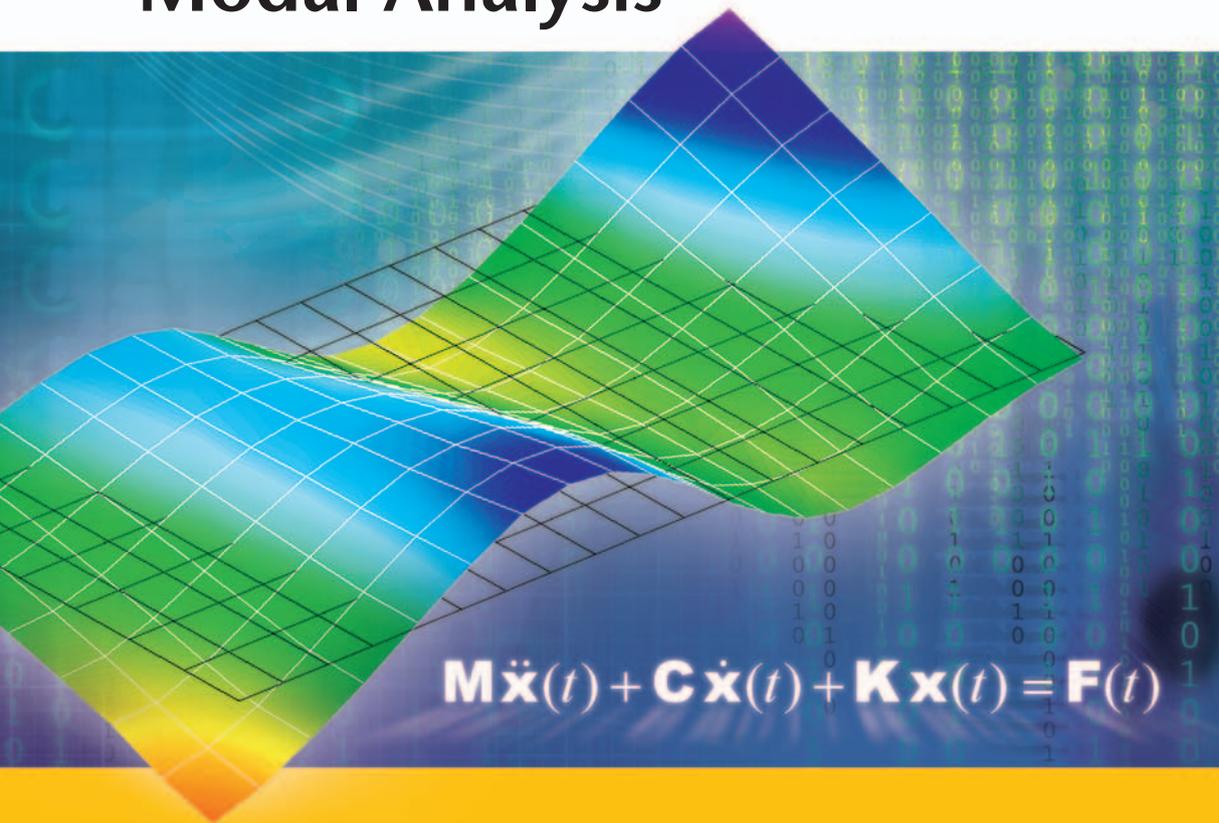


# Basics of Experimental Modal Analysis



*Modal analysis is a method to describe a structure in terms of its natural characteristics which are the frequency, damping and mode shapes – its dynamic properties. Without using a rigorous mathematical treatment, this article will introduce some concepts about how structures vibrate and some of the mathematical tools used to solve structural dynamic problems.*

## What Good are Modal Data?

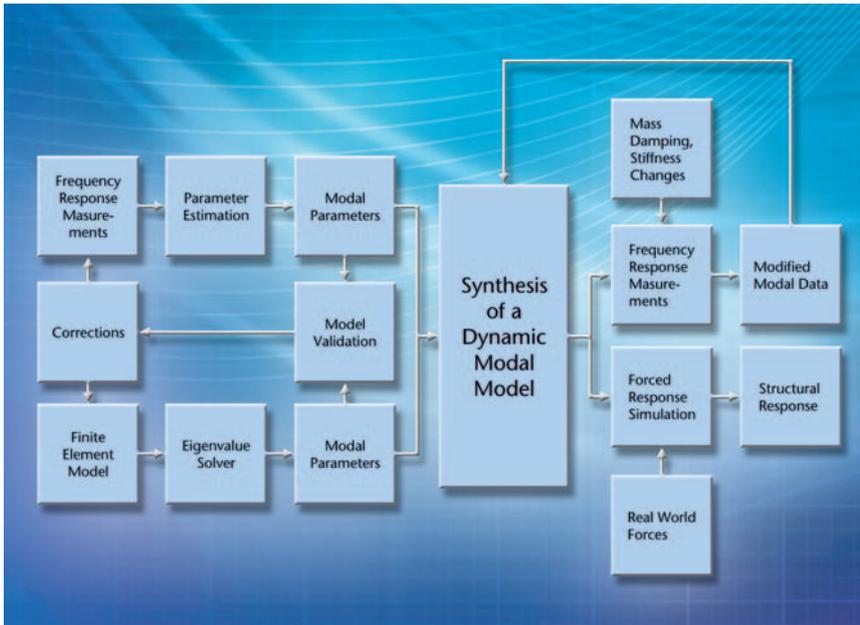
Modal data are extremely useful information that can assist in the design of almost any structure. The understanding and visualization of mode shapes is invaluable in the design process. It helps identify weakness in the design and areas where improvement is needed.

The development of a modal model, from either frequency response measurements or from a finite element model, is useful for simulation and design studies. One of these studies is structural dynamics modification.

This is a mathematical process which uses modal data (frequency, damping and mode shapes) to determine the effects of changes in the system characteristics due to physical structural changes.

These calculations can be performed without actually having to physically modify the actual structure until a suitable set of design changes is achieved.

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**Figure 1: Dynamic modal model development and modal testing in the design process.**

In addition to structural dynamic modification studies, other simulations can be performed such as force response simulation to predict system response due to applied forces. Another very important aspect of modal testing is the correlation and correction of an analytical model such as a finite element model. These are a few of the more important aspects related to the use of a modal model. A schematic is shown in Figure 1.

### A Simple Model

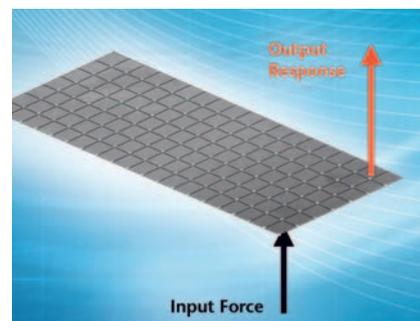
Modal analysis is often explained in terms of the modes of vibration of a simple, freely supported flat plate (Figure 2). A force that varies in a sinusoidal fashion is applied to one corner of the plate. While the rate of oscillation of the frequency is changed, the peak force will always be the same value. The response of the plate due to the excitation is measured with a laser vibrometer or with accelerometers. In the following, the response signal at one corner of the plate is analyzed.

When measuring the response on a point of the plate, the amplitude changes as the rate of oscillation of

the input force is changed (Figure 3). There will be increases as well as decreases in amplitude at different points when sweeping up in time.

Even with a constant input force to the system, the response amplitude varies depending on the frequency of oscillation of the input force. The response is larger when a force is applied with a rate of oscillation that gets close to a natural frequency (or resonant frequency) of the system and reaches a maximum when the frequency of oscillation is exactly matched to the resonant frequency. Transforming the time data to the frequency domain using the Fast

**Figure 2. Simple plate excitation/response model.**



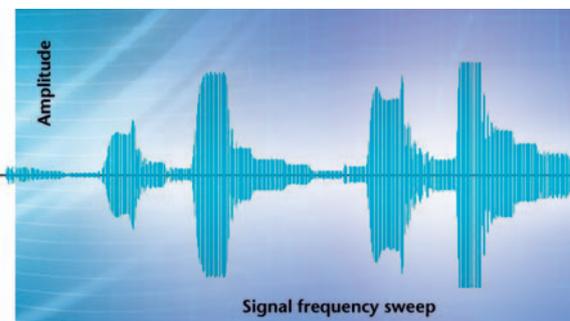
Fourier Transform generates something called the frequency response function (Figure 4). There are peaks in this function which occur at the resonant frequencies of the system.

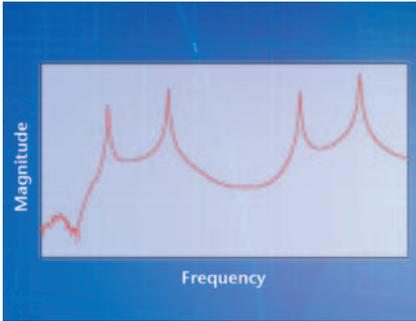
When overlaying the time trace with the frequency trace, the frequency of oscillation at the time at which the time trace reaches its maximum value corresponds to the frequency where peaks in the frequency response function reach a maximum (Figure 5), provided that the frequency change is linear over time.

Regarding the deformation patterns at the natural frequencies, they take on a variety of different shapes depending on which frequency is used for the excitation force. The shapes can be measured by using either a non-contact scanning laser vibrometer or by placing a set of distributed accelerometers on the plate and measure the amplitude of the response of the plate with different excitation frequencies. At each one of the natural frequencies a deformation pattern shows up that exists in the structure (Figure 6).

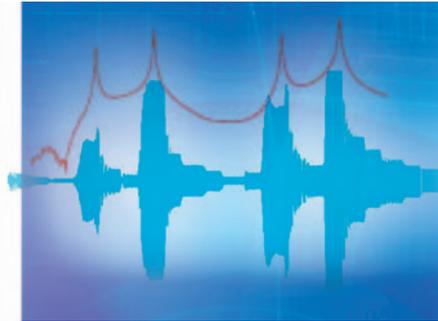
The figure shows the deformation patterns that will result when the excitation coincides with one of the natural frequencies of the system. At the first natural frequency, there is a first bending deformation pattern in the plate (mode 1). At the second natural frequency, there is a first twisting deformation pattern (mode 2). At the third and fourth natural frequencies, the second bending and second twisting

**Figure 3. Simple plate response upon a excitation frequency sweep.**

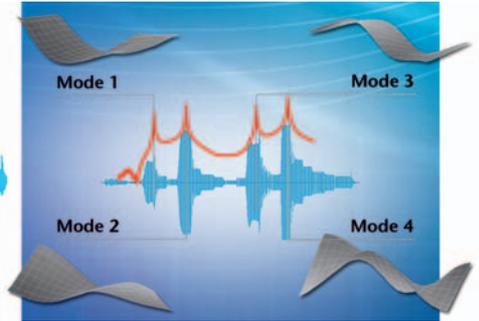




**Figure 4. Simple plate frequency response function.**



**Figure 5. Overlay of time and frequency response functions.**



**Figure 6. Simple plate sine dwell responses. The deformation patterns have been generated using PSV Software.**

deformation patterns are seen (mode 3 and 4, respectively). Such natural frequencies and mode shapes occur in all structures to some extent.

Basically, structure characteristics such as mass and stiffness determine where these natural frequencies and mode shapes will exist. Design engineers need to identify these frequencies and know how they might affect the response of the structure when a force excites the structure.

### Time Domain, Frequency Domain and Modal Space

In Figure 7, a simple cantilever beam is shown that is excited at the tip of the beam. The response at the tip of the beam will contain the response of all the modes of the system. This time response at the tip of the beam can be converted to the frequency domain by performing a Fourier Transform of the time signal. As described before, the frequency domain representation of this converted time signal is often referred to as the frequency response function, or FRF for short.

The cantilever beam will have many natural frequencies of vibration: there is a first, second and third bending mode as shown in Figure 7, and there are also other higher modes not shown.

Now the physical beam could also be approximated using an analytical lumped mass model or finite element model (shown in black in the upper

right part of the figure). This model will generally be evaluated using some set of equations where there is an interrelationship, or coupling, between the different masses, or degrees of freedom (DOF), used to model the structure.

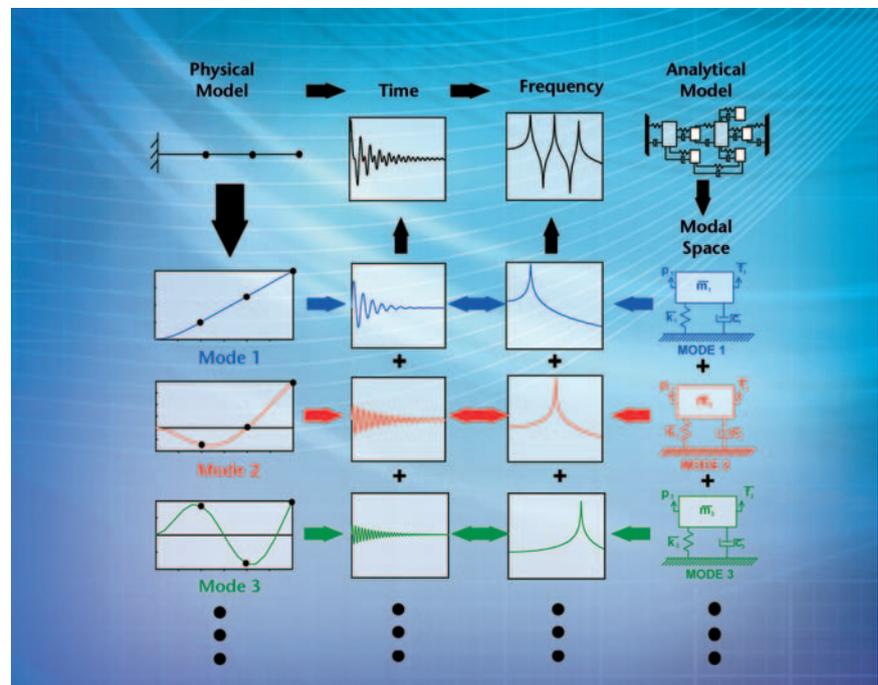
This means that if one pulls on one of the DOFs in the model, the other DOFs are also affected and also move. This coupling means that the equations are more complicated in order to determine how the system behaves.

As typically a large number of equations must be solved, matrices are often used for organizing them. The matrix representation of all of the equations of motion describing how the system behaves will look like

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{K}$  are the mass, damping and stiffness matrices respectively;  $\ddot{\mathbf{x}}$ ,  $\dot{\mathbf{x}}$  and  $\mathbf{x}$  the corresponding acceleration, velocity and displacement and  $\mathbf{F}$  is the force applied to the system.

**Figure 7: Composing time and frequency response from simple modal models.**



Usually the mass is a diagonal matrix and the damping and stiffness matrices are symmetric with off-diagonal terms indicating the degree of coupling between the different equations or DOFs describing the system. The size of the matrices is dependent on the number of equations used to describe our system. Mathematically, something called an eigensolution is performed and the modal transformation equation is used to convert these coupled equations into a set of uncoupled single DOF systems described by diagonal matrices of modal mass, modal damping and modal stiffness in a new coordinate system called modal space described as

$$\begin{bmatrix} \backslash \\ \mathbf{M} \\ \backslash \end{bmatrix} \ddot{\mathbf{p}} + \begin{bmatrix} \backslash \\ \mathbf{C} \\ \backslash \end{bmatrix} \dot{\mathbf{p}} + \begin{bmatrix} \backslash \\ \mathbf{K} \\ \backslash \end{bmatrix} \mathbf{p} = \mathbf{U}^T \mathbf{F}(t)$$

where  $\mathbf{p}$  is the vector of the modal coordinates. So the transformation from physical space to modal space using the modal transformation equation is a process whereby a complicated set of coupled physical equations is converted into a set of simple uncoupled single DOF systems. As is shown in Figure 7, the analytical model can be broken down into a set of single DOF systems where the single DOF describing mode 1 is shown in blue, mode 2 is shown in red and mode 3 is shown in green. Modal space allows the system to be described easily using simple, single DOF systems.

### Multiple Reference Analysis (MIMO)

MIMO means Multiple Input/Multiple Output and is a measuring technique where multiple inputs (excitations, references) are used to excite the measurement object at the same time and multiple outputs (responses) of the vibration are measured to obtain a matrix of FRF's that describe how the vibrating system reacts to an excitation.

Multiple reference tests are an important tool when the structure under test exhibits a high modal density, closely coupled modes or repeated roots. The dynamic MIMO model is a linear frequency domain model where spectra (Fourier transforms) of multiple inputs are multiplied by elements of an FRF matrix to yield spectra of multiple outputs. The FRF matrix model can be written as:

$$\mathbf{v}_n(f) = \mathbf{H}_{nm}(f) \mathbf{r}_m(f)$$

Here  $m$  is the number of inputs (references),  $n$  the number of outputs (responses),  $f$  the frequency,  $\mathbf{v}_n(f)$  the vector of outputs,  $\mathbf{r}_m(f)$  the vector of inputs and  $\mathbf{H}_{nm}(f)$  the matrix of the FRF's describing the system. If  $\mathbf{H}_{nm}(f)$  is known, e.g. as a result from scanning vibrometer measurements where  $n$  is the number of scan points and  $m$  the number of the reference channels, the response of the system to a set of inputs can be predicted.

### Modal Data and Operating Data

Modal data requires that the force is measured in order to determine the frequency response function and resulting modal parameters. Only modal data will give the true principal characteristics of the system. In addition, structural dynamic modifications and forced response can only be studied using modal data (operating data cannot be used for these types of studies). Also, correlation with a finite element model is best performed using modal data. However, modal data alone does not identify whether a structure is adequate for an intended service or application since modal data are independent of the forces on the system. Operating data on the other hand is an actual depiction of how the structure behaves in service.

This is extremely useful information. However, many times the operating shapes are confusing and do not necessarily provide clear guidance as to how to solve or correct an operating problem (and modification and response tools cannot be utilized on operating data). The best situation exists when both operating data and modal data are used in conjunction to solve structural dynamic problems.

Laser scanning vibrometry is ideally suited for modal tests because it provides an unambiguous phase reference, highly precise measurement data without mass loading problems and a high spatial resolution for detailed FEM correlations. Both complete and partial data sets can be exported to commercially available software packages for experimental modal analysis (LMS, ME'scope, and others).

### Acknowledgement

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