTI, DOPPLER, LPI, SAR, UWB
- PRF : LPRF, MPRF, HPRF
- OBJECT : A/C, SHIP, MISSILE, VEHICLE,
WEATHER, Human Body
- PLATFORM : GROUND, SHIPBORNE, AIRBORNE
SPACEBORNE, VEHICLE

Radar Classifications

■ **RADAR** : RAdio Detection And Ranging.

- transmit electromagnetic energy into a specific volume to search for targets.
- targets will reflect portions of this energy back to the radar.



■ **Classification**

Type : Platform, Frequency Band, Antenna Type, Waveform, Mission, Function

- 1) Platform : Ground based, airborne, spaceborne, ship based radar.
- 2) Mission : weather, acquisition and search, tracking, TWS, fire control, Early warning, Over the Horizon, Terrain Following, Terrain Avoidance Radar.
- 3) Phased Array Radar : Active Array, Passive Array
- 4) Waveform type : CW, FMCW, Pulsed (Doppler) Radar-LPRF, MPRF, HPRF

Radar Frequency Band

5) Operating frequency

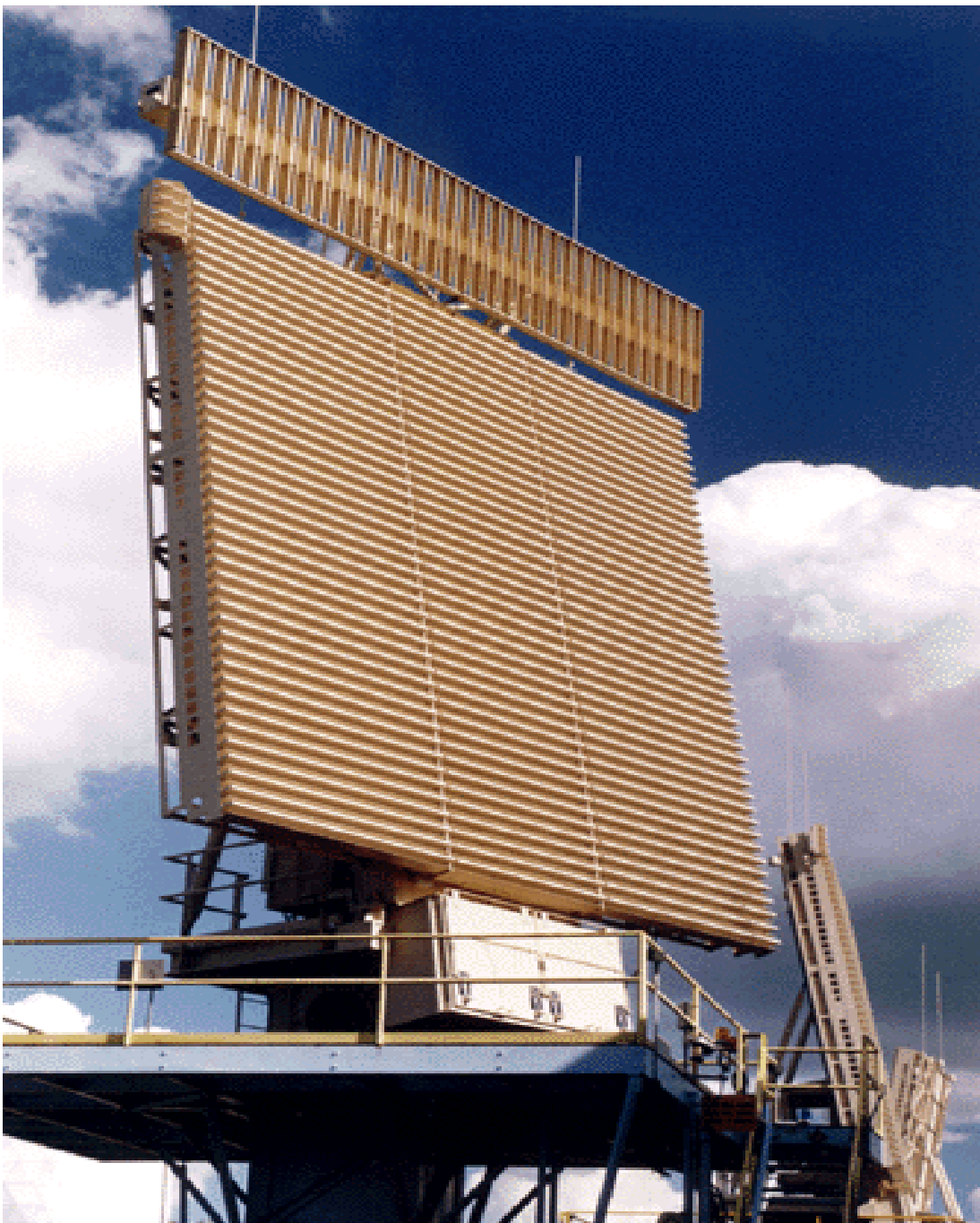
Letter Designation	Frequency (GHz)	New band designation	Letter Designation	Frequency (GHz)	New band designation
HF	0.003-0.03	A	X-band	8.0-12.5	I<10.0; J>10.0
VHF	0.03-0.3	A<0.25, B>0.25	Ku-band	12.5-18.0	J
UHF	0.3-1.0	B<0.5, C>0.5	K-band	18.0-26.5	J<20.0; K>20.0
L-band	1.0-2.0	D	Ka-band	26.5-40.0	K
S-band	2.0-4.0	E<3.0, F>3.0	MMW	Normally>34.0	L<60.0; M>60.0
C-band	4.0-8.0	G<6.0, H>6.0			

- L-band : primarily ground based and ship based systems,
long range military and air traffic control search operation.
- S-band : Most ground and ship based medium range radar
- C-band : Most weather detection radar systems,
medium range search, fire control and metric instrumentation radar.
- X-band : Small Size of the antenna → Airborne Radar
- Ku, K, Ka - band : severe weather and atmospheric attenuation,
short range applications police traffic radar, terrain avoidance.

최신 레이다 시스템 소개

- **Ground Based Radar : PAC3 – 페트리어트 미사일
다목적 레이다 (MFR)**
- **Shipborne Radar : EGIS 구축함 레이다**
- **Airborne Radar : AWACS 조기경보기**
- **Spaceborne Radar : RadarSat 위성 SAR**

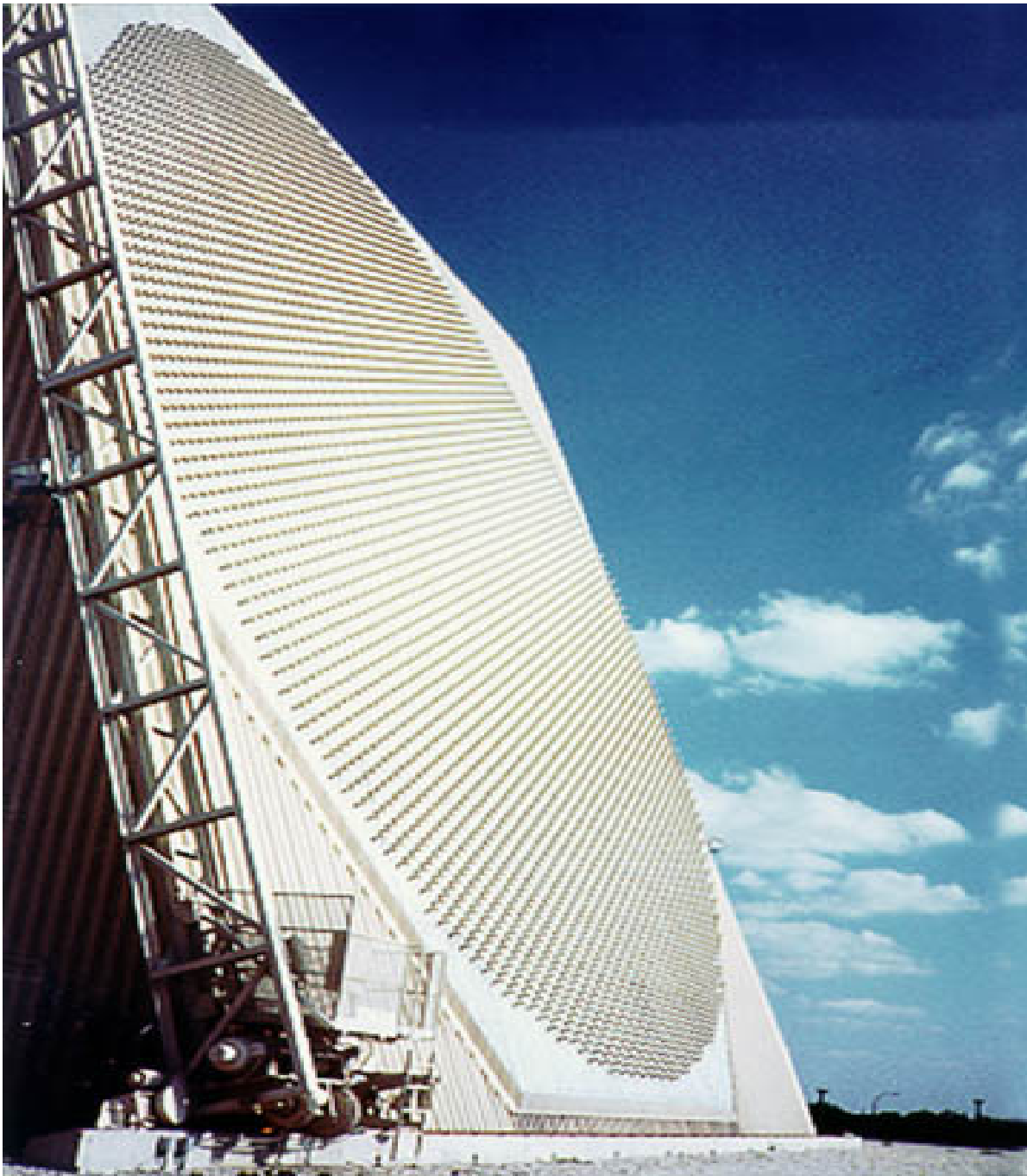
$$\therefore R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 L_{\text{sys}} L_{\text{pro}} S_{\min}} \right]^{1/4}$$



AN/FPS – 117

장거리 탐지 레이다

- 3D phased array antenna radar
- Frequency : L-band
- Detection range : 200-250nm
- Coverage (Az/EI/Altitude)
: 360deg/100k ft/-6 to 20deg
- Peak power : 24.75kW



BMEWS

탄도 미사일 조기경보 레이다

- Phase steered array Radar
- Frequency : UHF
- Diameter : 84ft
- 2560 Active Elements



AN/MPQ-53

PATRIOT Radar 다목적 레이다

- Frequency : G/H-band
- Detection range : 3-170km
- Max No. of target tracks : 100
- Search Sector :
120deg(Az)/90deg(EI)

ASR 23SS Primary Surveillance Radar



공항 감시 레이다

- L-band(1250-1350MHz)
- Range : 185 – 463km
- Peak power : 21/40 KW
- Beamwidth : 25deg(Az)
- Antenna gain : 36dBi
- Builder : Raytheon



NEXRAD (WSR-88D)

기상 레이다

- **Next Generation Weather Radar**
- **Frequency : S-band(2.7–3GHz)**
- **Peak power : 750kW**
- **Detection range : 248nm(460km)**
- **Antenna type : center-feed, Parabolic dish**
- **Diameter : 9m**



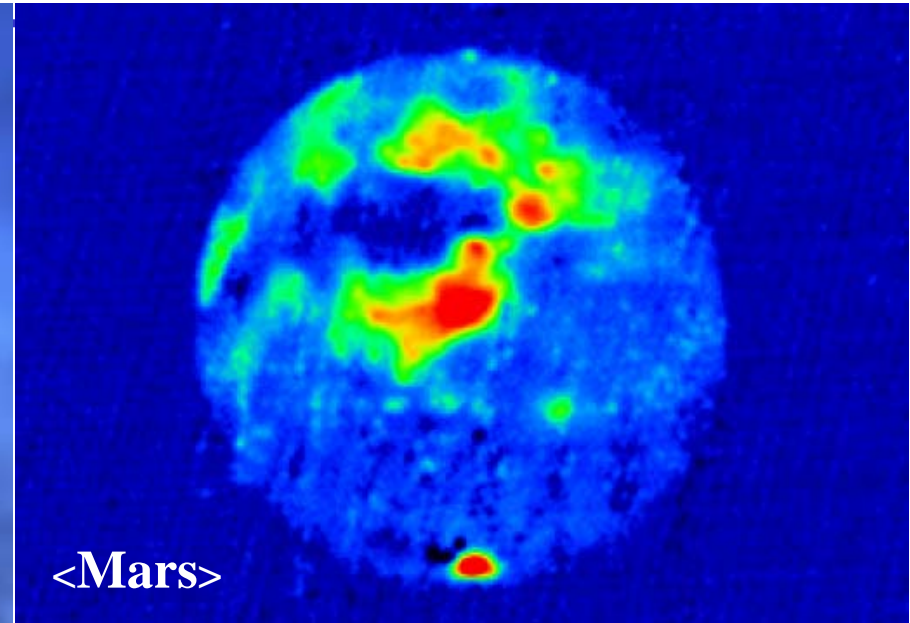
AN/FPS-118

OTH Radar

수평선 이상
탐지 레이다

- FM/CW Bistatic Doppler Radar
- Frequency : 5-28 MHz
- Coverage : 2.2 million square miles
- Max CW radiated power : 1,000kW
- Tx and Rx separation : 160km

Planetary Radar - Deep space station



- Mission : observations of nearby asteroids
- Frequency : S/X-band
- Antenna Diameter : 64m
- Range : 16 billion kilometers
- Accuracy : 3,850m² surface is maintained within 1cm.
- NASA

Shipborne Radar - CG-62 AEGIS

- Radar : AN/SPS-49(V)1(air search)
- Frequency : L-band
- Detection range : 250nm
- PRF : 280, 800, 1000 Hz



E-3 Sentry AWACS



- Radar : AN/APY-1/2
multi-mode surveillance radar
- Detection range : 200mile(375.5km)
- Frequency : S-band
- Northrop Grumman

F-16 Fighting Falcon



- Radar : AN/APG-66 (F-16A) ,
AN/APG-68 (F-16C)
- Frequency : I/J – band
- Detection Range : 48km(downlook), 72km(uplook)
- Beamwidth : 3.2deg(Az) X 4.86deg(EI)
- Antenna size : 74cm(length) X 48cm(width)

F-18 Hornet



- Radar : AN/APG-65 , AN/APG-73
(upgrade of APG-65)
- Frequency : I/J – band
- Detection Range : 80nm(Maximum)

- Max No. of target tracks : 10
- Beamwidth : 3.3deg(Az) X 5.3deg(EI)
- Raytheon

Helicopter MMW Radar - Apache



Millimeter Wave – Longbow Ka Band Fire Control Radar for the US Army's Apache Helicopter mounted in a radome on top of helicopter mast

UAV Radar - TESAR



RQ-1 Predator

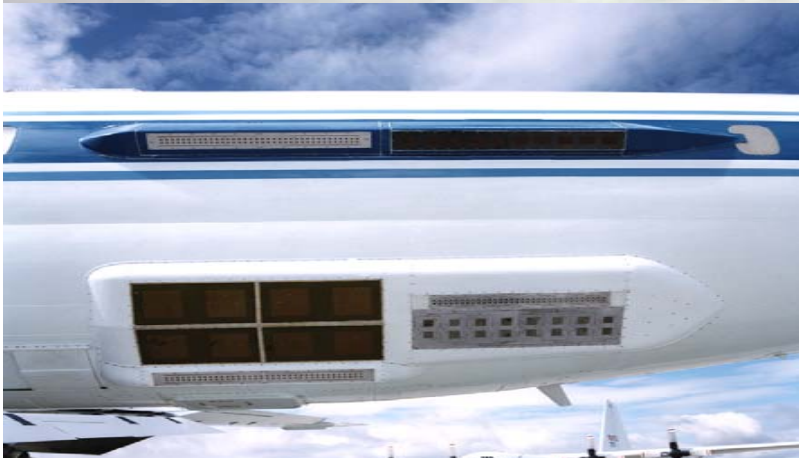
- Radar : AN/ZPQ-1 Tactical Endurance SAR
- Impulse response 3dB width : 0.3m +/-10%
- Range : 4.4 - 10.8km
- Squint angle : 70 – 110 deg.
- Swath width : 800m at 25-35m/s



Airborne Radar - SAR



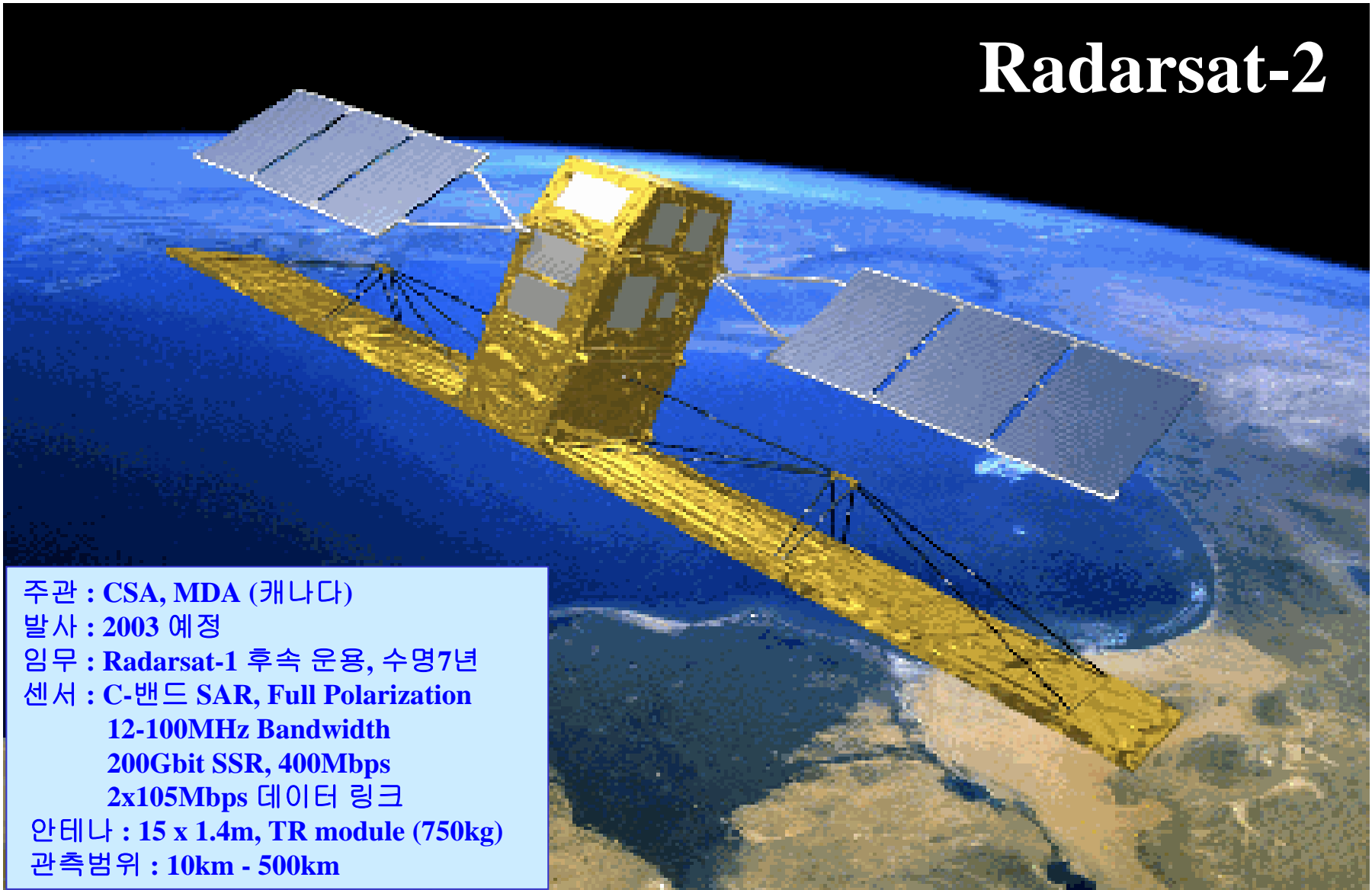
NASA-AIRSAR



- Platform : DC-8 aircraft
- Frequency : P/L/C-band
- Range resolution : 7.5 / 3.75 / 1.875m
- Peak power : 1/6/2 kW (P/L/C)
- Swath width : 10km(nominal) / 17km(max)

Spaceborne Radar - SAR

Radarsat-2



주관 : CSA, MDA (캐나다)

발사 : 2003 예정

임무 : Radarsat-1 후속 운용, 수명 7년

센서 : C-밴드 SAR, Full Polarization

12-100MHz Bandwidth

200Gbit SSR, 400Mbps

2x105Mbps 데이터 링크

안테나 : 15 x 1.4m, TR module (750kg)

관측범위 : 10km - 500km

Space Shuttle Radar - SAR



주관 : NASA/JPL, NIMA, DLR(미국, 독일)
발사 : 2000. 2. 11 17:43 GMT
임무시간 : 11일 5시간 38분
임무 : Global DTM 3차원 맵(Interferometry)
60m baseline 안테나 마스터 설치
관측범위 : 북위 +60 ~ -56도, 225km swath
센서 : C-band, X-band SAR
고도정확도 : 20m(수평), 10m(수직)
성과 : 지구표면의 80% DEM자료 획득

SRTM Project – Interferometry SAR Image

SAR Image – Seoul (RadarSat1)



Over The Horizon Radar

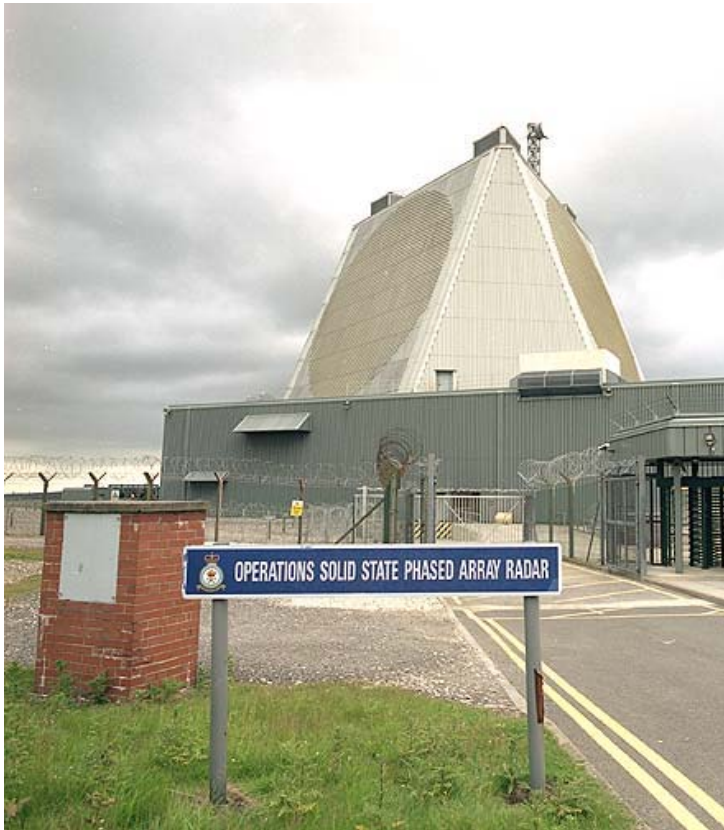
U.S. Navy ROTH
2.6-km Receiving Array

➔ Relocatable Over the
Horizon Radar (ROTHR)



< U.S. Navy Over The Horizon Radar >
Frequency range : 5 ~ 28MHz

BMEWS

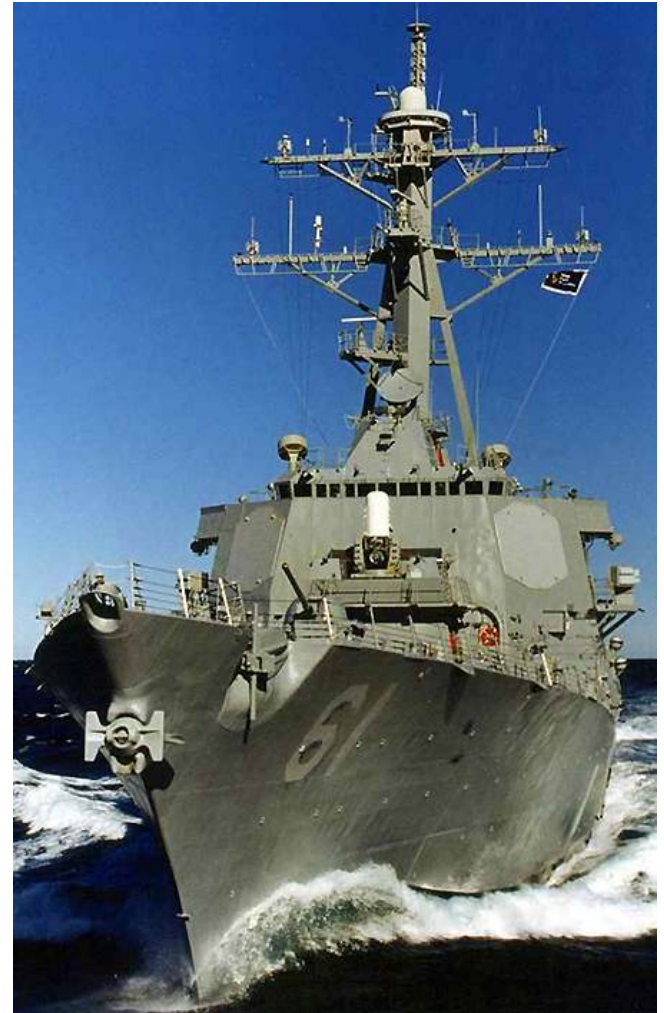


< Ballistic Missile Early Warning System >
Operating Frequency : 245MHz

AEGIS



< U.S. Navy AEGIS >
Operating frequency : S-band



AWACS



< Airborne Warning And Control System >
Operating frequency : S-band

Radar Sensor Information

Detection Range

$$R = \frac{cT}{2}$$

Az/EL

$$\Delta R = R\theta$$

Resolution

Doppler Velocity

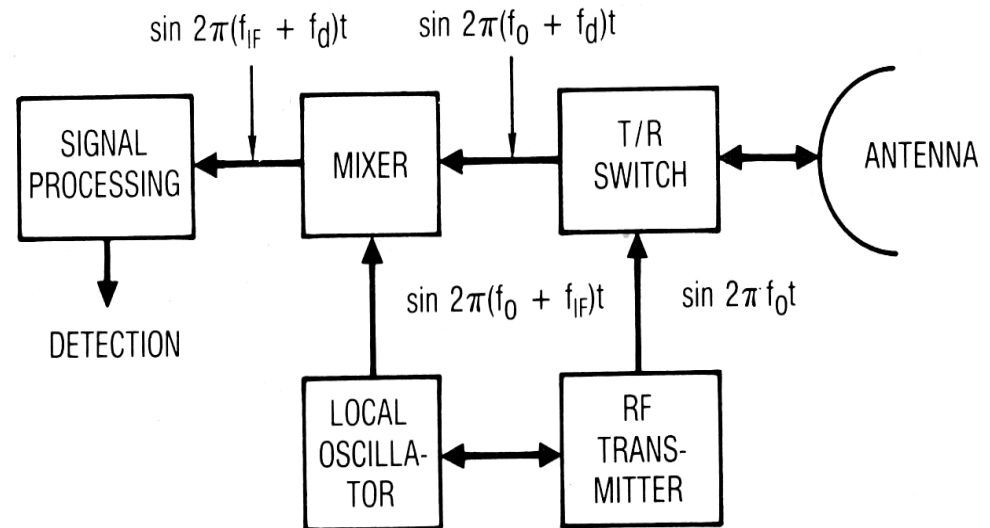
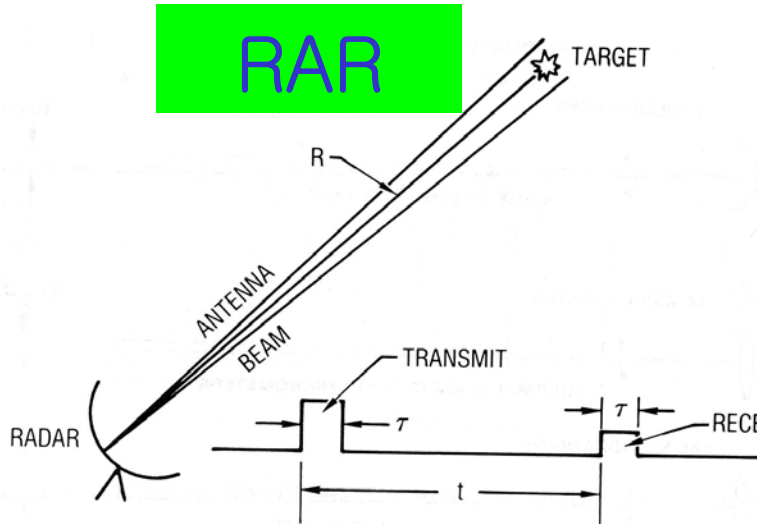
$$f_d = \frac{2R'}{\lambda}$$

High Range/Azimuth
Resolution Image

레이다 영상

SAR

RAR

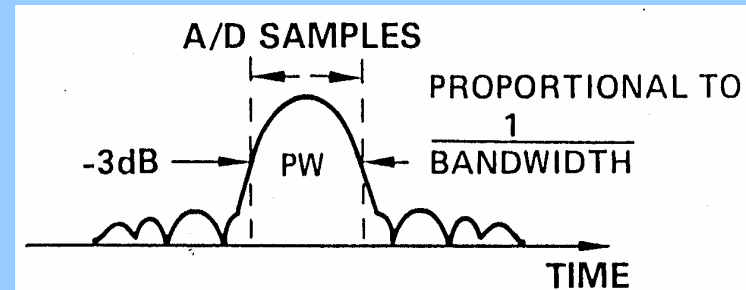


PHYSICAL RESOLUTION CELL

☐ RANGE

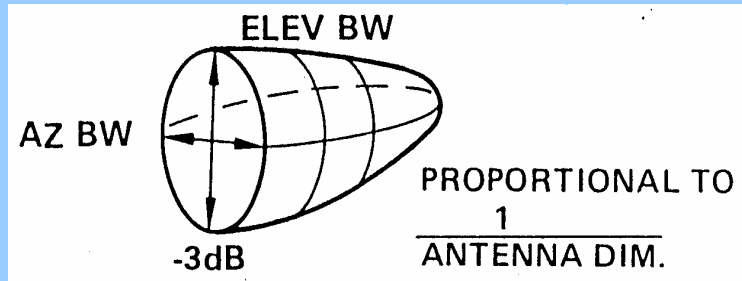
(A/D SAMPLING PERIOD)

PW=PULSE WIDTH



☐ ANGLE

(BEAMWIDTH)



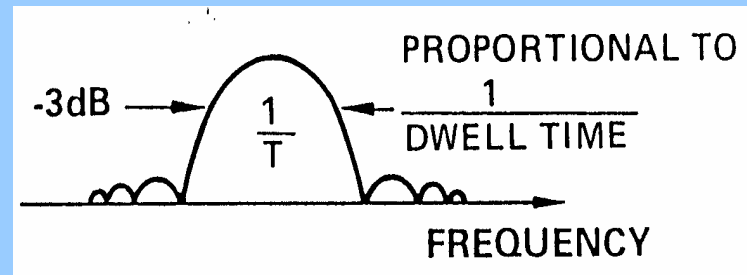
☐ DOPPLER FREQUENCY

(DOPPLER FILTER)

DWELL TIME

= TIME OF ENERGY

TRANSMISSION

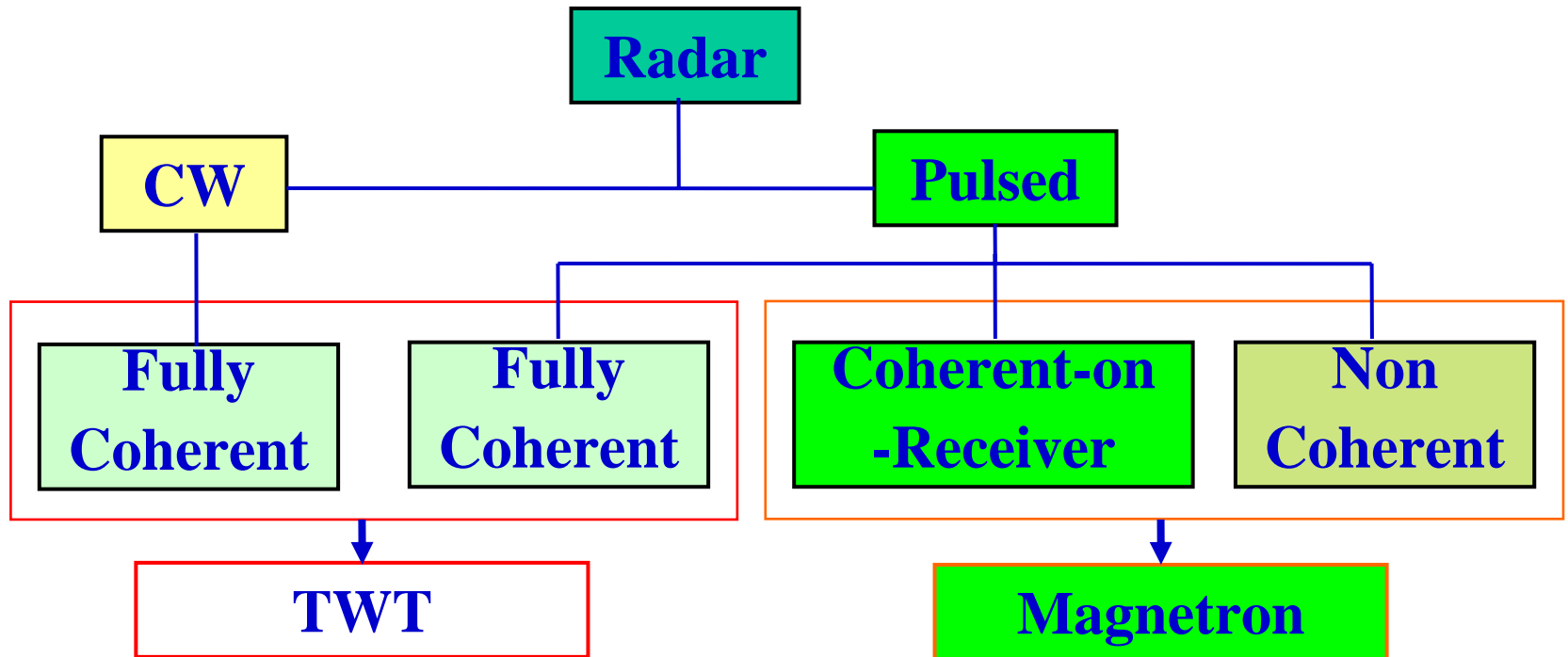


Radar Design Requirement

◆ Requirement

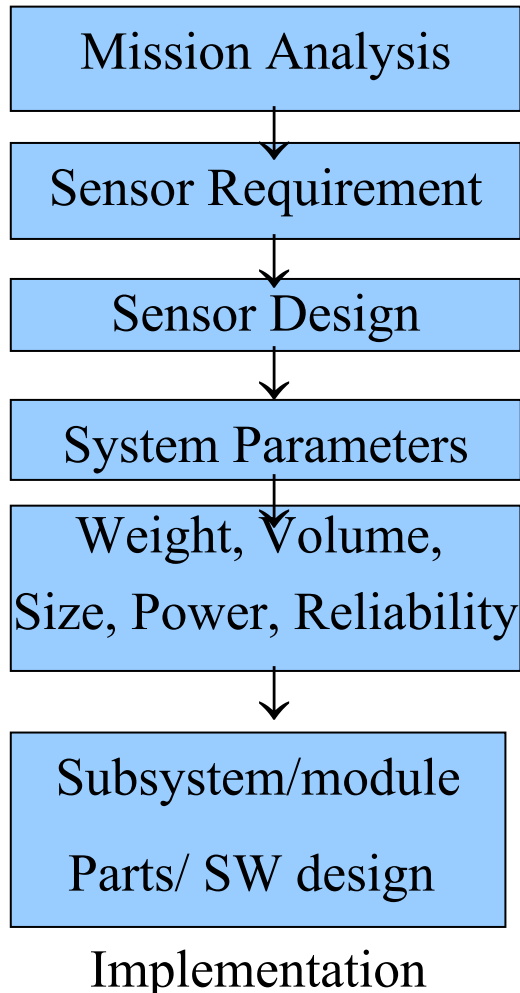
- * Mission requirement – Target RCS
- * Detection : high P_d , low P_{fa}
- * Accuracy : Range, Angle, Doppler
- * Resolution : Range / Azimuth / Elevation
- * Clutter Rejection : Waveform,
Signal Processor

Radar Design Type : Trade-Off



Type	Information	Characteristics
Coherent	Range, Doppler	Precise System, Complicate & Expensive
Non Coherent	Range Only	Simple, Low Cost

Radar Design Procedure



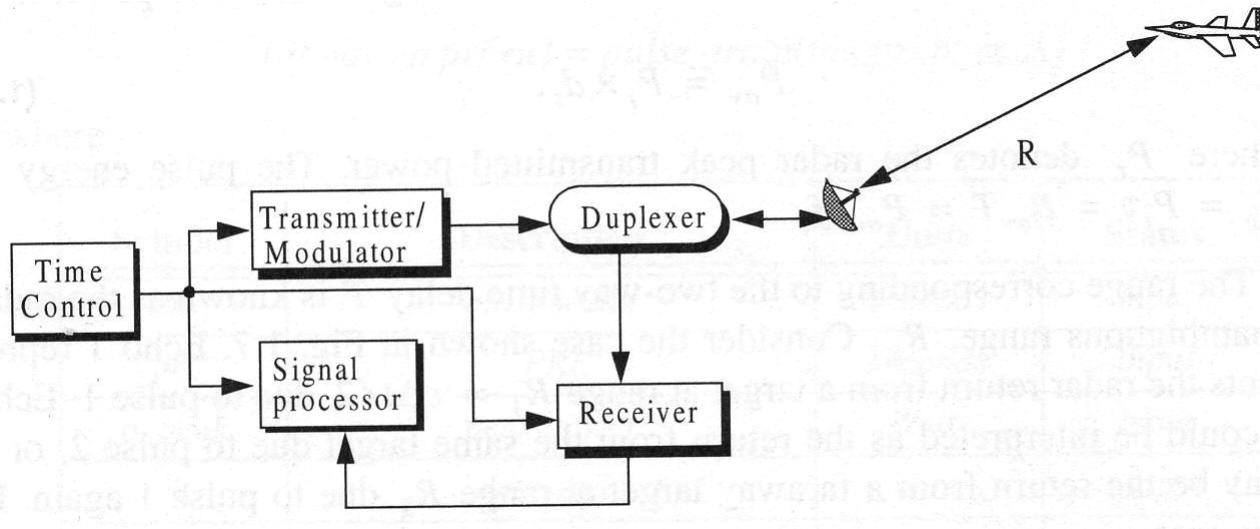
- Environmental limits
- Applicable technology & components limits
 - * Radar frequency selection
 - * mechanical or electrical scan Ant.
 - * Choice of polalization
 - * Radar waveform
 - * Type of processing : MTI or pulse Doppler MTD
 - * Transmitting power : Tube/MPM or Solid-state

Mission Based Top-Down Approach



Radar Range Measurement

■ Pulse



<A simplified pulsed radar block diagram>

- Target's range R , is computed by measuring the time delay Δt ,

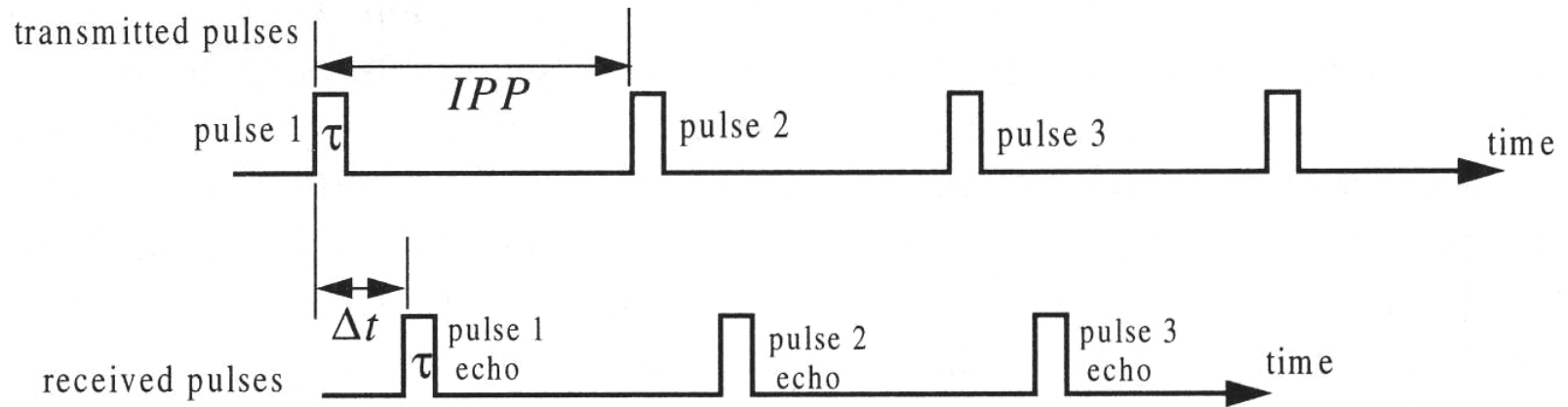
$$R = \frac{c\Delta t}{2} \quad (1.1)$$

* $c = 3 \times 10^8$ m/s

* factor $\frac{1}{2}$ is needed to account for the two-way time delay

Train of pulses for Measurement

- In general, a pulsed radar transmits and receives a train of pulse.



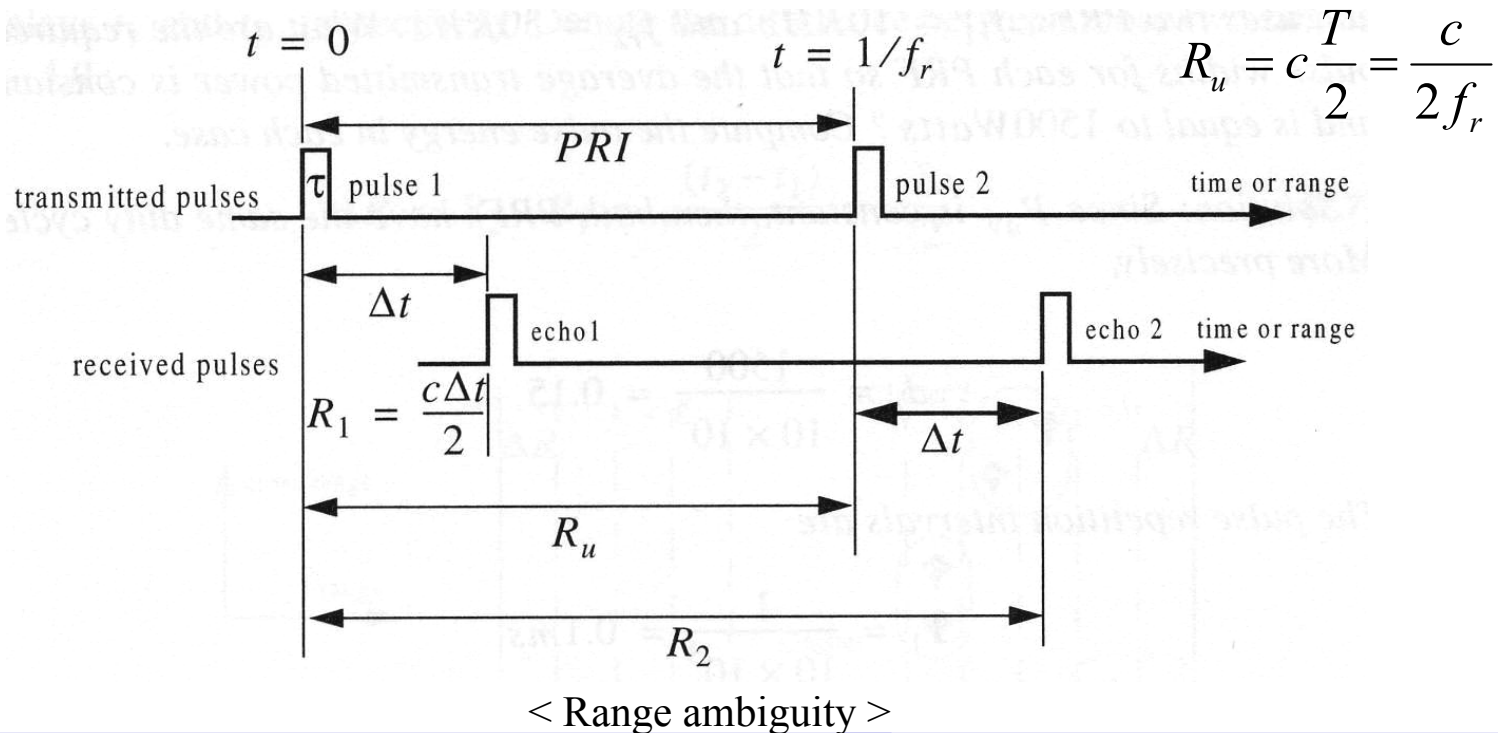
< Train of transmitted and received pulses >

- IPP : inter pulse period T , τ : pulse width
- IPP is referred to as the Pulse Repetition Interval (PRI)
- PRF = Inverse of the PRI (f_r).

$$f_r = \frac{1}{PRI} = \frac{1}{T} \quad (1.2)$$

Range Ambiguity

- Radar transmitting duty cycle (factor) d_t is defined, $d_t = \tau / T$
- Radar average transmitted power is $P_{av} = P_t \times d_t$
- Pulse energy is $E_p = P_t \tau = P_{av} T = P_{av} / f_r$
- Unambiguous Range R_u : Range corresponding to the two-way time delay T ,



Example 1.1

EX1.1) A airborne pulsed radar has peak power $P_t=10\text{KW}$, and uses two PRF $f_{r1}=10\text{KHz}$, $f_{r2}=30\text{KHz}$, What are the required pulse width so that $P_{av}=1500\text{W}$? And compute pulse energy.

Sol)

$$d_t = \frac{1500}{10 \times 10^3} = 0.15$$

The pulse repetition interval are

$$T_1 = \frac{1}{10 \times 10^3} = 0.1\text{ms}$$

$$T_2 = \frac{1}{30 \times 10^3} = 0.0333\text{ms}$$

$$\tau_1 = 0.15 \times T_1 = 15\mu\text{s}$$

$$\tau_2 = 0.15 \times T_2 = 5\mu\text{s}$$

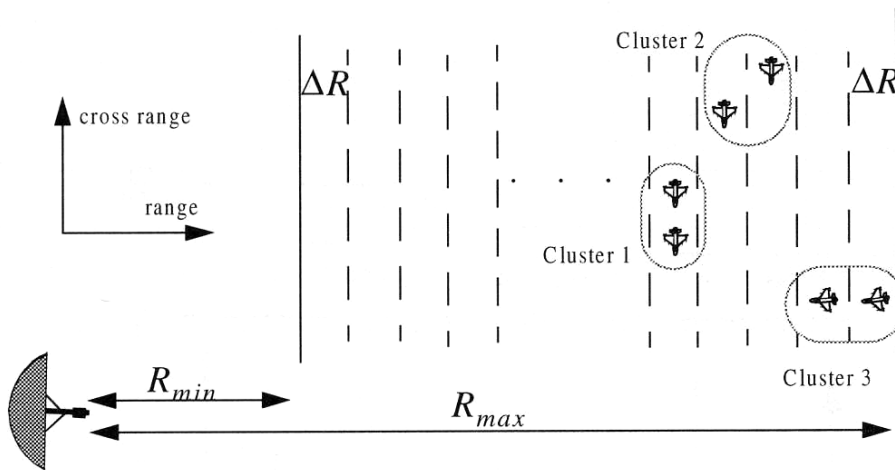
$$E_{p1} = P_t \tau_1 = 10 \times 10^3 \times 15 \times 10^{-6} = 0.15\text{J}$$

$$E_{p2} = P_t \tau_2 = 10 \times 10^3 \times 5 \times 10^{-6} = 0.05\text{J}$$

Range Resolution

- Range resolution ΔR , is radar metric that describes its ability to detect target in close proximity to each other as distinct objects.
- The distance between minimum range R_{\min} and maximum range R_{\max} is divided into M range bin, each of ΔR ,

$$M = \frac{R_{\max} - R_{\min}}{\Delta R} \quad (1.6)$$



< Resolving targets in range and cross range >

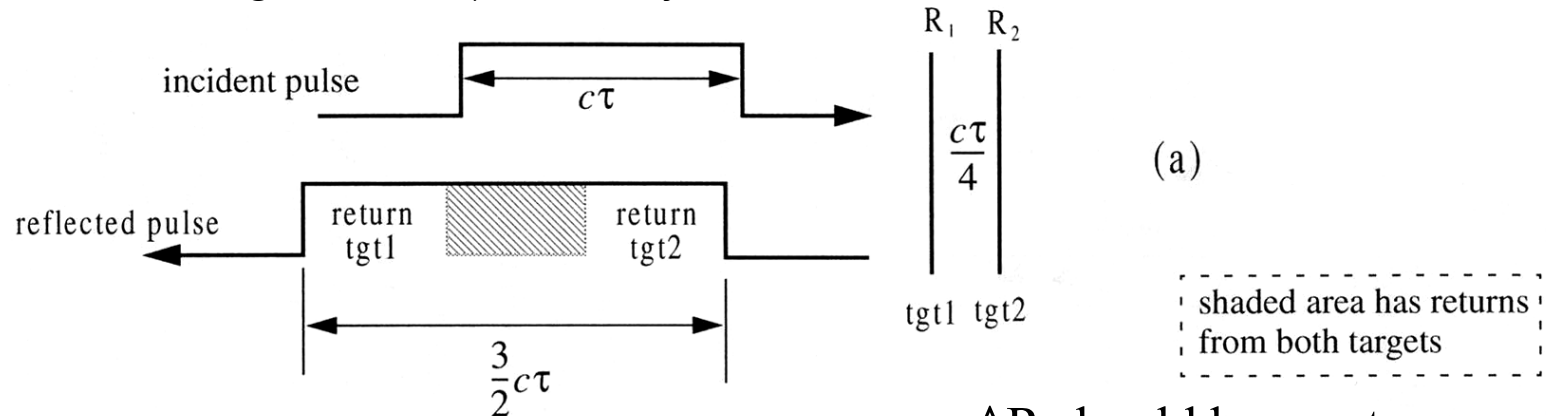
* target within the same range bin can be resolved in cross range (azimuth)

- Two target located at range R_1 and R_2 , the difference those two ranges as ΔR

$$\Delta R = R_2 - R_1 = c \frac{(t_2 - t_1)}{2} = c \frac{\delta t}{2} \quad (1.7)$$

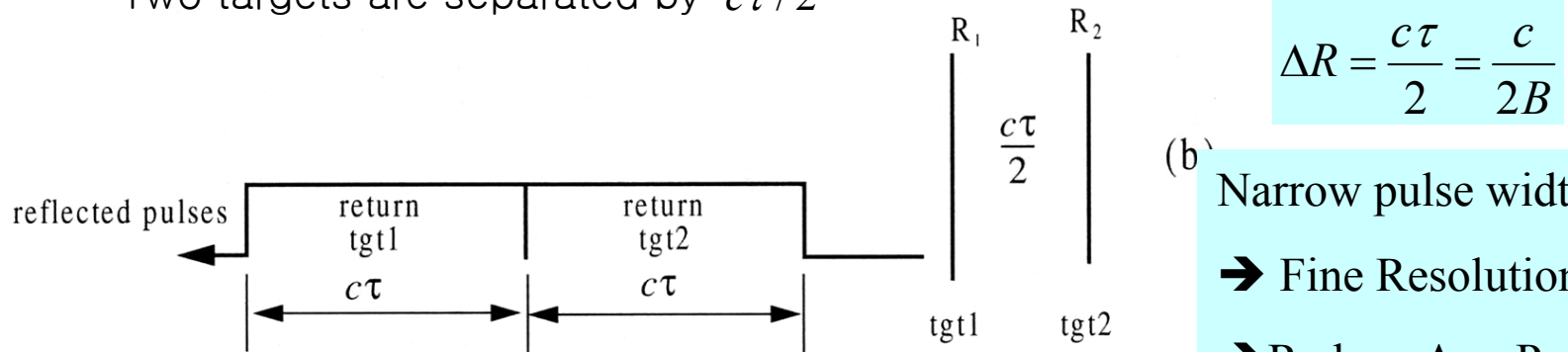
Range Resolution

- Two targets are separated by $c\tau/4$



ΔR should be greater or equal to $\frac{c\tau}{2}$

- Two targets are separated by $c\tau/2$



$$\Delta R = \frac{c\tau}{2} = \frac{c}{2B}$$

Narrow pulse width

→ Fine Resolution

→ Reduce Avg Power

→ Pulse Compression

(a) Two unresolved targets. (b) Two resolved targets

Example 1.2

EX 1.2) unambiguous range of 100 km, and a bandwidth 0.5Mhz,
Compute the required PRF, PRI, ΔR , and τ .

Sol)

$$PRF = \frac{c}{2r_u} = \frac{3 \times 10^8}{2 \times 10^5} = 1500 \text{ Hz}$$

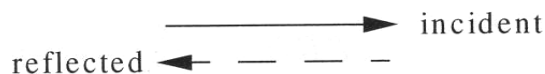
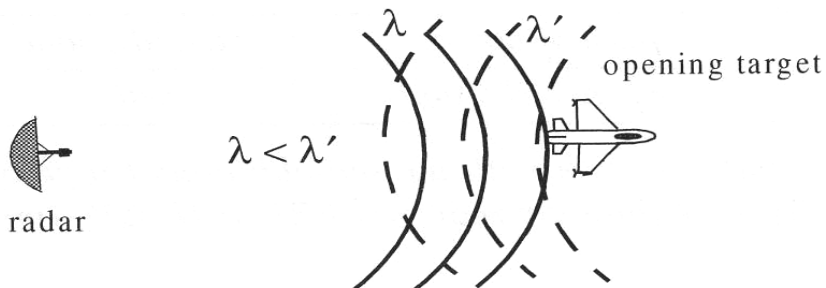
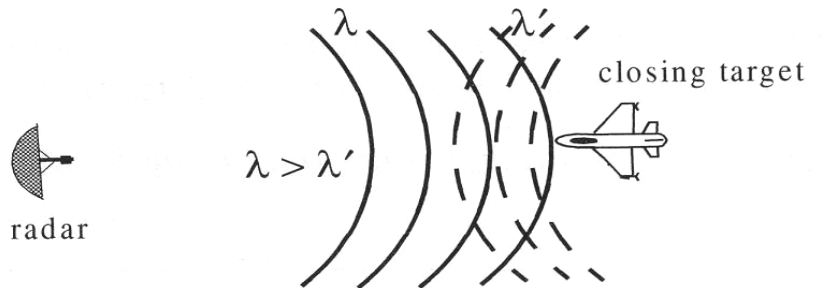
$$PRI = \frac{1}{PRF} = \frac{1}{1500} = 0.6667 \text{ ms}$$

$$\Delta R = \frac{c}{2B} = \frac{3 \times 10^8}{2 \times 0.5 \times 10^6} = 300 \text{ m}$$

$$\tau = \frac{2\Delta R}{c} = \frac{2 \times 300}{3 \times 10^8} = 2 \mu\text{s}$$

Doppler Effect of Target Motion

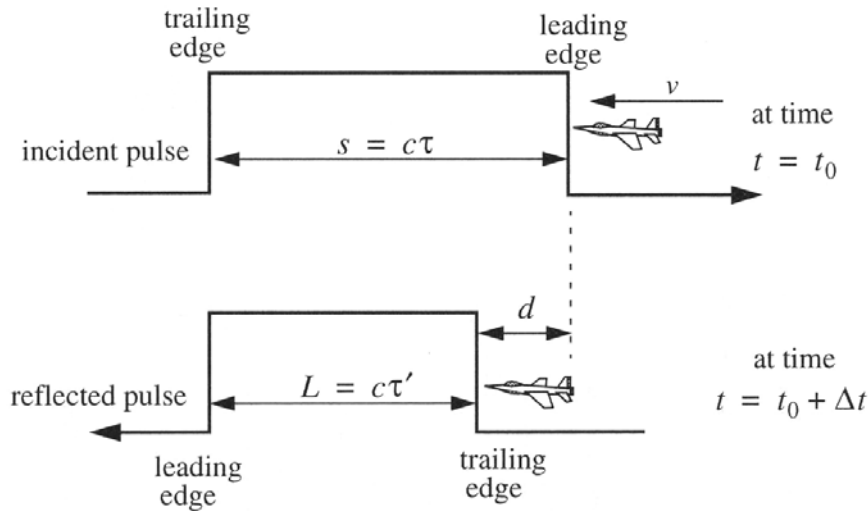
- Doppler frequency to extract target radial velocity (range rate) and to distinguish between moving and stationary targets (MTI)



- A closing target will cause the reflected equiphase wavefronts to get closer to each other. (smaller wavelength)
- An opening target will cause the reflected equiphase wavefronts to expand. (larger wavelength)

< Effect of target motion on the reflected equiphase waveforms >

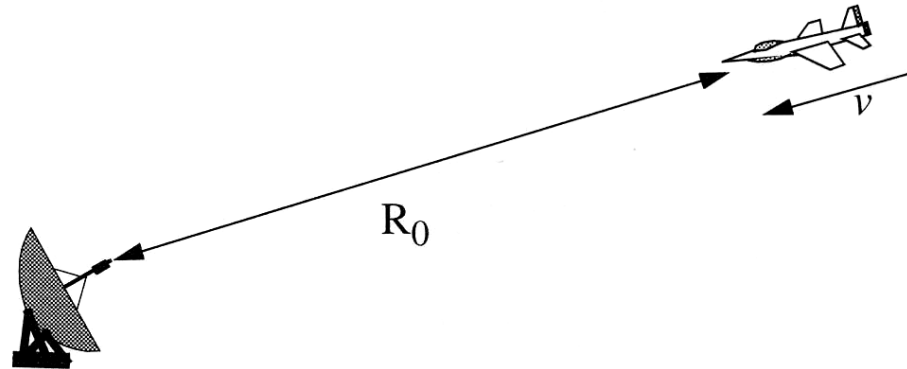
Doppler Frequency Derivation (1)



$$f_d = f_0' - f_0 = \frac{c+v}{c-v} f_0 - f_0 = \frac{2v}{c-v} f_0 \quad (1.25)$$

$$f_d \approx \frac{2v}{c} f_0 = \frac{2v}{\lambda} \quad (\Theta v \ll c, \quad c = \lambda f_0) \quad (1.26)$$

Doppler Frequency Derivation (2)



< Closing target with velocity v >

- the range to the target at any time t , $R(t)$

$$R(t) = R_0 - v(t - t_0) \quad (1.27) \quad R_0 : \text{the range at time } t_0 \text{ (time reference)}$$

- the signal received by the radar $x_r(t)$

$$x_r(t) = x(t - \psi(t)) \quad (1.28) \quad x(t) : \text{transmitted signal}$$

$$\psi(t) = \frac{2}{c}(R_0 - vt + vt_0) \quad (1.29)$$

Doppler Frequency Derivation (2)

- substituting Eq.(1.29) into Eq.(1.28)

$$x_r(t) = x\left(\left(1 + \frac{2v}{c}\right)t - \psi_0\right) \quad (1.30) \quad \psi_0 : \text{constant phase}$$

$$\psi_0 = \frac{2R_0}{c} + \frac{2v}{c}t_0 \quad (1.31)$$

- compression or scaling factor γ

$$\gamma = 1 + \frac{2v}{c} \quad (1.32)$$

- using Eq.(1.32), rewrite Eq.(1.30)

$$x_r(t) = x(\gamma t - \psi_0) \quad (1.33)$$

- a time-compressed version of the returned signal from a stationary target
- based on the scaling property of the Fourier transform
 - the spectrum of the received signal will be expanded in frequency by a factor of γ

Doppler Frequency Derivation (2)

- consider the special case

$$x(t) = y(t) \cos w_0 t \quad (1.34)$$

w_0 : radar center frequency in radians per second

- received signal $x_r(t)$

$$x_r(t) = y(\gamma t - \psi_0) \cos(\gamma w_0 t - \psi_0) \quad (1.35)$$

- Fourier transform of Eq.(1.35)

$$X_r(w) = \frac{1}{2\gamma} \left(Y\left(\frac{w}{\gamma} - w_0\right) + Y\left(\frac{w}{\gamma} + w_0\right) \right) \quad (1.36)$$

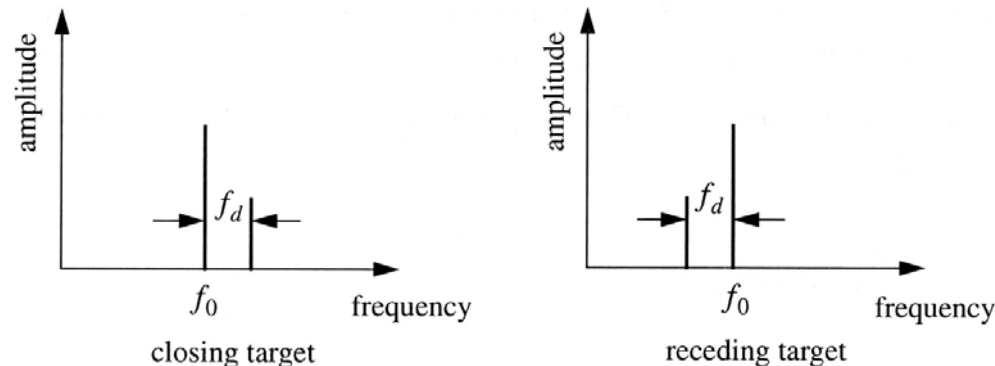
Doppler Frequency Derivation (2)

- where for simplicity the effects of the constant phase ψ_0 have been ignored
- band pass spectrum \rightarrow centered at ω_0 instead of ω_0
- difference between the two values incurred due to the target motion

$$\omega_d = \omega_0 - \gamma \omega_0 \quad (1.37) \quad \longleftarrow \quad \gamma = 1 + \frac{2v}{c}, \quad \omega = 2\pi f$$

$$f_d = \frac{2v}{c} f_0 = \frac{2v}{\lambda} \quad (1.38) \quad \longrightarrow \quad \text{same as Eq.(1.26)}$$

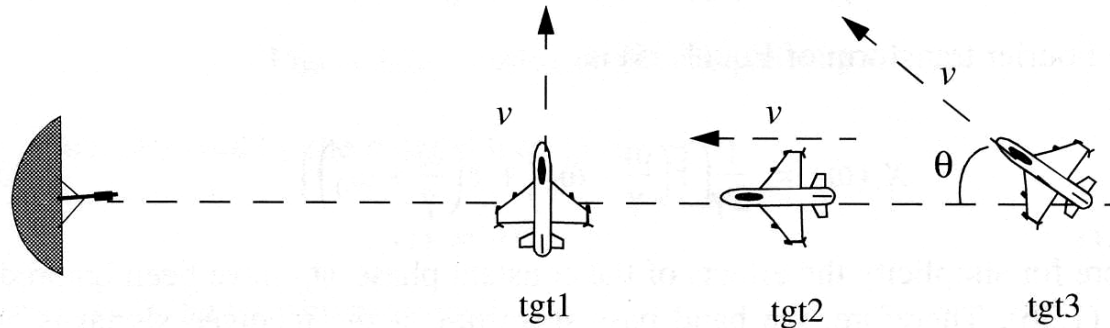
- for a receding target the Doppler shift $f_d = -2v/\lambda$



< Spectra of radar received signal >

Doppler Frequency Effect

- Doppler frequency depends on radial velocity



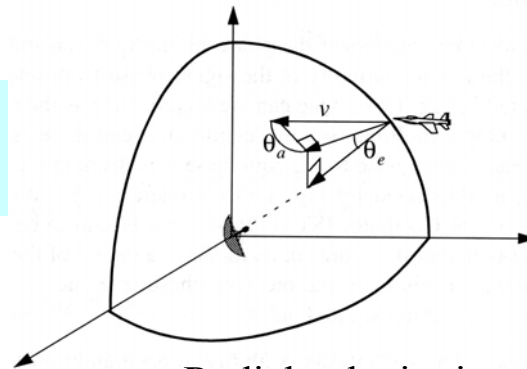
< Target1 generates zero Doppler. Target2 generates maximum Doppler. Target3 is in-between >

- General expression for f_d

$$f_d = \frac{2v}{\lambda} \cos \theta \quad (1.39)$$

for an opening target

$$f_d = -\frac{2v}{\lambda} \cos \theta \quad (1.40)$$



$$\cos \theta = \cos \theta_e \cos \theta_a$$

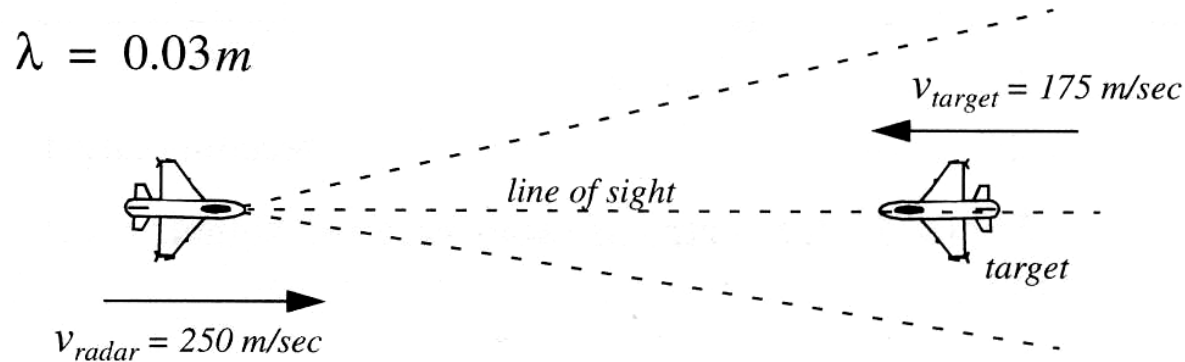
θ_e : elvation angle

θ_a : azimuth angle

< Radial velocity is proportional to the azimuth and elevation angles >

Example 1.3

- Compute the Doppler frequency measured by the radar shown in the figure



$$f_d = 2 \frac{(250 + 175)}{0.03} = 28.3 \text{ KHz}$$

Similarly, if the target were opening the Doppler frequency is

$$f_d = 2 \frac{(250 - 175)}{0.03} = 5 \text{ KHz}$$

MATLAB Function “doppler_freq.m”

$$[f_d, tdr] = \text{doppler_freq}(freq, ang, tv, indicator)$$

Symbol	Description	Units	Status
<i>freq</i>	<i>radar operating frequency</i>	<i>Hz</i>	<i>input</i>
<i>ang</i>	<i>aspect angle</i>	<i>degrees</i>	<i>input</i>
<i>tv</i>	<i>target velocity</i>	<i>m/sec</i>	<i>input</i>
<i>indicator</i>	<i>1 for closing target, 0 otherwise</i>	<i>none</i>	<i>input</i>
<i>fd</i>	<i>Doppler frequency</i>	<i>Hz</i>	<i>output</i>
<i>tdr</i>	<i>time dilation factor ratio τ' / τ</i>	<i>none</i>	<i>output</i>

1. $freq = 10\text{GHz}$, $ang = 0^\circ$, $tv = 175\text{m/s}$, $indicator = 1$

Output $\rightarrow tdr = 0.99999883333401$

2. $freq = 10\text{GHz}$, $ang = 0^\circ$, $tv = 175\text{m/s}$, $indicator = 0$

Output $\rightarrow tdr = 1.00000116666735$

Coherence – Continuity of Phase

- COHERENT

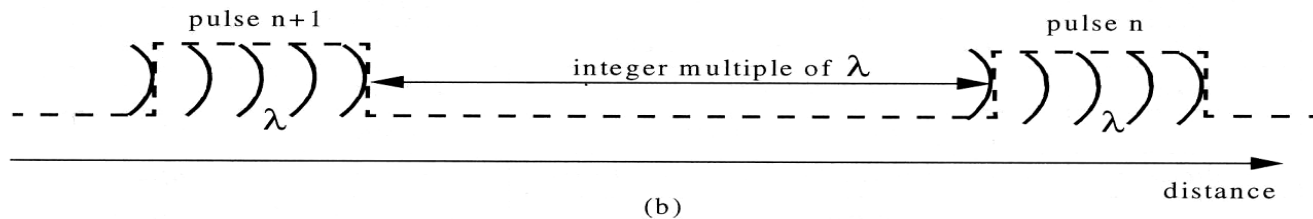
- the phase of any two transmitted pulse is consistent (Fig .a)
- to maintain an integer multiple of wavelengths between the equiphase wavefront (Fig .b) using STALO

- COHERENT-ON-RECEIVER (or quasi-coherent)

- stores a record of the phase of transmitted phase



(a) Phase continuity between consecutive pulses.



- (b) Maintaining an integer multiple of wavelengths between the equiphase wavefronts of any two successive pulses guarantees coherency.

Doppler Frequency Extraction

- coherence : refer to extract the received signal phase
- *only coherent or coherent-on-receiver radars* → extract Doppler inform.

$$f_i = \frac{1}{2\pi} \frac{d}{dt} \phi(t) \quad (1.14) \quad f_i : \text{instantaneous frequency}$$

$\phi(t) : \text{signal phase}$

Ex) signal

$$x(t) = \cos(\gamma w_0 t - \varphi_0) \quad (1.42) \quad \gamma : \text{scaling factor}$$

$\varphi_0 : \text{constant phase}$

$$f_i = f_0 \quad (1.43) \quad \leftarrow \quad w_0 = 2\pi f_0$$

$$f_i = \gamma \left(1 + \frac{2v}{c} \right) = f_0 + \underbrace{\frac{2v}{\lambda}}_{\text{Doppler shift}} \quad (1.44) \quad \leftarrow \quad c = \lambda f$$

Radar System Parameters

- Frequency (f)
- Detection Range(R)
- PRF (Pulse Repetition Frequency)
- Pulse Width (τ)
- System Bandwidth (B_n)
- Range Resolution(ΔR)
- Peak Power (P_t)
- Max Average Power (P_{av})

- Scan Coverage
- Scan Rate
- Antenna Beam Width (Θ_3)
- Antenna Gain (G)
- Receiver Noise Figure (F_n)
- RCS (Radar Cross Section, σ)
- Prob of False Alarm (P_{fa})
- Prob. of Detection (P_D)

$$R = \left(\frac{P_T G^2 \lambda^2 \sigma T_D f_r \tau}{(4\pi)^3 (SNR)_n k T_e F L} \right)^{1/4}$$

Radar Equation – Derivation

(1) peak power density (P_D) in case of omni antenna

$$P_D = \frac{\text{Peak transmitted power}}{\text{area of a sphere}} \quad \frac{\text{watts}}{\text{m}^2} \quad (1.45)$$

$$= \frac{P_t}{4\pi R^2} \quad (1.46) \quad (\text{assuming a lossless propagation medium})$$

- case of directional antenna

$$A_e = \frac{G\lambda^2}{4\pi} \quad (1.47) \quad A_e: \text{ant. effective aperture} \quad G: \text{ant. gain}$$

$$A_e = \rho A \quad (1.48) \quad 0 \leq \rho \leq 1 \quad \rho: \text{aperture efficiency}$$

$$\rho \approx 0.7$$

Power Density at R

(3) power density P_D (distant R , antenna gain G)

$$P_D = \frac{P_t G}{4\pi R^2} \quad (1.49)$$

- the radar radiated energy impinges on a target
→ the amount of the radiated energy is proportional to target RCS

(4) RCS (Radar Cross Section)

: defined as the ratio of the power reflected back to the radar to the power density incident on the target

$$\sigma = \frac{P_r}{P_D} \text{ m}^2 \quad (1.50) \quad P_r : \text{reflected power}$$

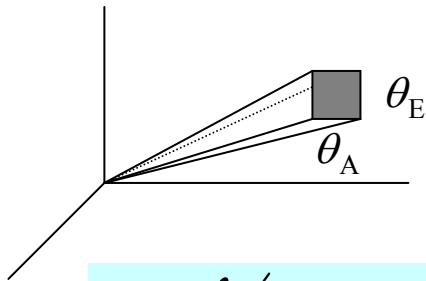
(5) total power delivered to the radar signal processor by the ant.

$$P_{Dr} = \frac{P_t G \sigma}{(4\pi R^2)^2} A_e \quad (1.51) \quad \leftarrow \quad A_e = \frac{G \lambda^2}{4\pi}$$

$$= \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (1.52)$$

Radar Range Equation

$$\therefore R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right]^{1/4}$$



$$G = \frac{4\pi}{\theta_E \theta_A}$$

$$\theta_E = \lambda / D_E, \quad \theta_A = \lambda / D_A$$

$$\therefore G = \frac{4\pi D_A D_E}{\lambda^2} = \frac{4\pi A_e}{\lambda^2}$$

$$P_R = \frac{P_T G_T \sigma A_E}{(4\pi)^2 R^4}$$

$$= \frac{K_R \sigma}{R^4 L_A}$$

where $K_R = \frac{P_T G_T A_E}{(4\pi)^2 L_S}$

L_S = radar system loss

L_A = propagation path loss

Maximum Radar Range → MDS

(6) maximum radar range R_{\max}

$$R_{\max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right)^{1/4} \quad (1.53)$$

S_{\min} : minimum detectable signal power

- in order to double the radar maximum range → P_t sixteen times
→ A_e four times

(7) In practical, the returned signal received corrupted with noise

- noise : random, described by Power Spectral Density function
- noise power N

$$N = \text{Noise PSD} \times B \quad (1.54) \quad B: \text{radar operating bandwidth}$$

- input noise power to a lossless ant.

$$N_i = k T_e B \quad (1.55) \quad k : 1.38 \times 10^{-23} \text{ joule/degree Kelvin (Boltzman's constant)}$$

T_e : effective noise temperature in degree Kelvin

Radar Equation with SNR

(8) noise figure(F) : the fidelity of a radar receiver is described by a figure of merit

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i/N_i}{S_o/N_o} \quad (1.56)$$

$(SNR)_i, (SNR)_o$: signal to noise ratio (SNR) at input and output of the receiver

- Eq.(1.55) rearranging

$$S_i = kT_e B F (SNR)_o \quad (1.57)$$

$$S_{\min} = kT_e B F (SNR)_{o_{\min}} \quad (1.58)$$

- substituting Eq.(1.58) into Eq.(1.53)

$$R_{\max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F (SNR)_{o_{\min}}} \right)^{1/4} \quad (1.59)$$

$$(SNR)_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F R^4} \quad (1.60)$$

$$(SNR)_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F L R^4}$$

Radar losses

Example 1.4

- A certain C-band radar with the following parameters:

Peak power $P_t = 1.5MW$, operating frequency $f_0 = 5.6GHz$, antenna gain $G = 45dB$, effective temperature $T_e = 290K$, pulse width $\tau = 0.2\mu sec$.

The radar threshold is $(SNR)_{min} = 20dB$. Assume target cross section $\sigma = 0.1m^2$.

Compute the maximum range.

solution : the radar bandwidth is $B = \frac{1}{\tau} = \frac{1}{0.2 \times 10^{-6}} = 5MHz$

the wavelenth is $\lambda = \frac{c}{f_0} = \frac{3 \times 10^8}{5.6 \times 10^9} = 0.054m$

$$(R^4)_{dB} = (P_t + G^2 + \lambda^2 + \sigma - (4\pi)^3 - kT_e B - F - (SNR)_{o_{min}})_{dB}$$

P_t	λ^2	G^2	$kT_e B$	$(4\pi)^3$	F	$(SNR)_{o_{min}}$	σ
61.761	-25.421	90dB	-136.987	32.976	3dB	20dB	-10

$$R^4 = 61.761 + 90 - 25.352 - 10 - 32.976 + 136.987 - 3 - 20 = 197.420dB$$

$$R^4 = 10^{197.420/10} = 55.208 \times 10^{18} m^4$$

$$R = 86.199km$$

The maximum detection range is 86.2 Km

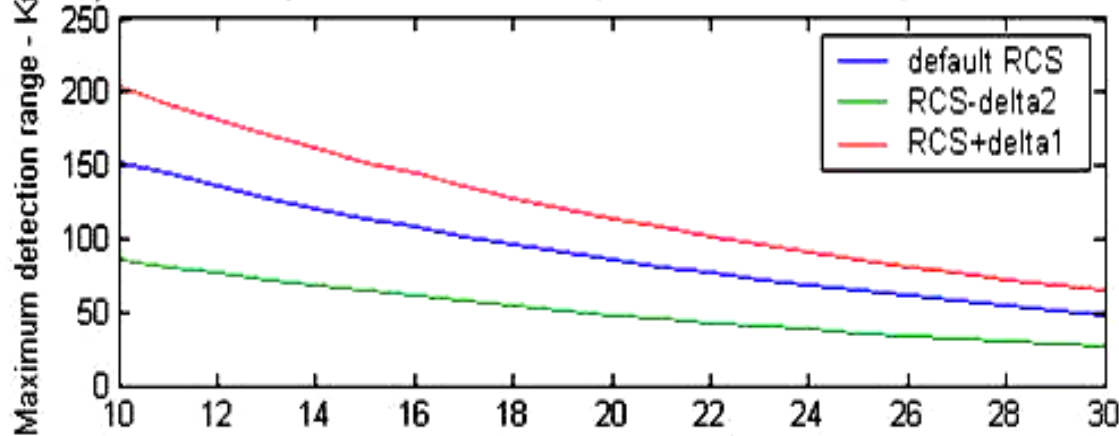
MATLAB Function “radar_eq.m”

$[out_par] = radar_eq(pt, freq, g, sigma, te, b, nf, loss, input_par, option, rcs_delta1, rcs_delta2, pt_percent1, pt_percent2)$

Symbol	Description	Units	Status
<i>pt</i>	<i>peak power</i>	<i>KW</i>	<i>input</i>
<i>freq</i>	<i>frequency</i>	<i>Hz</i>	<i>input</i>
<i>g</i>	<i>antenna gain</i>	<i>dB</i>	<i>input</i>
<i>sigma</i>	<i>target cross section</i>	<i>m²</i>	<i>input</i>
<i>te</i>	<i>effective temperature</i>	<i>Kelvin</i>	<i>input</i>
<i>b</i>	<i>bandwidth</i>	<i>Hz</i>	<i>input</i>
<i>nf</i>	<i>noise figure</i>	<i>dB</i>	<i>input</i>
<i>loss</i>	<i>radar losses</i>	<i>dB</i>	<i>input</i>
<i>input_par</i>	<i>SNR, or R_{max}</i>	<i>dB, or Km</i>	<i>input</i>
<i>option</i>	<i>1 means input_par = SNR 2 means input_par = R</i>	<i>none</i>	<i>input</i>
<i>rcs_delta1</i>	<i>rcs delta1 (sigma - delta1)</i>	<i>dB</i>	<i>input</i>
<i>rcs_delta2</i>	<i>rcs delta2 (sigma + delta2)</i>	<i>dB</i>	<i>input</i>
<i>pt_percent1</i>	<i>pt * pt_percent1%</i>	<i>none</i>	<i>input</i>
<i>pt_percent2</i>	<i>pt * pt_percent2%</i>	<i>none</i>	<i>input</i>
<i>out_par</i>	<i>R for option = 1 SNR for option = 2</i>	<i>Km, or dB</i>	<i>output</i>

MATLAB Function “radar_eq.m”

< peak power=61.8dB, default RCS=-10dBsm, RCS-delta2=-20dBsm, RCS+delta1=-5dBsm >



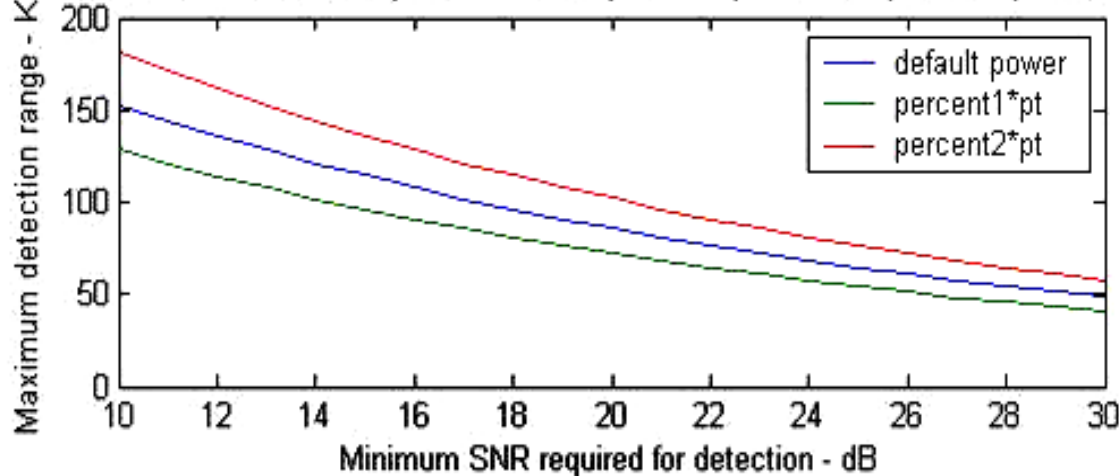
option 1

default RCS : 0.1m²

delta1 : 5dB

delta2 : 10dB

< RCS=-10dBsm, default power=61.8dB, percent1*pt=58.8dB, percent2*pt=64.8dB >



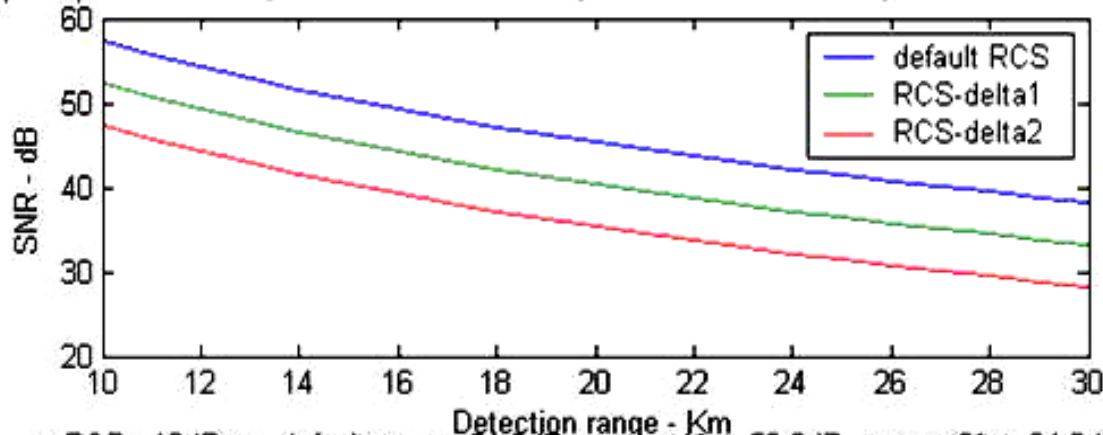
default Power : 1.5MW

percent1 : 0.5

percent2 : 2

MATLAB Function “radar_eq.m”

< peak power=61.8dB, default RCS=-10dBsm, RCS-delta1=-15dBsm, RCD-delta2=-20dBsm >



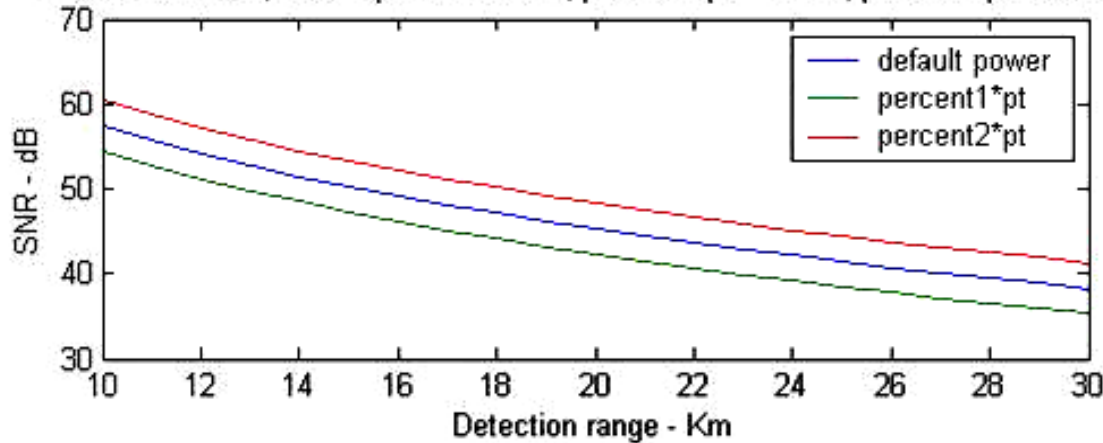
option 2

default RCS : 0.1m²

delta1 : 5dB

delta2 : 10dB

< RCS=-10dBsm, default power=61.8dB, percent1*pt=58.8dB, percent2*pt=64.8dB >



default Power : 1.5MW

percent1 : 0.5

percent2 : 2

Low PRF Radar Equation

Parameters

$P_{av} = P_t d_t$: Average transmitted power

$d_t = \tau/T$: Transmission duty factor

$d_r = \frac{T-\tau}{T} = 1 - \tau f_r$ (1.62) : Receiving duty factor

for low PRF radars ($T \gg \tau$) receiving duty factor is $d_r \approx 1$.

$T_i = \frac{n_p}{f_r} \Rightarrow n_p = T_i f_r$ (1.63) : Time on target = Dwell Time

n_p : number of pulses that strikes the target

f_r : radar PRF

Low PRF Radar Equation

-Single pulse radar equation

$$(SNR)_1 = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_e BFL} \quad (1.64)$$

-Integrated pulses

$$(SNR)_{n_p} = \frac{P_t G^2 \lambda^2 \sigma n_p}{(4\pi)^3 R^4 k T_e BFL} \quad (1.65)$$

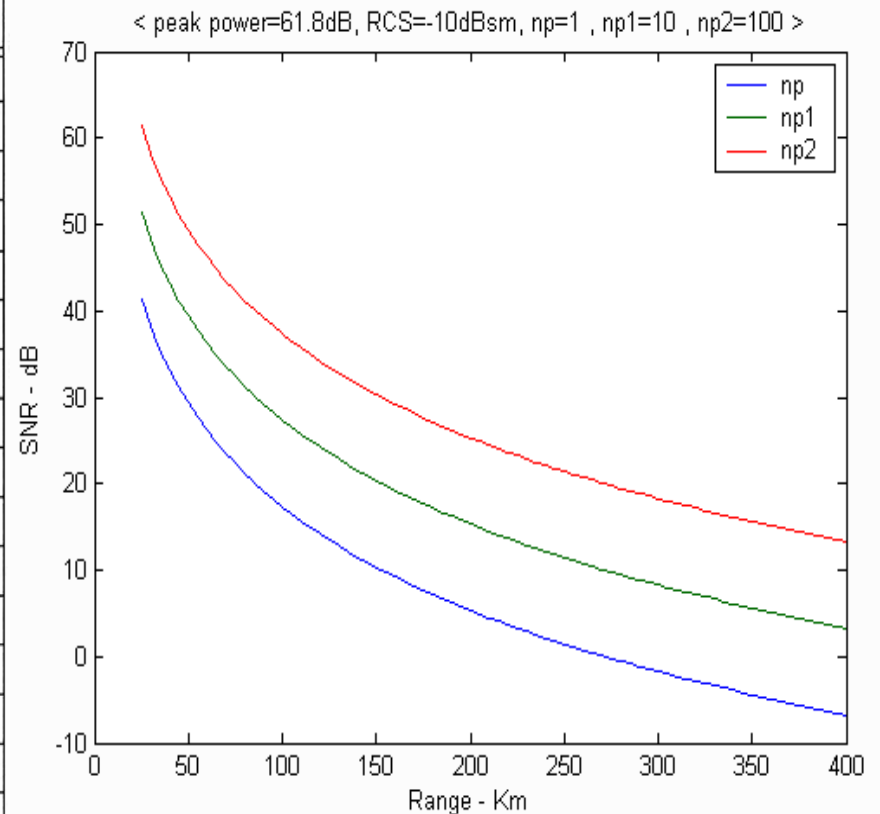
-Using Eq.(1.63) and $B=1/\tau$

$$(SNR)_{n_p} = \frac{P_t G^2 \lambda^2 \sigma T_i f_r \tau}{(4\pi)^3 R^4 k T_e FL} \quad (1.66)$$

MATLAB “lprf_req.m”

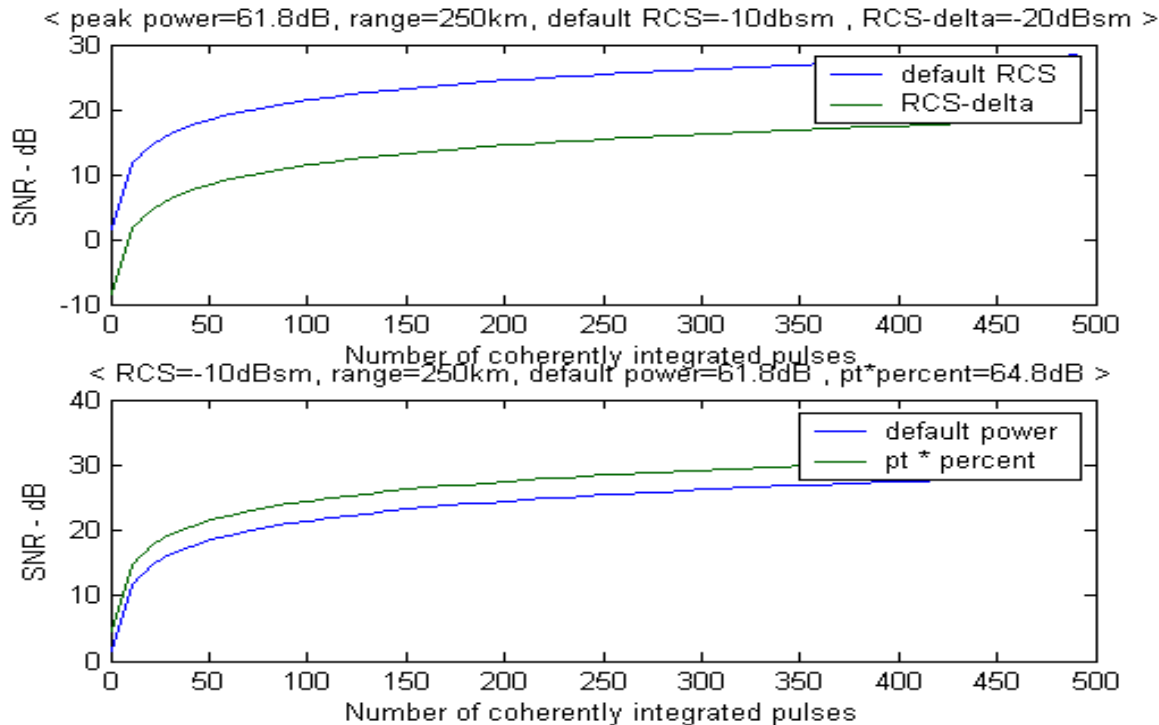
- The function “lprf_req.m” computes $(\text{SNR})_{np}$.
- Plot SNR vs range for three sets of coherently integrated pulses

Symbol	Description	Units	Status
<i>pt</i>	<i>peak power</i>	<i>KW</i>	<i>input</i>
<i>freq</i>	<i>frequency</i>	<i>Hz</i>	<i>input</i>
<i>g</i>	<i>antenna gain</i>	<i>dB</i>	<i>input</i>
<i>sigma</i>	<i>target cross section</i>	<i>m²</i>	<i>input</i>
<i>te</i>	<i>effective temperature</i>	<i>Kelvin</i>	<i>input</i>
<i>b</i>	<i>bandwidth</i>	<i>Hz</i>	<i>input</i>
<i>nf</i>	<i>noise figure</i>	<i>dB</i>	<i>input</i>
<i>loss</i>	<i>radar losses</i>	<i>dB</i>	<i>input</i>
<i>range</i>	<i>target range</i>	<i>Km</i>	<i>input</i>
<i>prf</i>	<i>PRF</i>	<i>Hz</i>	<i>input</i>
<i>np</i>	<i>number of pulses</i>	<i>none</i>	<i>input</i>
<i>np1</i>	<i>choice 1 for np</i>	<i>none</i>	<i>input</i>
<i>np2</i>	<i>choice 2 for np</i>	<i>none</i>	<i>input</i>
<i>rcs_delta</i>	<i>rcs delta1 (sigma - delta)</i>	<i>dB</i>	<i>input</i>
<i>pt_percent</i>	<i>pt * pt_percent%</i>	<i>none</i>	<i>input</i>
<i>snr_out</i>	<i>SNR</i>	<i>dB</i>	<i>output</i>



MATLAB ‘lprf_req.m’

- Plot of SNR vs number of coherently integrated pulses for two choices of the default RCS and Peak power



- Integrating a limited number of pulses can significantly enhance the SNR; however, integrating large amount of pulses does not provide any further major improvement.

High PRF Radar Equation

-Single pulse radar equation for a high PRF Radar

$$SNR = \frac{P_t G^2 \lambda^2 \sigma d_t^2}{(4\pi)^3 R^4 k T_e B F L d_r} \quad (1.67)$$

$$-d_r \approx d_t = \tau f_r \quad B = 1/T_i$$

$$SNR = \frac{P_t G^2 \lambda^2 \sigma \tau f_r T_i}{(4\pi)^3 R^4 k T_e F L} \quad (1.68)$$

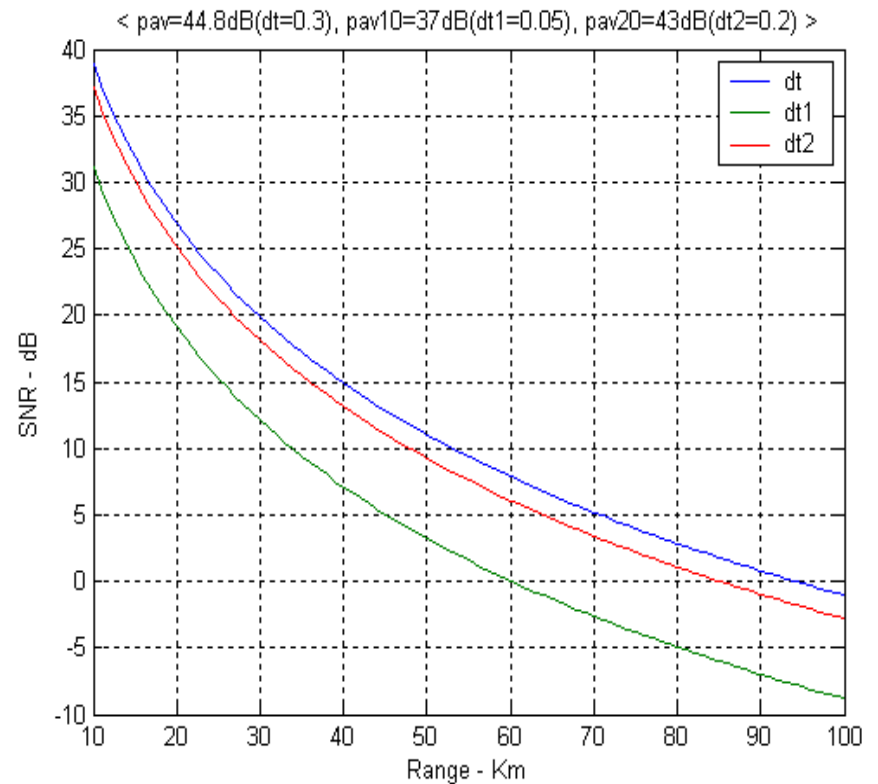
- finally

$$SNR = \frac{P_{av} G^2 \lambda^2 \sigma T_i}{(4\pi)^3 R^4 k T_e F L} \quad (1.69)$$

MATLAB ‘hprf_req.m’

- Plot of SNR vs range for three duty cycle choices

Symbol	Description	Units	Status
<i>pt</i>	<i>peak power</i>	<i>KW</i>	<i>input</i>
<i>freq</i>	<i>frequency</i>	<i>Hz</i>	<i>input</i>
<i>g</i>	<i>antenna gain</i>	<i>dB</i>	<i>input</i>
<i>sigma</i>	<i>target cross section</i>	<i>m²</i>	<i>input</i>
<i>dt</i>	<i>duty cycle</i>	<i>none</i>	<i>input</i>
<i>ti</i>	<i>time on target</i>	<i>seconds</i>	<i>input</i>
<i>range</i>	<i>target range</i>	<i>Km</i>	<i>input</i>
<i>te</i>	<i>effective temperature</i>	<i>Kelvin</i>	<i>input</i>
<i>nf</i>	<i>noise figure</i>	<i>dB</i>	<i>input</i>
<i>loss</i>	<i>radar losses</i>	<i>dB</i>	<i>input</i>
<i>prf</i>	<i>PRF</i>	<i>Hz</i>	<i>input</i>
<i>tau</i>	<i>pulse width</i>	<i>seconds</i>	<i>input</i>
<i>dt1</i>	<i>duty cycle choice 1</i>	<i>none</i>	<i>input</i>
<i>dt2</i>	<i>duty cycle choice 2</i>	<i>none</i>	<i>input</i>
<i>snr_out</i>	<i>SNR</i>	<i>dB</i>	<i>output</i>



Example 1.5

- Compute the single pulse SNR for a high PRF radar with the following Parameters: peak power $P_t=100\text{KW}$, antenna gain $G=20\text{dB}$, operating frequency $f_0=5.6\text{GHz}$, losses $L=8\text{dB}$, noise figure $F=5\text{dB}$, effective temperature $T_e=400\text{K}$, dwell interval $T_i=2\text{s}$, duty factor $dt=0.3$. The range of interest is $R=50\text{Km}$. Assume target RCS $\sigma=0.01\text{m}^2$.

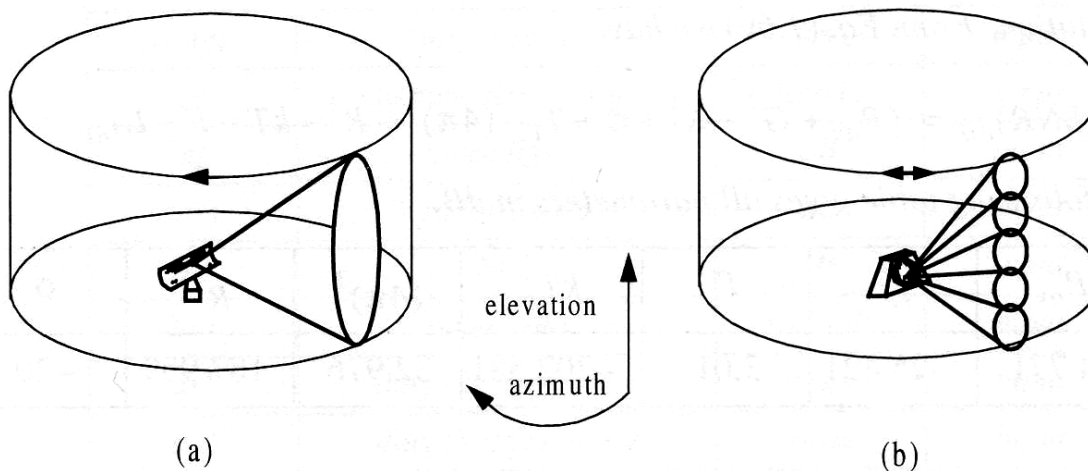
$$(SNR)_{dB} = (P_{av} + G^2 + \lambda^2 + \sigma + T_i - (4\pi)^3 - R^4 - kT - F - L)_{dB}$$

solution

$$(SNR)_{dB} = 44.771 + 40 - 25.42 - 20 + 3.01 - 32.976 + 202.581 - 187.959 - 5 - 8 = 11.006\text{dB}$$

Surveillance Radar Equation

- Surveillance or search radars continuously scan a specified volume in space searching for targets.
- 2D Radar → (a): fan search pattern , (b): stacked search pattern



(a) pattern radar → steered in azimuth.

(b) pattern radar → steered in azimuth and elevation.

(employed by phased array radar)

Surveillance Radar Equation

- Search volume : search solid angle Ω
- Antenna 3dB beam width : θ_a and θ_e
- number of antenna beam position (n_B)

$$n_B = \frac{\Omega}{\theta_a \theta_e} = \frac{\Omega}{\theta_{3dB}^2} \quad (1.70)$$

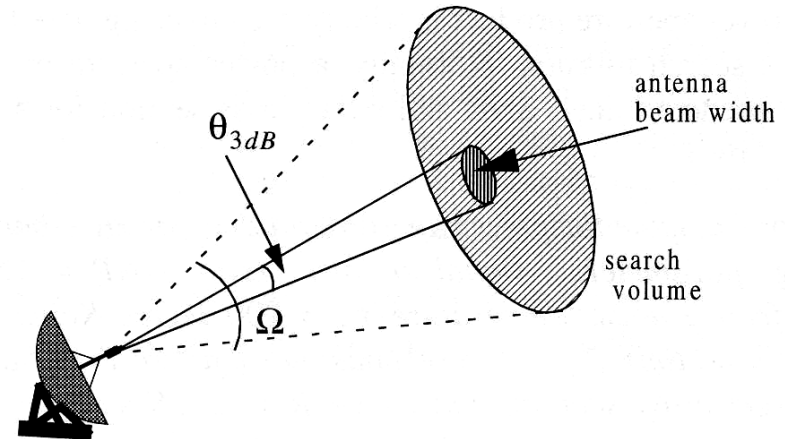
- for a circular aperture of diameter D

$$\theta_{3dB} \approx \frac{\lambda}{D} \quad (1.71)$$

- when aperture tapering is used, $\theta_{3dB} = 1.25 / D$

Substituting Eq.(1.71) into Eq.(1.70)

$$n_B = \frac{D^2}{\lambda^2} \Omega \quad (1.72)$$



< A cut in space showing the antenna beam width and the search volume >

Surveillance Radar Equation

- Time on target (expressed in terms of T_{sc} :scan time)

$$T_i = \frac{T_{sc}}{n_B} = \frac{T_{sc} \lambda^2}{D^2 \Omega} \quad T_{sc} : \text{Scan time} \quad (1.73)$$

- Search Radar Equation

$$SNR = \frac{P_{av} G^2 \lambda^2 \sigma T_{sc} \lambda^2}{(4\pi)^3 R^4 k T_e F L D^2 \Omega} \quad (1.74)$$

- using Eq.(1.47) in Eq.(1.74)

$$SNR = \frac{P_{av} A \sigma T_{sc}}{16 R^4 k T_e L F \Omega} \quad A = \pi D^2 / 4 \text{ (aperture area)} \quad (1.75)$$

- Power aperture product : $P_{av} A$
- Computed to meet predetermined SNR and RCS for a given search volume defined by Ω

Example 1.6

- Compute the power aperture product for an X-band radar

Parameter \Rightarrow SNR = 15dB; L=8dB; $T_e=900$ degree Kelvin; $\Omega=2^\circ$; $T_{sc}=2.5$ sec; F=5dB. Assume a -10dBsm target cross section, and R=250Km.

Compute the Peak transmitted power corresponding to 30% duty factor, if the antenna gain is 45dB.

Solution: *Solid angle coverage* : $\Omega = \frac{2 \times 2}{(57.23)^2} = -29.132 \text{ dB}$

$$(SNR)_{dB} = (P_{av} + A + \sigma + T_{sc} - 16 - R^4 - kT_e - L - F - \Omega)_{dB}$$

$$15 = P_{av} + A - 10 + 3.979 - 12.041 - 215.918 + 199.054 - 5 - 8 + 29.133$$

power aperture product : $P_{av} + A = 33.793 \text{ dB}$

radar wavelength : $\lambda = 0.03 \text{ m}$

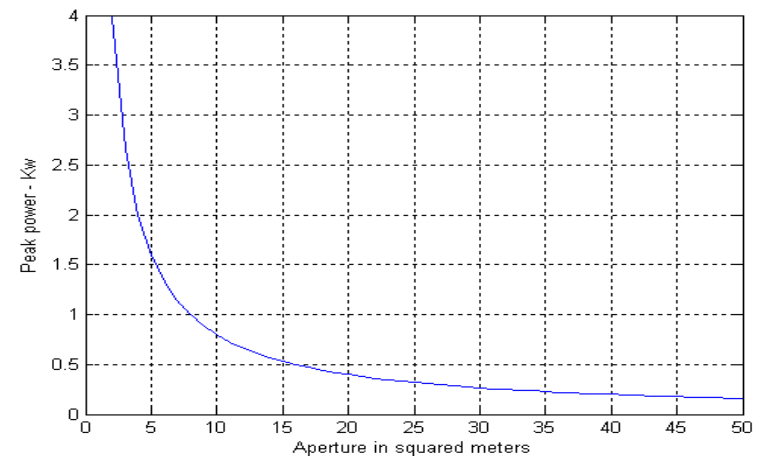
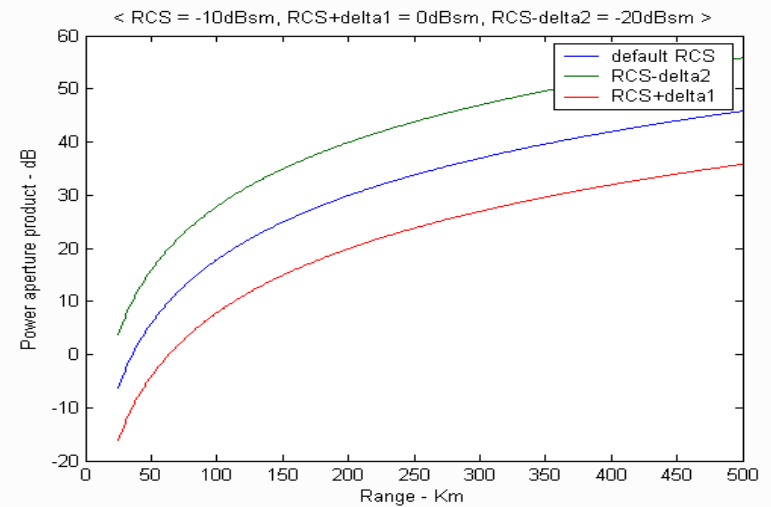
$$A = \frac{G\lambda^2}{4\pi} = 3.550 \text{ dB} ; \quad \Rightarrow \quad P_{av} = -A + 33.793 = 30.243 \text{ dB} = 10^{3.0243} = 1057.548 \text{ W}$$

$$P_t = \frac{P_{av}}{d_t} = \frac{1057.548}{0.3} = 3.52516 \text{ KW}$$

MATLAB “power_aperture_eq.m”

-Plots of peak power vs. aperture area and the power aperture product vs. range

Symbol	Description	Units	Status
<i>snr</i>	sensitivity snr	dB	input
<i>freq</i>	frequency	Hz	input
<i>tsc</i>	scan time	seconds	input
<i>sigma</i>	target cross section	m ²	input
<i>dt</i>	duty cycle	none	input
<i>range</i>	target range	Km	input
<i>te</i>	effective temperature	Kelvin	input
<i>nf</i>	noise figure	dB	input
<i>loss</i>	radar losses	dB	input
<i>az_angle</i>	search volume azimuth extent	degrees	input
<i>el_angle</i>	search volume elevation extent	degrees	input
<i>g</i>	antenna gain	dB	input
<i>rcs_delta1</i>	rcs delta 1 (<i>sigma</i> - <i>delta1</i>)	dB	input
<i>rcs_delta2</i>	rcs delta2 (<i>sigma</i> + <i>delta2</i>)	dB	input
<i>p_a_p</i>	power aperture product	dB	output
<i>aperture</i>	antenna aperture	m ²	output
<i>pt</i>	peak power	KW	output
<i>pav</i>	average power	KW	output



Radar Equation with Jamming

■ ECM (Electronic Countermeasure)

→ chaff, radar decoys, radar RCS alteration, and radar jamming

■ Jammers

1) Barrage jammers

: Attempt to increase the noise level across the entire radar operating BW.

Can be deployed in the main beam or in side lobes of the radar antenna.

2) Deceptive jammers (repeaters)

: Carry receiving devices on board in order to analyze the radar's transmission, and then send back false target-like signals in order to confuse the radar.

(1) spot noise repeaters – measures the transmitted radar signal BW and then jams only a specific range of frequencies.

(2) deceptive repeaters – sends back altered signals that make the target appear in some false position (ghosts).

Self-Screen Jammers (SSJ)

- Escort jammers can also be treated as SSJs if they appear at the same range as that of the targets.
- Single pulse power received by radar at R

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L} \quad (1.76)$$

- Received Power from an SSJ jammer at R

$$P_{SSJ} = \frac{P_J G_J}{4\pi R^2} \frac{AB}{B_J L_J} \quad (1.77)$$

- Substituting Eq.(1.47) into Eq.(1.77)

$$P_{SSJ} = \frac{P_J G_J}{4\pi R^2} \frac{\lambda^2 G}{4\pi} \frac{B}{B_J L_J} \quad (1.78)$$

Self-Screen Jammers (SSJ)

- Radar Eq. for a SSJ case

$$\frac{S}{S_{SSJ}} = \frac{P_t G \sigma B_J L_J}{4\pi P_J G_J R^2 B L} \quad (1.79)$$

- ratio S/S_{SSJ} is less than unity since the jamming power is greater than the target signal power.
- as the target becomes closer to the radar, there will be a certain range such that the ratio S/S_{SSJ} is equal to unity. This range is the crossover or burn-through range.

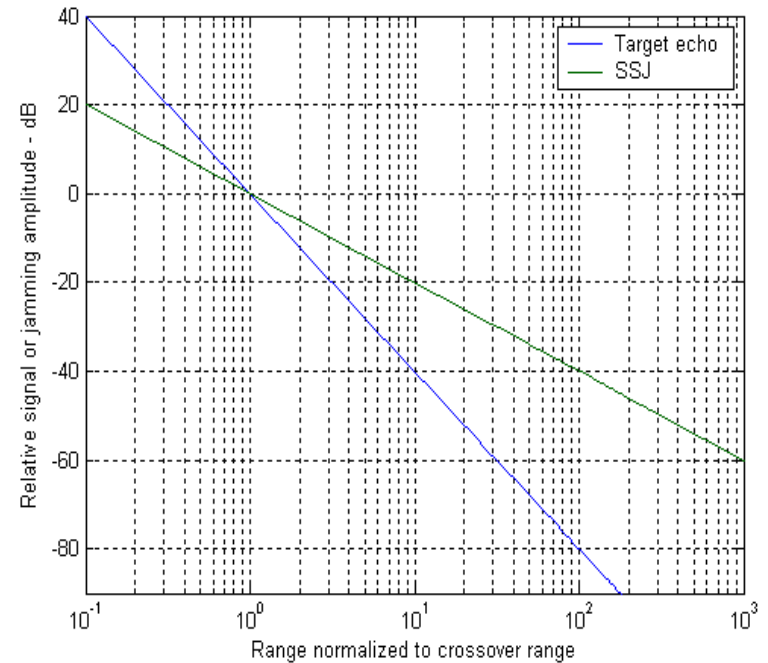
$$(R_{CO})_{SSJ} = \left(\frac{P_t G \sigma B_J L_J}{4\pi P_J G_J B L} \right)^{1/2} \quad (1.80)$$

R_{CO} : crossover range

MATLAB “ssj_req.m”

- calculates the crossover range and generates plots of relative S and S_{SSJ} versus range and generates plots of relative S and S_{SSJ} .

Symbol	Description	Units	Status
<i>pt</i>	radar peak power	KW	input
<i>g</i>	radar antenna gain	dB	input
<i>sigma</i>	target cross section	m^2	input
<i>freq</i>	radar operating frequency	Hz	input
<i>b</i>	radar operating bandwidth	Hz	input
<i>loss</i>	radar losses	dB	input
<i>pj</i>	jammer peak power	KW	input
<i>bj</i>	jammer bandwidth	Hz	input
<i>gj</i>	jammer antenna gain	dB	input
<i>lossj</i>	jammer losses	dB	input
<i>BR_range</i>	burn-through range	Km	output



Stand-Off Jammer (SOJ)

- SOJ emit ECM signals from long ranges which are beyond the defense's lethal capability. Received power from an SOJ jammer at range R_j is

$$P_{SOJ} = \frac{P_J G_J}{4\pi R_J^2} \cdot \frac{\lambda^2 G'}{4\pi} \cdot \frac{B}{B_J L_J} \quad (1.81)$$

- SOJ Radar equation is

$$\frac{S}{S_{SOJ}} = \frac{P_t G^2 R_J^2 \sigma B_J L_J}{4\pi P_J G_J G' R^4 B L} \quad \Leftrightarrow (S = S_{SOJ}) \quad (1.82)$$

$$(R_{co})_{SOJ} = \left(\frac{P_t G^2 R_J^2 \sigma B_J L_J}{4\pi P_J G_J G' B L} \right)^{1/4} \quad (1.83)$$

- Detection range is

$$R_D = \frac{(R_{co})_{SOJ}}{\sqrt[4]{(S / S_{SOJ})_{\min}}} \quad (1.84)$$

where $(S/S_{SOJ})_{\min}$ = min. value of the signal – to – jammer power ratio
such that target detection can occur.

Range Reduction Factor

- Consider a radar system whose detection range R in the absence of jamming,

$$(SNR)_0 = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e B F L R^4} \quad (1.85)$$

- Range Reduction Factor (RRF) refers to the reduction in the radar detection range due to jamming. In the presence of jamming the effective detection range is,

$$R_{dj} = R \times RRF \quad (1.86)$$

- Jammer power in the radar receiver is,

$$P_j = J_o B = k T_j B \quad (1.87)$$

where J_o = output power spectral density of barrage jammer

T_j = jammer effective temperature

- Total jammer plus noise power in the radar receiver is

$$N_i + P_j = k T_e B + k T_j B \quad (1.88)$$

Range Reduction Factor

- The radar detection range is limited by the receiver signal-to-noise plus interference ratio rather than SNR.

$$\left(\frac{S}{P_{SSJ} + N} \right) = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k(T_e + T_J) BFLR^4} \quad (1.89)$$

- The amount of reduction in the signal-to-noise plus interference ratio because of the jammer effect can be computed from the difference between Eqs.(1.85) and (1.89)

$$\gamma = 10.0 \times \log \left(1 + \frac{T_J}{T_e} \right) (dBs) \quad (1.90)$$

- The RRF is

$$RRF = 10^{\frac{-\gamma}{40}} \quad (1.91)$$

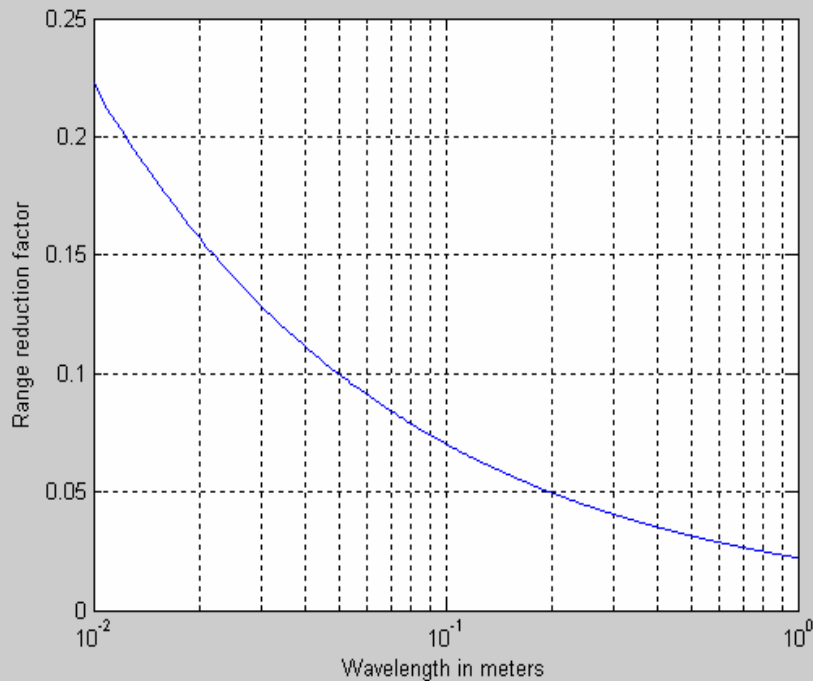
Range Reduction Factor

The function “*range_red_fac.m*” implements Eqs.(1.90) and (1.91)

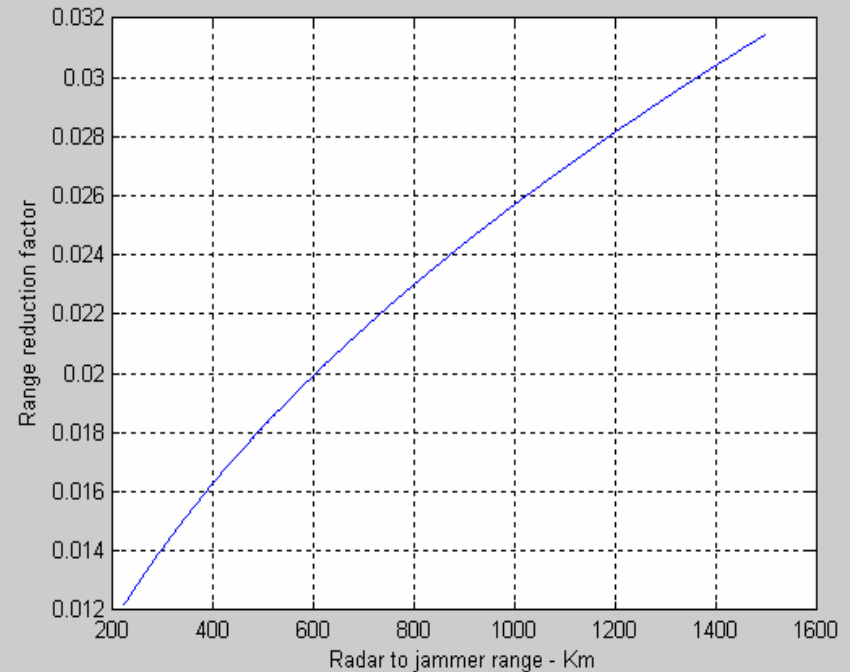
Symbol	Description	Units	Status
<i>te</i>	<i>radar effective temperature</i>	<i>K</i>	<i>input</i>
<i>pj</i>	<i>jammer peak power</i>	<i>KW</i>	<i>input</i>
<i>gj</i>	<i>jammer antenna gain</i>	<i>dB</i>	<i>input</i>
<i>g</i>	<i>radar antenna gain on jammer</i>	<i>dB</i>	<i>input</i>
<i>freq</i>	<i>radar operating frequency</i>	<i>Hz</i>	<i>input</i>
<i>bj</i>	<i>jammer bandwidth</i>	<i>Hz</i>	<i>input</i>
<i>rangej</i>	<i>radar to jammer range</i>	<i>Km</i>	<i>input</i>
<i>lossj</i>	<i>jammer losses</i>	<i>dB</i>	<i>input</i>

<i>te</i>	<i>pj</i>	<i>gj</i>	<i>g</i>	<i>freq</i>	<i>bj</i>	<i>rangej</i>	<i>lossj</i>
730K	150KW	3dB	40dB	10GHz	1MHz	40Km	1dB

MATLAB Function “*range_red_fac.m*”



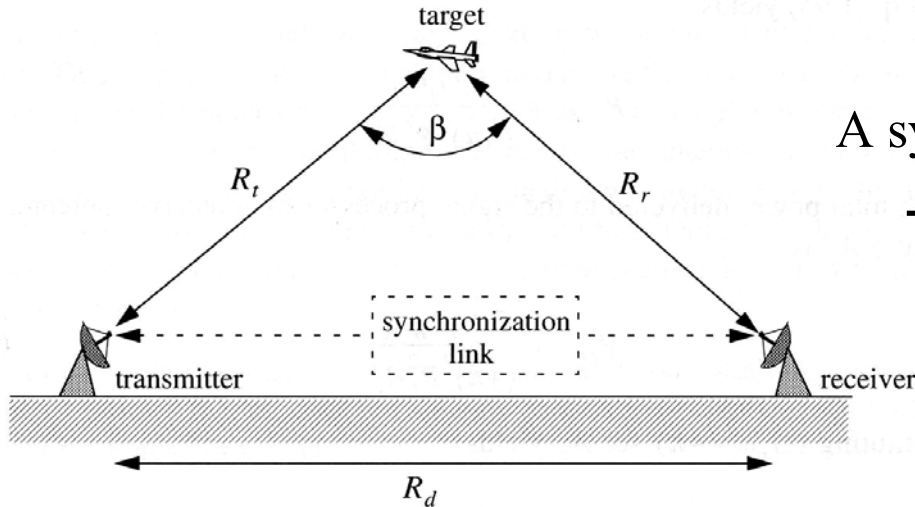
< Range reduction factor versus
radar operating wavelength >



< Range reduction factor versus
radar to jammer range >

Bi-static Radar Equation

- Monostatic radar : use the same ant. for both transmitting and receiving.
- Bi-static radar : use transmit and receive ant. placed in different locations.



A synchronization link

→ extract maximum target
information at Rx

<Bistatic radar geometry>

- Bistatic radar → measured bistatic RCS(σ_B)

Case1. small bistatic angle → bistatic RCS \approx monostatic RCS

Case2. bistatic angle approaches 180° → bistatic RCS becomes large and approximated by

$$\sigma_{B_{\max}} \approx \frac{4\pi A_t^2}{\lambda^2}$$

Bistatic Radar Equation

(1) The power density at the target is

$$P_D = \frac{P_t G_t}{4\pi R_t^2} \quad (1.93)$$

(2) The effective power scattered off a target with bistatic RCS σ_B is

$$P' = P_D \sigma_B \quad (1.94)$$

(3) The power density at the receiver ant. is

$$P_{refl} = \frac{P'}{4\pi R_r^2} = \frac{P_D \sigma_B}{4\pi R_r^2} \quad (1.95)$$

where R_t = range from the radar transmitter to the target

R_r = range from the target to the receiver

$$P_{refl} = \frac{P_D \sigma_B}{4\pi R_r^2} = \frac{P_t G_t \sigma_B}{(4\pi)^2 R_t^2 R_r^2} \quad (1.96)$$

Bistatic Radar Equation

(4) The total power delivered to the signal processor by a receiver ant. with A_e

$$P_{Dr} = \frac{P_t G_t \sigma_B A_e}{(4\pi)^2 R_t^2 R_r^2} \quad (1.97)$$

Substituting $(G_r \lambda^2 / 4\pi)$ for A_e yields

$$P_{Dr} = \frac{P_t G_t G_r \lambda^2 \sigma_B}{(4\pi)^3 R_t^2 R_r^2} \quad (1.98)$$

(5) when transmitter and receiver losses, L_t and L_r , are taken into consideration, the bi-static radar equation is

$$P_{Dr} = \frac{P_t G_t G_r \lambda^2 \sigma_B}{(4\pi)^3 R_t^2 R_r^2 L_t L_r L_p} \quad (1.99)$$

where L_p = medium propagation loss

Radar Losses

■ Radar Losses

- Receiver SNR $\propto (1 / \text{losses})$
- Losses increase \rightarrow drop in SNR \rightarrow decreasing the probability of detection.

(1) Transmit and Receive Losses (typically, 1 to 2 dBs)

- Occur between the radar Tx and ant. Input port and between the ant. output port and receiver front end. \rightarrow often called plumbing losses

(2) Antenna Pattern Loss and Scan Loss

- Radar equation assumed maximum ant. gain.
 - \rightarrow target is located along the ant. boresight axis.
- The loss in the SNR due to not having max. ant. gain on the target at all time is called ant. pattern (shape) loss.
- Consider a $\sin x/x$ ant. radiation pattern (next page), average ant. gain over $\pm \theta/2$. about the boresight axis is \rightarrow next page!!

Atmospheric & Collapsing Losses

(3) Atmospheric Loss

- Atmospheric attenuation is a function of the radar operating frequency, target range, and elevation angle. Atmospheric attenuation can be as high as a few dBs.

(4) Collapsing loss

- When the number of integrated returned noise pulses is larger than the target returned pulses, a drop in the SNR occurs. The collapsing loss factor is

$$\rho_c = \frac{n + m}{n} \quad (1.102)$$

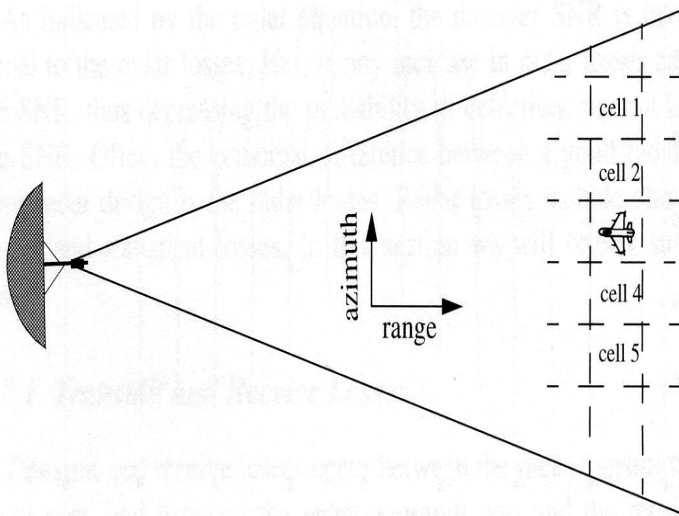
where n = the number of pulses containing
both signal and noise

m = the number of pulses containing noise only.

< Illustration of collapsing loss. Noise source

In cells 1,2,4, and 5 converge to increase

the noise level in cell3>



Processing Losses

(5) Processing Losses

a. Detector Approximation :

- The output voltage signal of a radar receiver (linear detector) is

$$v(t) = \sqrt{v_I^2(t) + v_Q^2(t)} \quad \text{where } (v_I, v_Q) = \text{in-phase and quadrature components.}$$

- For a radar using a square law detector,

$$v^2(t) = v_I^2(t) + v_Q^2(t)$$

- Since in real hardware the operation of squares and square roots are time consuming, many algorithms have been developed for detector approximation. \rightarrow typically 0.5 to 1 dB

CFAR Losses

b. Constant False Alarm Rate (CFAR) Losses

- Radar detection threshold is constantly adjusted as a function of the receiver noise level
 - maintain a constant false alarm rate.
- CFAR processor : keep the number of false alarms under control in a changing and unknown background of interference.
- CFAR processing can cause a loss in the SNR level on the order of 1dB.
- Adaptive CFAR / Nonparametric CFAR / Nonlinear receiver techniques.

Quantization Loss & Range Gate Straddle

c. Quantization Loss

- Finite word length (number of bits) and quantization noise cause and increase in the noise power density at the output of the ADC.
- A/D noise level is $q^2/12$ (q :quantization level)

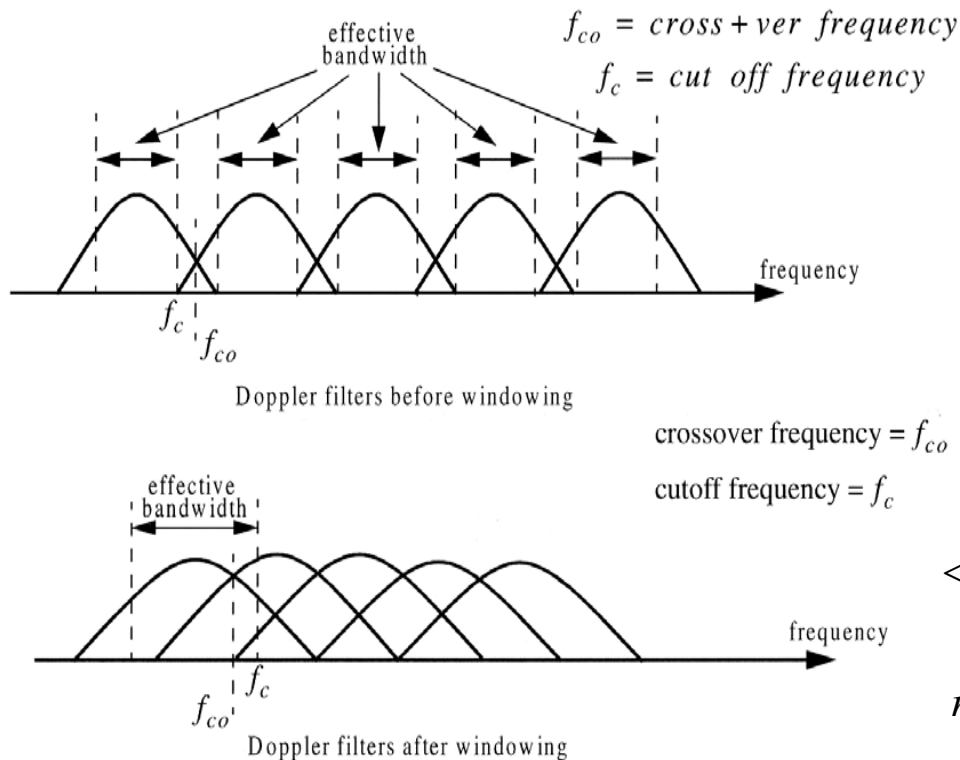
d. Range gate straddle

- Radar receiver is mechanized as a series of contiguous range gate.
- Each range gate is implemented as an integrator matched to the Tx pulse width.
- The smoothed target return envelope is normally straddled to cover more than one range gate.

Doppler Filter Straddle

e. Doppler Filter Straddle

- Doppler filter spectrum is spread (widened) due to weighting functions.
- The target doppler freq. can fall anywhere between two doppler filters, signal loss occurs.



< due to weighting, the crossover freq. f_{co} is smaller than the filter cutoff freq. f_c which normally corresponds to the 3dB power point >

MATLAB Program and Function

Matlab-based Source Code : www.crcpress.com

1.1 pulse train

1.2 range resolution

1.3 doppler frequency

1.4 radar equation

1.5 LPRF radar equation

1.6 HPRF radar equation

1.7 power-aperture radar equation

1.8 SSJ radar equation

1.9 SOJ radar equation

1.10 range reduction factor