A

MICROWAVE FORMULAS AND TABLES

A.1 GENERAL

TABLE A.1 General

Decibel (dB) = 10 log(P_O/P_I) = 20 log(E_O/E_I)
Neper = 1/2 Ln (P_O/P_I) = Ln (E_O/E_I)
Neper = 0.1151 [dB Value]
dB = 8.686 [Neper value]

P_O = Power at the output

P_I = Power at the input

E_O = Voltage at the output

E_I = Voltage at the input

If $\log_B X = A$ then $B^A = X = \text{Antilog}_B A$

log(x) = common (Brigg's) logarithm = $\log_{10}(x) = \log_e(x)/\log_e(10)$

ln (x) = natural (Napierian) logarithm = $\log_e(x) = \log_{10}(x)/\log_{10}(e)$

$e \cong 2.7182818284\ldots$

dBW = 10 log (power measured in watts)
dBm = 10 log (power measured in milliwatts) = dBW + 30
dBrn = 10 log (power measured in picowatts) = dBW + 90

Adding 3dB (actually 3.01) to any dB value doubles the power (ratio).
Subtracting 3dB (actually 3.01) from any dB value halves the power (ratio).
Adding 10dB to any dB value multiplies the power (ratio) by 10.
Subtracting 10dB to any dB value divides the power (ratio) by 10.
Changing the sign of a dB value inverts the power (ratio).
Adding 1dB to any dB value multiplies the power (ratio) by 5/4 (actually 1.26).
Subtracting 1dB from any dB value multiplies the power (ratio) by 4/5 (actually 0.794).

Note: For this document the following abbreviations will be used:

Abs (x) = $|x|$ = absolute value = magnitude ignoring sign or phase

Cerfc (x) = complementary error function

Ln (x) = natural (Napierian) logarithm = $\log_e(x)$

log(x) = common (Brigg's) logarithm = $\log_{10}(x)$

sqrt (x) = $[x]^{1/2}$ = square root function

Arabic Numeral	Roman Numeral	Arabic Numeral	Roman Numeral	Arabic Numeral	Roman Numeral	Arabic Numeral	Roman Numeral
0	Nulla (N)	5	V	50	L	500	D
1	I	10	X	100	C	1000	M

Vertical lines on both sides of the numeral multiply the value by 100. A horizontal bar over the Roman numeral multiplies the value by 1000. Two horizontal bars over the numeral or one horizontal bar below the numeral multiplies the value by 1,000,000.

Digital Microwave Communication: Engineering Point-to-Point Microwave Systems, First Edition. George Kizer.

TABLE A.2 Scientific and Engineering Notation

Symbol	Prefix	Name	Multiplication Factor
		Googolplex	$10^{10^{100}}$
		(centillion)	10^{600}
		Centillion	10^{303}
		(vigintillion)	10^{120}
		(novemdecillion)	10^{114}
		(octodecillion)	10^{108}
		(septendecillion)	10^{102}
		Googol	10^{100}
		(sexdecillion)	10^{96}
		(quindecillion)	10^{90}
		(quattuordecillion)	10^{84}
		(tredecillion)	10^{78}
		(duodecillion)	10^{72}
		(undecillion)	10^{66}
		Vigintillion	10^{63}
		Novemdecillion (decillion)	10^{60}
		Octodecillion	10^{57}
		Septdecillion (nonillion)	10^{54}
		Sexdecillion	10^{51}
		Quindecillion (octillion)	10^{48}
		Quattuordecillion	10^{45}
		Tredecillion (septillion)	10^{42}
		Duodecillion	10^{39}
		Undecillion (sextillion)	10^{36}
		Decillion	10^{33}
		Nonillion (quintillion)	10^{30}
		Octillion	10^{27}
Y	Yotta	Septillion (quadrillion)	10^{24}
Z	Eta	Sextillion	10^{21}
E	Exa	Quintillion (trillion)	10^{18} = 1,000,000,000,000,000,000
P	Peta	Quadrillion (billiard)	10^{15} = 1,000,000,000,000,000
T[a]	Tera	Trillion (billion)	10^{12} = 1,000,000,000,000
G[a]	Giga	Billion (milliard)	10^{9} = 1,000,000,000
M[a]	Mega	Million	10^{6} = 1,000,000
Ma	Myria		10^{4} = 10,000
k[a]	Kilo	Thousand	10^{3} = 1000
h	Hecto	Hundred	10^{2} = 100
da	Deka (deca)	Ten	10^{1} = 10
		One (unity)	10^{0} = 1
d	Deci	Tenth	10^{-1} = 0.1
c	Centi	Hundredth	10^{-2} = 0.01
m	Milli	Thousandth	10^{-3} = 0.001
μ	Micro	Millionth	10^{-6} = 0.000,001
n	Nano	Billionith	10^{-9} = 0.000,000,001
Å		Ángstrom	10^{-10} m
p	Pico	Trillionth	10^{-12} = 0.000,000,000,001
f	Femto	Quadrillionth	10^{-15} = 0.000,000,000,000,001
a	Atto	Quintillionth	10^{-18} = 0.000,000,000,000,000,001
z	Zepto	Sextillionth	10^{-21}
y	Yocto	Septillionth	10^{-24}

[a]Note: k (kilo), in computer usage, the prefix indicates $2^{10} = 1024$; M (mega), in computer usage, the prefix indicates $2^{20} = 1{,}048{,}576$; G (giga), in computer usage, the prefix indicates $2^{30} = 1{,}073{,}741{,}824$; T (tera), in computer usage, the prefix indicates $2^{40} = 1{,}099{,}511{,}627{,}776$.

Scientific notation is a multiplicative factor of 10 to the *n*th power. Engineering notation limits the exponent *n* to multiples of 3.

In Europe, the usage of decimal points and commas are reversed relative to US usage. Common usage is to replace the US comma or European decimal point with a space.

The names in the table are usage in the United States and France. Usage in Great Britain and Germany is shown in parentheses ( ).

TABLE A.3 Emission Designator

An emission designator is a coded word defining the type of signal modulation and its bandwidth. The FCC and ITU-R format for the emission designator is **three numerals and a capital letter** to express necessary bandwidth followed by **three capital letters** describing the form of modulation. The **necessary bandwidth** (usually considered to be the channel bandwidth) uses the letter to indicate the magnitude and the decimal location.

Examples:		
	60 Hz	= 60H0
	100 kHz	= 100K
	70 MHz	=70M0
	1.99 GHz	= 1G99
	10.74 GHz	= 10G7
	10.75 GHz	= 10G8

The radio signal's **form of modulation** is described by three symbols as follows:

The **first symbol** describes the manner in which the main carrier is modulated and is one of the following:

Unmodulated Carrier	N
Amplitude Modulation	
Double Sideband	A
Single Sideband, Full Carrier	H
Single Sideband, Reduced or Variable Carrier	R
Single Sideband, Suppressed Carrier	J
Independent Sideband	B
Vestigial Sideband	C
Angle Modulation	
Frequency Modulation	F
Phase Modulation	G
Combination of Amplitude and Angle Modulation	D
Pulse Modulation	
Unmodulated Pulses	P
Pulse Amplitude Modulation	K
Pulse Width Modulation	L
Pulse Position Modulation	M
Angle Modulation during pulse period	Q
Combinations of the above or other	V
Combinations of two or more modes	W
Cases not otherwise covered	X

For current generation QAM digital radios, the usual choice is D.

The **second symbol** describes the nature of the signal(s) modulating the carrier. It is one of the following:

No Modulating Signal	0
A single channel containing quantized or digital information without the use of a modulating subcarrier excluding time division multiplex	1
A single channel containing quantized or digital information with the use of a modulating subcarrier excluding time division multiplex	2
A single channel containing analog information	3
Two or more channels containing quantized or digital information	7
Two or more channels containing analog information	8
Composite system with one or more channels containing quantized or digital information together with one or more channels containing analog information	9
Cases not otherwise covered	X

For current generation digital radios, the usual choices are 1 or 7.

The **third symbol** describes the type of information being transmitted and is one of the following:

No Information Transmitted	N
Telegraphy (aural reception)	A
Telegraphy (automatic reception)	B
Facsimile	C
Data Transmission (telemetry or telecommand)	D
Telephony (including sound broadcasting)	E
Television (video)	F
Combination of the above	W
Cases not otherwise covered	X

For current generation digital radios, the usual choice is W.

TABLE A.4 Typical Commercial Parabolic Antenna Gain (dBi)

	Diameter ft (m)								
Frequency, GHz	1 (0.3)	2 (0.6)	3 (0.9)	4 (1.2)	6 (1.8)	8 (2.4)	10 (3.0)	12 (3.7)	15 (4.6)
1.9		19.3	22.5	25.3	28.9	31.1	33.0	34.6	36.6
2.1		20.0	23.8	26.3	29.8	31.9	33.8	35.4	37.4
2.4		21.1	24.4	27.3	30.7	32.9	34.5	36.9	
3.9					34.5	37.2	39.0	40.7	42.7
4.7		26.6		32.8	36.4	38.6	40.4	42.2	44.1
5.85	23.0	28.0				41.0	42.6	44.2	45.4
6.175				35.0	38.6	41.2	43.0	44.8	46.5
6.775				35.7	39.2	41.8	43.4	45.3	47.1
7.438		30.8		36.6	40.4	42.8	44.6	46.4	48.0
7.813		31.0	34.4	37.2	40.8	43.1	45.1	46.7	48.5
8.125		31.6		37.6	41.4	43.7	45.6	47.2	49.0
8.35				37.6	41.1	43.7	45.7	47.2	48.7
10.6		34.6	37.6	39.8	43.4	45.9	47.8	49.3	
11.2		34.7	38.0	40.4	43.8	46.5	48.2	49.7	
14.8	32.1	36.8		42.7	46.2	48.6	50.5		
18.7	33.6	38.7	42.1	44.6	47.9	50.5			
22.4	35.5	40.3	43.6	46.1	49.4	51.6			
28.5	36.9	41.9							
38.5	40.1	45.1							
78.5	43.8	51.0							

TABLE A.5 Typical Rectangular Waveguide

Band Designation, GHz	Nominal Frequency Range, GHz	Typical Waveguide
2	1.7–2.5	Coaxial cable
4	3.7–4.2	WR-229
5	4.4–5.0	WR-187
Lower 6	5.9–6.4	WR-137/WR-159
Upper 6	6.5–6.9	WR-137
STL	6.9–7.1	WR-137
7	7.1–7.8	WR-112
8	7.8–8.5	WR-112
$10\frac{1}{2}$	10.6–10.7	WR-75
11	10.7–11.7	WR-75/WR-90
13	12.7–12.7	WR-75
15	14.0–15.4	WR-62
18	17.7–18.7	WR-42
23	21.2–23.6	WR-42
31	31.0–31.3	WR-28
38	38.6–40.00	WR-28

TABLE A.6 Typical Rectangular Waveguide Data

Operating Range for TE10 Mode Frequency, GHz	EIA Designation	Attenuation Over Operating Range dB/100 m	dB/100 ft	Inside Dimensions, in.	Cutoff for for TE10 Mode Frequency, GHz
1.70–2.60	WR 430	2.59–1.69 (B)	0.788–0.516 (B)	4.300–2.150	1.375
		1.64–1.08 (A)	0.501–0.330 (A)		
2.60–3.95	WR 284	4.85–3.31 (B)	1.478–1.008 (B)	2.840–1.340	2.080
		3.08–2.10 (A)	0.940–0.641 (A)		
3.30–4.90	WR 229	6.11–4.33 (B)	1.862–1.320 (B)	2.290–1.145	2.579
		3.91–2.77 (A)	1.192–0.845 (A)		
3.95–5.85	WR 187	9.15–6.33 (B)	2.79–1.93 (B)	1.872–0.872	3.155
		5.81–4.00 (A)	1.77–1.22 (A)		
4.90–7.05	WR 159	9.48–7.35 (B)	2.89–2.24 (B)	1.590–0.795	3.714
		6.04–4.66 (A)	1.84–1.42 (A)		
5.85–8.20	WR 137	12.6–10.1 (B)	3.85–3.08 (B)	1.372–0.622	4.285
		8.04–6.36 (A)	2.45–1.94 (A)		
7.05–10.00	WR 112	18.1–14.1 (B)	5.51–4.31 (B)	1.122–0.497	5.260
		11.5–8.99 (A)	3.50–2.74 (A)		
8.20–12.40	WR 90	28.3–19.8 (B)	8.64–6.02 (B)	0.900–0.400	6.560
		18.0–12.6 (A)	5.49–3.83 (A)		
10.00–15.00	WR 75	33.0–23.1 (B)	10.07–7.03 (B)	0.750–0.375	7.873
		21.2–14.8 (A)	6.45–4.50 (A)		
12.4–18.00	WR 62	41.9–36.6 (B)	12.76–11.15 (B)	0.622–0.311	9.490
		20.1–17.6 (S)	6.14–5.36 (S)		
15.00–22.00	WR 51	56.8–41.3 (B)	17.30–12.60 (B)	0.510–0.255	11.578
		27.5–20.0 (S)	8.37–6.10 (S)		
18.00–26.50	WR 42	90.9–65.0 (B)	27.7–19.8 (B)	0.420–0.170	14.080
		43.6–31.2 (S)	13.30–9.50 (S)		
22.00–33.00	WR 34	109–75.8 (B)	33.3–23.1 (B)	0.340–0.170	17.368
		52.8–36.7 (S)	16.1–11.2 (S)		
26.50–40.00	WR 28	71.9–49.2 (S)	21.9–15.0 (S)	0.280–0.140	21.100
33.00–50.00	WR 22	102–68.6 (S)	31.0–20.9 (S)	0.224–0.112	26.350
40.00–60.00	WR 19	127–89.2 (S)	38.8–27.2 (S)	0.188–0.094	31.410
50.00–75.00	WR 15	174–128 (S)	52.9–39.1 (S)	0.148–0.074	39.900
60.00–90.00	WR 12	306–171 (S)	93.3–52.2 (S)	0.122–0.061	48.400
75.00–110.00	WR 10	328–231 (S)	100.0–70.4 (S)	0.100–0.050	59.050
90.00–140.00	WR 8	499–325 (S)	152.0–99.0 (S)	0.080–0.040	73.840

B, brass; A, aluminum; S, silver.

TABLE A.7 Typical Copper Corrugated Elliptical Waveguide Loss

Frequency, GHz	Waveguide Type CommScope/RFS	Loss dB/100 m	dB/100 ft
1.9	EW20/E20	2.0/1.3	0.61/0.39
2.1	EW20/E20	1.7/1.1	0.51/0.35
2.4	EW20/—	1.5/—	0.45/—
3.7	EW34/E38	2.2/2.3	0.68/0.70
3.9	EW34/E38	2.2/2.2	0.66/0.67
4.0	EW34/E38	2.1/2.2	0.65/0.66

Continued

TABLE A.7 (*Continued*)

Frequency, GHz	Waveguide Type CommScope/RFS	Loss dB/100 m	Loss dB/100 ft
4.7	EW43/E46	2.8/2.8	0.88/0.85
5.9	EW52/E60	4.0/4.0	1.2/1.2
6.2	EW52/E60	3.9/3.9	1.2/1.2
6.8	EW63/E65	4.4/4.4	1.4/1.3
7.4	EW64/E70	4.8/4.9	1.5/1.5
7.8	EW77/E78	5.9/5.8	1.8/1.8
8.1	EW77/E78	5.7/5.7	1.7/1.7
8.4	EW77/E78	5.6/5.6	1.7/1.7
10.6	EW90/E105	10.4/9.3	3.2/2.8
11.2	EW90/E105	10.1/9.0	3.1/2.8
12.7	EW127/E130	11.6/11.3	3.6/3.4
13.0	EW127/E130	11.5/11.2	3.5/3.4
14.8	EW132/E150	15.7/13.8	4.8/4.2
18.7	EW180/E185	19.4/19.3	5.9/5.9
22.4	EW220/E220	28.2/28.3	8.6/8.6

TABLE A.8 Typical Copper Circular Waveguide Loss

Frequency, GHz	Waveguide Type	Loss dB/100 m	Loss dB/100 ft
3.7	WC-281/−269	1.2/1.3	0.39/0.45
4.2	WC-281/−269	1.1/1.3	0.34/0.39
4.7	WC-281/−269	1.0/1.1	0.32/0.35
5.9	WC-281/−269	0.91/0.99	0.28/0.30
6.4	WC-281/−269/−205	0.91/0.98/1.6	0.28/0.30/0.50
6.8	WC-281/−269/−166	0.89/0.97/2.5	0.27/0.30/0.76
7.4	WC-281/−166	0.89/2.3	0.27/0.70
8.1	WC-281/−166	0.89/2.1	0.27/0.65
8.4	WC-281/−166	0.89/2.1	0.27/0.64
10.7	WC-281/−166/−109	0.91/1.9/4.5	0.28/0.57/1.4
11.7	WC-281/−166/−109	0.92/1.9/4.3	0.28/0.57/1.3
17.7	WC-109	3.6	1.1
20.0	WC-109	3.6	1.1

Waveguide Attenuation (Loss)

$$\text{Attn}\left(\frac{\text{dB}}{100\ \text{m}}\right) = \frac{A\left(\frac{f}{f_C}\right)^2 + B}{\sqrt{\left(\frac{f}{f_C}\right)\left[\left(\frac{f}{f_C}\right)^2 - 1\right]}} \tag{A.1}$$

$$\text{Attn}\left(\frac{\text{dB}}{100\ \text{ft}}\right) = 0.3048\text{Attn}\left(\frac{\text{dB}}{100\ \text{m}}\right) \tag{A.2}$$

f = frequency of interest (GHz);

f_C = cutoff frequency (GHz).

See Chapter 5 for general methods of determining A and B.
A and B are coefficients listed in Table A.9.

TABLE A.9 Rectangular Waveguide Attenuation Factors

Waveguide Designation	Wall Metal	Cutoff Frequency, GHz	Lowest Frequency, GHz	Highest Frequency, GHz	*A*	*B*
WR 430	Brass	1.375	1.700	2.600	0.79912	0.87227
WR 430	Aluminum	1.375	1.700	2.600	0.51666	0.53603
WR 284	Brass	2.080	2.600	3.950	1.61319	1.54625
WR 284	Aluminum	2.080	2.600	3.950	1.02226	0.98538
WR 229	Brass	2.579	3.300	4.900	2.09096	2.09398
WR 229	Aluminum	2.579	3.300	4.900	1.33704	1.34170
WR 187	Brass	3.155	3.950	5.850	3.07195	2.89720
WR 187	Aluminum	3.155	3.950	5.850	1.92858	1.87417
WR 159	Brass	3.714	4.900	7.050	3.74082	2.85964
WR 159	Aluminum	3.714	4.900	7.050	2.35605	1.86956
WR 137	Brass	4.285	5.850	8.200	5.06772	4.23786
WR 137	Aluminum	4.285	5.850	8.200	3.12730	2.90245
WR 112	Brass	5.260	7.050	10.000	7.00480	6.11684
WR 112	Aluminum	5.260	7.050	10.000	4.48911	3.81714
WR 90	Brass	6.560	8.200	12.400	9.91582	8.23680
WR 90	Aluminum	6.560	8.200	12.400	6.31386	5.22805
WR 75	Brass	7.873	10.000	15.000	11.19789	11.06060
WR 75	Aluminum	7.873	10.000	15.000	7.14940	7.17723
WR 62	Brass	9.490	12.400	18.000	21.66835	3.28603
WR 62	Silver	9.490	12.400	18.000	10.44449	1.49119
WR 51	Brass	11.578	15.000	22.000	20.04607	19.60623
WR 51	Silver	11.578	15.000	22.000	9.71048	9.48393
WR 42	Brass	14.080	18.000	26.500	31.61794	30.18296
WR 42	Silver	14.080	18.000	26.500	15.19172	14.43432
WR 34	Brass	17.368	22.000	33.000	36.60789	36.64440
WR 34	Silver	17.368	22.000	33.000	17.71324	17.78239
WR 28	Silver	21.100	26.500	40.000	23.74185	23.77586
WR 22	Silver	26.350	33.000	50.000	32.63928	34.86860
WR 19	Silver	31.410	40.000	60.000	43.23420	42.89257
WR 15	Silver	39.900	50.000	75.000	67.35574	41.32939
WR 12	Silver	48.400	60.000	90.000	60.35912	156.85742
WR 10	Silver	59.050	75.000	110.000	110.96517	110.45313
WR 8	Silver	73.840	90.000	140.000	159.76070	146.55681

TABLE A.10 CommScope Elliptical Waveguide Attenuation Factors

Waveguide Designation	Metal	Cutoff Frequency, GHz	Lowest Frequency, GHz	Highest Frequency, GHz	*A*	*B*
EW 17	Copper	1.364	1.700	2.400	0.49424	0.48626
EW 20	Copper	1.570	1.900	2.700	0.69877	0.48371
EW 28	Copper	2.200	2.600	3.400	0.77964	0.86249
EW 34	Copper	2.376	3.100	4.200	1.05075	0.76711
EW 37	Copper	2.790	3.300	4.300	1.01620	1.29877
EW 43	Copper	2.780	4.400	5.000	1.45111	0.97967
EW 52	Copper	3.650	4.600	6.425	1.81424	1.68203
EW 63	Copper	4.000	5.850	7.125	2.07262	1.94243
EW 64	Copper	4.320	5.300	7.750	2.23080	2.17665
EW 77	Copper	4.720	6.100	8.500	2.56220	2.90547
EW 85	Copper	6.460	7.700	9.800	3.98638	4.31452
EW 90	Copper	6.500	8.300	11.700	4.62183	4.78385
EW 127	Copper	7.670	10.000	13.250	5.52354	4.58571
EW 132	Copper	9.220	11.000	15.350	7.46516	5.79189
EW 180	Copper	11.150	14.000	19.700	8.52820	9.89320
EW 220	Copper	13.340	21.000	23.600	14.52826	8.35617
EW 240	Copper	15.200	22.000	26.500	14.95157	15.98567

TABLE A.11 RFS Elliptical Waveguide Attenuation Factors

Waveguide Designation	Metal	Cutoff Frequency, GHz	Lowest Frequency, GHz	Highest Frequency, GHz	*A*	*B*
E 20	Copper	1.380	1.700	2.300	0.45746	0.51945
E 30	Copper	1.800	2.500	3.100	0.51252	1.05598
E 38	Copper	2.400	3.100	4.200	0.90171	1.19017
E 46	Copper	2.880	4.400	5.000	1.37676	0.95528
ES 46	Copper	3.080	4.400	5.000	1.99642	0.42518
EP 58	Copper	3.560	4.400	6.200	1.76406	1.42354
E 60	Copper	3.650	5.600	6.425	1.77577	1.80124
E 65	Copper	4.010	5.900	7.125	1.89412	2.34046
EP 70	Copper	4.340	6.400	7.750	2.24551	2.32916
E 78	Copper	4.720	7.100	8.500	2.77861	2.24383
EP 100	Copper	6.430	8.500	10.000	3.14271	4.84624
E 105	Copper	6.490	10.300	11.700	4.55513	3.12052
E 130	Copper	7.430	10.700	13.250	5.71860	3.75977
E 150	Copper	8.640	13.400	15.350	6.93307	4.83389
E 185	Copper	11.060	17.300	19.700	9.20008	7.95227
E 220	Copper	13.360	21.200	23.600	16.17421	3.89422
E 250	Copper	15.060	24.250	26.500	16.57892	10.08753

TABLE A.12 Elliptical Waveguide Cutoff Frequencies

EW/EWP-	Range, GHz	Cutoff, GHz
CommScope		
17	1.70–2.40	1.36
20	1.90–2.70	1.57
28	2.60–3.40	2.20
34	3.10–4.20	2.38
37	3.30–4.30	2.79
43	4.40–5.00	2.78
52	4.60–6.425	3.65
63	5.85–7.125	4.00
64	5.30–7.75	4.32
77	6.10–8.50	4.72
85	7.70–9.80	6.46
90	8.30–11.70	6.50
127	10.00–13.25	7.67
132	11.00–15.35	9.22
180	14.00–19.70	11.15
220	17.00–23.60	13.34
240	18.00–26.50	15.20

Continued

TABLE A.12 (*Continued*)

E/EP-	Range, GHz	Cutoff, GHz
RFS		
20	1.70–2.30	1.38
30	2.30–3.10	1.80
38	3.00–4.20	2.40
46	3.65–5.00	2.88
S46	3.90–5.00	3.08
58	4.40–6.20	3.56
60	4.50–6.425	3.65
65	5.00–7.125	4.01
70	5.40–7.75	4.34
78	5.90–8.50	4.72
100	8.00–10.00	6.43
105	8.10–11.70	6.49
130	9.30–13.25	7.43
150	10.80–15.35	8.64
185	13.70–19.70	11.06
220	16.70–23.60	13.36
250	19.00–26.50	15.06
300	24.00–33.40	19.05
380	29.00–39.50	23.45

TABLE A.13 Circular Waveguide Cutoff Frequencies (GHz)

Waveguide	TE_{11}	TM_{01}	TE_{21}	TM_{11}/TE_{01}	TE_{31}	TM_{21}	TE_{41}
WC-281	2.460	3.213	4.083	5.121	5.612	6.863	7.105
WC-269	2.571	3.359	4.268	5.353	5.866	7.175	7.428
WC-205	3.374	4.407	5.600	7.024	7.698	9.415	9.746
WC-166	4.167	5.443	6.916	8.675	9.506	11.627	12.036
WC-109	6.346	8.289	10.532	13.211	14.477	17.706	18.330
WC-75	9.223	12.047	15.307	19.200	21.040	25.733	26.640
Waveguide	TE_{12}	TM_{02}	TM_{31}	TE_{51}	TE_{22}	TM_{12}/TE_{02}	TE_{61}
WC-281	7.123	7.376	8.524	8.570	8.962	9.374	10.021
WC-269	7.446	7.710	8.911	8.959	9.368	9.799	10.476
WC-205	9.771	10.117	11.693	11.756	12.293	12.859	13.746
WC-166	12.066	12.494	14.440	14.518	15.181	15.880	16.976
WC-109	18.376	19.028	21.991	22.110	23.119	24.183	25.853
WC-75	26.707	27.653	31.960	32.133	33.600	35.147	37.573

TABLE A.13 (*Continued*)

Waveguide	TM_{41}	TE_{32}	TM_{22}	TE_{13}	TE_{71}	TM_{03}	TM_{51}
WC-281	10.139	10.708	11.245	11.405	11.462	11.561	11.721
WC-269	10.599	11.193	11.755	11.922	11.981	12.086	12.253
WC-205	13.907	14.688	15.424	15.644	15.722	15.859	16.078
WC-166	17.175	18.139	19.048	19.319	19.416	19.584	19.855
WC-109	26.156	27.624	29.009	29.422	29.569	29.826	30.239
WC-75	38.013	40.147	42.160	42.760	42.973	43.347	43.947
Waveguide	TE_{42}	TE_{81}	TM_{32}	TM_{61}	TE_{23}	TM_{13}/TE_{03}	TE_{52}
WC-281	12.400	12.891	13.041	13.275	13.321	13.592	14.054
WC-269	12.963	13.476	13.632	13.877	13.926	14.208	14.691
WC-205	17.010	17.683	17.888	18.210	18.273	18.644	19.278
WC-166	21.006	21.837	22.090	22.488	22.566	23.024	23.807
WC-109	31.991	33.257	33.642	34.248	34.367	35.064	36.257
WC-75	46.493	48.333	48.893	49.773	49.947	50.960	52.693

Formula

For coaxial cable velocity factor is related to group velocity using the following formulas:

$$V_G = \text{Group velocity} = V_O V_F \tag{A.3}$$

V_O = velocity of propagation in free space;

= 0.9833 ft/ns;

= 0.2998 m/ns;

V_F = velocity factor = 1/sqrt [dielectric constant (relative permittivity)]. See Table A.15.

The absolute delay D of a transmission line is given by

$$D = \frac{L}{V_G} \tag{A.4}$$

L = physical length of the transmission line.

The effective length of the cable (when compared to an RF signal traveling in free space) is given by

$$L_{EFF} = \text{Effective length} = \frac{L}{V_F} \tag{A.5}$$

The cutoff frequency for coaxial cable is given by

$$F_{CO} = \frac{7.50 \times V_F}{D(\text{in.}) + d(\text{in.})} = \frac{190 \times V_F}{D(\text{mm}) + d(\text{mm})} \tag{A.6}$$

D = inside diameter of outer conductor;

d = outside diameter of inner conductor.

Cable operation should be limited to a frequency no higher than 1/2 of the cutoff frequency.

TABLE A.14 Typical Coaxial Microwave Connectors

Connector	Description	Frequency Limit, GHz	Wrench Size, in.	Recommended Torque
BNC	Bayonet type N Connector (or Neill–Concelman)	4		
SMB	Subminiature type B	4		
N (common)	Named for Paul Neil	11	13/16 (hex nut)	14 in-lb
SMC	Subminiature type C	10	1/4	Brass: 3 in-lb
TNC (common)	Threaded type N Connector (or Neill–Concelman)	10	5/8 (hex nut)	14 in-lb
7 mm (or APC-7)	Sexless connector	18	3/4	14 in-lb
N (precision)	Named for Paul Neil	18	13/16 (hex nut)	24 in-lb
TNC (precision)	Threaded type N Connector (or Neill–Concelman)	15	5/8 (hex nut)	24 in-lb
SMA	Subminiature type A	25	5/16	Stainless steel/thick wall brass: 8 in-lb Thin wall brass: 4 in-lb
3.5 mm	Mates with SMA	27	5/16	Same as SMA
2.9 mm (or K)	Mates with SMA	40	5/16	Same as SMA
GPO	Gilbert Push On	40		
SSMA	Smaller SMA	38	1/4	Stainless steel: 8 in lb
2.4 mm	Mates with 1.85 mm	50		
1.85 mm (or V)	Mates with 2.4 mm	60		
1 mm		110		

To convert from inch-pounds (in-lbs) to Newton-meters (N-m), multiply inch-pounds by 0.113.
Caution: Both BNC and N connectors have 50- and 75-ohm versions. The 50-ohm versions have larger diameter center pins. Do not attempt to mate the 50- and 75-ohm connectors. Either a poor connection or permanently deformed connector will result.

50-Ohm Coaxial Cable Attenuation (Loss)

$$\text{Attn}\left(\frac{\text{dB}}{100\ \text{m}}\right) = A\sqrt{f} + Bf \tag{A.7}$$

$$\text{Attn}\left(\frac{\text{dB}}{100\ \text{ft}}\right) = 0.3048\text{Attn}\left(\frac{\text{dB}}{100\ \text{m}}\right) \tag{A.8}$$

$$f = \text{frequency of interest (MHz)}.$$

See Chapter 5 for general methods of determining A and B.
A and B are coefficients listed in Table A.16.

TABLE A.15 Coaxial Cable Velocity Factors

Dielectric	Air	9913	Foam FEP Teflon	Foam Polyethylene	TFE Teflon	Polyethylene	Silicon	PVC	Solid Nylon
Velocity Factor	0.9997	0.84	0.80	0.78–0.80	0.69–0.71	0.66–0.67	0.58	0.55	0.45

TABLE A.16 50 Ohm Coaxial Cable Attenuation Factors

Cable Type	Diameter, in.	*A*, Conductive	*B*, Dielectric	Maximum Frequency, MHz
Flexible Foam				
1/4″	0.25″	0.565	0.00161	20,000
3/8″	0.38″	0.379	0.00177	14,000
1/2″	0.50″	0.325	0.00154	12,000
Semirigid Foam				
1/4″	0.25″	0.394	0.00120	16,000
3/8″	0.38″	0.331	0.00109	14,000
1/2″	0.50″	0.211	0.000619	9000
5/8″	0.63″	0.158	0.000597	6000
7/8″	0.88″	0.126	0.000487	5000
1–1/4″	1.25″	0.0847	0.000523	3300
1–5/8″	1.63″	0.0626	0.000453	2500
2–1/4″	2.25″	0.0517	0.000476	2200
Air Dielectric				
1/2″	0.50″	0.267	0.00241	11,000
5/8″	0.63″	0.172	0.000251	7000
1–5/8″	1.63″	0.0638	0.000282	2700
2–1/4″	2.25″	0.0528	0.000263	2300
Other				
MIL-C-17/28 (RG-58)	0.20″	1.35	0.00413	1000
MIL-C-17/60 (RG-142)	0.20″	1.21	0.00394	8000
MIL-C-17/74 (RG-213/214)	0.42″	0.531	0.00413	11,000
MIL-C-17/79 (RG-218/219)	0.87″	0.220	0.00413	1000
MIL-C-17/84 (RG-223)	0.21″	1.25	0.00413	12,400
Belden 8237	0.41″	0.555	0.00669	7000
Belden 8240	0.19″	1.12	0.0124	7000
Belden 9913	0.41″	0.395	0.00217	7000
LMR-200	0.20″	1.05	0.00108	7000
LMR-240	0.24″	0.794	0.00108	7000
LMR-300	0.30″	0.630	0.00108	7000
LMR-400	0.41″	0.401	0.000853	7000
LMR-500	0.50″	0.317	0.000854	7000
LMR-600	0.59″	0.248	0.000853	7000
LMR-900	0.87″	0.170	0.000525	7000
LMR-1200	1.20″	0.123	0.000525	3500
LMR-1700	1.67″	0.0868	0.000525	3500

LMR is the registered trade mark of Times Microwave Systems.

TABLE A.17 Frequency Bands, General Users

Band Designation	Nominal Frequency Range	ITU Designation
ULF	300 Hz–3 kHz	Hectokilometric waves
VLF	3–30 kHz	Myriametric waves
LF	30–300 kHz	Kilometric waves
MF	300 kHz–3 MHz	Hectometric waves
HF	3–30 MHz	Decametric waves
VHF	30–300 MHz	Metric waves
UHF	300 MHz–3 GHz	Decimetric waves
SHF	3–30 GHz	Centimetric waves
EHF	30–300 GHz	Millimetric waves
	300 GHz–3 THz	Decimillimetric waves
	3–30 THz	Centimillimetric waves
	30–300 THz	Micrometric waves
	300 THz–3 PHz	Decimicrometric waves

Infrared light begins about 1 THz.
Source: FCC Title 47 CFR Part 97.3, NTIA Manual for Federal Radio Frequency Management, Chapter 6.2, ITU-T B.15, ITU-R V.431-7.

TABLE A.18 Frequency Bands, Fixed Point to Point Operators

Band Designation	Nominal Frequency Range, GHz	Users
2 GHz	1.850–2.690	FCC, NTIA, Canada
2.4 GHz	2.400–2.4835	FCC (unlicensed, CFR Part 15)
4 GHz[a]	3.700–4.200	FCC
5 GHz[a]	4.400–4.940	NTIA
5.2 GHz	5.150–5.350	FCC (unlicensed, CFR Part 15)
5.8 GHz	5.725–5.850	FCC (unlicensed, CFR Part 15)
Lower 6 GHz[a]	5.925–6.425	FCC, Canada
Upper 6 GHz	6.525–6.875	FCC
STL	6.875–7.125	FCC (CFR Parts 74 and 101)
7 GHz	7.125–7.725	NTIA, Canada
8 GHz	7.725–8.500	NTIA, Canada
10½ GHz[a]	10.550–10.680	FCC
11 GHz[a]	10.700–11.700	FCC
CARS	12.700–13.250	FCC (CFR Parts 74 and 101)
15 GHz	14.500–15.350	NTIA, Canada
18 GHz	17.700–19.700	FCC, NTIA
23 GHz[a]	21.200–23.600	FCC, NTIA
LMDS[a]	27.500–31.300	FCC, NTIA
38 GHz	38.600–40.000	FCC
60 GHz	57.000–64.000	FCC (unlicensed, CFR Part 15)
70 GHz[a]	71.000–76.000	FCC, NTIA
80 GHz[a]	81.000–86.000	FCC, NTIA
90 GHz	92.000–95.000	FCC (licensed and unlicensed), NTIA

[a] Shared with satellite service
FCC, US Commercial (CFR Part 101), NTIA, US Federal Government.

TABLE A.19 Frequency Bands, Radar, Space, and Satellite Operators

Band Designation	Nominal Frequency Range
L	1–2 GHz
S	2–4 GHz
C	4–8 GHz
X	8–12 GHz
K_u	12–18 GHz
K[a]	18–27 GHz
K_a[a]	27–40 GHz
V	40–75 GHz
W	75–110 GHz
mm (millimeter)	110–300 GHz
submm (sub-millimeter)	300 GHz–3 THz

[a]For space radio communications, the K and K_a bands are often designated by the single symbol K_a.
Source: IEEE Std 521–2002, ITU-R V.431-7.

TABLE A.20 Frequency Bands, Electronic Warfare Operators

Band Designation	Nominal Frequency Range
A	0 Hz–250 MHz
B	250–500 MHz
C	500 MHz–1 GHz
D	1–2 GHz
E	2–3 GHz
F	3–4 GHz
G	4–6 GHz
H	6–8 GHz
I	8–10 GHz
J	10–20 GHz
K	20–40 GHz
L	40–60 GHz
M	60–100 GHz

EU-NATO-US ECM Band Designations.

TABLE A.21 Frequency Bands, Great Britain Operators

Band Designation	Nominal Frequency Range, GHz
L	1–2
S	2–4
C	4–8
X	8–12
K_u	12–18
K	18–26.5
K_a	26.5–40
Q	30–50
U	40–60
V	50–75
E	60–90
W	75–110
F	90–140
D	110–170

Radio Society of Great Britain (RSGB) Frequency Bands.

TABLE A.22 Signal-to-Noise Ratio for Demodulator 10^{-6} BER

Receiver Type and Number of Symbols	S/N (dB) for 10^{-6} BER	Information Density, bits/s/Hz	Transmitter Peak-to-Average Ratio, dB
CPSK (Phase Shift Keying, Coherent Demodulation)			
2	13.6	1	0.0
4	13.8	2	0.0
8	19.3	3	0.0
16	25.1	4	0.0
32	31.1	5	0.0
64	37.2	6	0.0
DPSK (Phase Shift Keying, Differential Demodulation)			
2	15.9	1	0.0
4	16.1	2	0.0
8	22.1	3	0.0
16	28.1	4	0.0
32	34.1	5	0.0
64	40.2	6	0.0
QPR (Quadrature Partial Response)			
3	17.6	1	2.0
9	17.6	2	2.0
25	22.3	3	3.3
49	24.6	4	4.6
121	28.5	5	4.3
225	30.8	6	5.7
529	34.3	7	5.2
961	37.0	8	6.3
2209	40.8	9	5.4
QAM (Quadrature Amplitude Modulation, Coherent Demodulation)			
2	13.5	1	0.0
4	13.5	2	0.0
8	17.1	3	1.3
16	20.2	4	2.6
32	23.2	5	2.3
64	26.2	6	3.7
128	29.1	7	3.2
256	32.0	8	4.3
512	34.9	9	3.4
1024	37.7	10	4.5
TCM 2D (Trellis Coded Modulation, Two Dimensions)			
32	20.9	4	2.3
128	27.2	6	3.2
256	30.3	7	4.3
512	33.3	8	3.4
TCM 4D (Trellis Coded Modulation, Four Dimensions)			
32	19.9	4.5	2.3
128	26.2	6.5	3.2
256	29.3	7.5	4.3
512	32.3	8.5	3.4

Sometimes BER performance is specified in E_b/N_o rather than S/N.

$$\begin{aligned}\frac{E_b}{N_o} &= \text{energy per bit to noise power spectral density ratio (dB)}\\ &= \text{S/N (dB)} - 10\ \log_{10}\left(\frac{\text{bits per symbol} \times \text{symbols per second}}{B\ \text{(Hz)}}\right)\\ &\approx \text{S/N (dB)} - 10\ \log_{10}\left[\text{spectral efficiency (bits/s/Hz)}\right]\\ &\quad \text{(assuming the modulated signal essentially fills the transmission channel)}\end{aligned} \tag{A.9}$$

S/N (dB) = average signal-to-noise power ratio;
B = channel bandwidth.

Signal-to-noise ratio is the ratio of average powers. Most transmitters are peak power limited. Practical radio systems must consider the ratio of peak (transmit) signal power to average noise power. The above transmitter peak-to-average ratios assume square root raised cosine filtering. They represent "typical" industry values. Actual peak-to-average ratio will depend on the filtering alpha value.

The S/N conversion from 10^{-6} to 10^{-3} or to 10^{-12} is 1–3 dB depending on modulation complexity. Forward error correction coding gain (S/N improvement) is typically 2–5 dB for the 10^{-6} BER. The above equation assumes synchronous demodulation; however, asynchronous demodulation is typically used in practice. This increases the S/N by 1 or 2 dB. This is compensated for with forward error correction. Practical considerations will degrade coding gain when measured performance is compared to theoretical.

Peak-to-average ratio is for the constellation with no filtering. Nyquist filtering will increase the peak-to-average ratio.

A.2 RADIO TRANSMISSION

A.2.1 Unit Conversions

$$\text{Watts} = 0.001 \times 10^{\text{dBm}/10} \tag{A.10}$$

$$\text{dBm} = 10\ \log(1000 \times \text{Watts}) \tag{A.11}$$

$$\text{dBm} = \text{dBW} + 30 \tag{A.12}$$

$$\text{dBm} = \text{dBrn} - 90 \tag{A.13}$$

$$\text{dBm0} = \text{dBm} - \text{TLP(dB)} \tag{A.14}$$

$$\text{dBrn0} = \text{dBrn} - \text{TLP(dB)} \tag{A.15}$$

$$\begin{aligned}P(\text{W/m}^2) &= 10\ P(\text{mW/cm}^2) = E(\text{V/m})H(\text{A/m})\\ &= \frac{E(\text{V/m})^2}{(120\pi)} = (120\pi)\ H(\text{A/m})^2\end{aligned} \tag{A.16}$$

$$\text{V/m} = 10^{[\text{dB}(\mu\text{V/m})-120]/20} \tag{A.17}$$

$$\text{A/m} = 10^{[\text{dB}(\mu\text{A/m})-120]/20} \tag{A.18}$$

$$\text{dBm(mW)} = \text{dB}(\mu\text{V}) - 107.0 \tag{A.19}$$

$$\text{dB(mW/m}^2) = \text{dB}(\mu\text{V/m}) - 115.8 \tag{A.20}$$

$$\text{dB}(\mu\text{A/m}) = \text{dB}(\mu\text{V/m}) - 20\ \log(120\pi) \tag{A.21}$$

$$\mathrm{dB}(\mathrm{W/m^2}) = 10\ \log\ [(\mathrm{V/m})(\mathrm{A/m})] \tag{A.22}$$

$$\mathrm{dB}(\mathrm{mW/m^2}) = \mathrm{dB}(\mathrm{W/m^2}) + 30.0 \tag{A.23}$$

A.2.2 Free Space Propagation Absolute Delay

1.0167 ns/ft

3.3356 ns/m

A.2.3 Waveguide Propagation Absolute Delay

$$\begin{aligned} &1.0167\ \mathrm{ns/ft}\ \left(1\Big/\left\{\mathrm{sqrt}\left[1-\left(\frac{f_C}{f}\right)^2\right]\right\}\right) \\ &3.3356\ \mathrm{ns/m}\ \left(1\Big/\left\{\mathrm{sqrt}\left[1-\left(\frac{f_C}{f}\right)^2\right]\right\}\right) \end{aligned} \tag{A.24}$$

f_C = cutoff frequency for mode of interest;
f = frequency of operation greater than or equal to f_C.

A.2.4 Coaxial Cable Propagation Absolute Delay

1.0167 ns/ft/Velocity factor;

3.3356 ns/m/Velocity factor;

Velocity factor (between 0 and 1) = 1/sqrt (Dielectric constant);

Dielectric constant [1 (air) or greater] = relative permittivity of dielectric.

$$\begin{aligned} \text{Polystyrene foam dielectric constant} &= 1.05 \\ \text{Teflon dielectric constant} &= 2.1 \\ \text{Polyethylene dielectric constant} &= 2.25 \end{aligned} \tag{A.25}$$

A.2.5 Free Space Propagation Wavelength

$$\begin{aligned} \lambda(\mathrm{ft}) &= \frac{0.98357}{F(\mathrm{GHz})} \\ \lambda(\mathrm{m}) &= \frac{0.29980}{F(\mathrm{GHz})} \end{aligned} \tag{A.26}$$

F = frequency of electromagnetic wave.

A.2.6 Dielectric Medium Propagation Wavelength

$$\begin{aligned} \lambda(\mathrm{ft}) &= \frac{\left[\dfrac{0.98357}{F\ (\mathrm{GHz})}\right]}{\mathrm{sqrt}(\varepsilon_r)} \\ \lambda(\mathrm{m}) &= \frac{\left[\dfrac{0.29980}{F\ (\mathrm{GHz})}\right]}{\mathrm{sqrt}(\varepsilon_r)} \end{aligned} \tag{A.27}$$

F = frequency of electromagnetic wave;
ε_r = medium dielectric constant (permittivity relative to free space).

A.2.7 Free Space Loss (dB)

$$96.58 + 20 \log F(\text{GHz}) + 20 \log D(\text{miles})$$
$$92.45 + 20 \log F(\text{GHz}) + 20 \log D(\text{km}) \tag{A.28}$$

F = frequency of radio wave;
D = path length.

A.2.8 Effective Radiated Power (ERP) and Effective Isotropic Radiated Power (EIRP)

$$\text{ERP(W)} = 10^{P(\text{dBW})/10} \times 10^{G(\text{dBd})/10} = 0.001[10^{P(\text{dBm})/10} \times 10^{G(\text{dBd})/10}] \tag{A.29}$$

$$\text{EIRP(W)} = 10^{P(\text{dBW})/10} \times 10^{G(\text{dBi})/10} = 0.001[10^{P(\text{dBm})/10} \times 10^{G(\text{dBi})/10}] \tag{A.30}$$

$$\text{ERP(W)} = \text{EIRP(W)} \times 10^{-2.15/10} = 0.61\ \text{EIRP(W)} \tag{A.31}$$

P = transmitter power (at the antenna input);
G(dBd) = antenna gain referenced to a half wave dipole;
G(dBi) = antenna gain referenced to an isotropic radiator.

A.2.9 Voltage Reflection Coefficient

$$\text{Voltage reflection coefficient} = R = \frac{(\text{VSWR} - 1)}{(\text{VSWR} + 1)} \tag{A.32}$$

Reflected voltage divided by incident voltage at point of reflection;
VSWR, voltage standing wave ratio.

A.2.10 Voltage Standing Wave Ratio Maximum

$$\text{VSWR maximum } (V_{\text{max}}) = \text{sqrt}\left[\frac{(1 + R)}{(1 - R)}\right] \tag{A.33}$$

A.2.11 Voltage Standing Wave Ratio Minimum

$$\text{VSWR minimum } (V_{\text{min}}) = \text{sqrt}\left[\frac{(1 - R)}{(1 + R)}\right] \tag{A.34}$$

A.2.12 Voltage Standing Wave Ratio

$$\text{VSWR} = \frac{(1 + R)}{(1 - R)} \tag{A.35}$$

Maximum voltage divided by minimum voltage in standing wave pattern

$$= \text{Absolute value} \left[\left(\frac{Z_L}{Z_G} \right) \text{ or } \left(\frac{Z_G}{Z_L} \right) \right], \text{whichever is larger} \tag{A.36}$$

Z_G = originating generator internal resistance;
Z_L = end (load) terminating resistance.

Note: If the generator and load are separated by a homogeneous transmission line, at the beginning of the signal's transmission, the transmission line is the load. At the other end of the transmission line, the transmission line is the generator.

A.2.13 Power Reflection Coefficient

$$\text{Power reflection coefficient} = R^2 \le 1 \tag{A.37}$$

Reflected power divided by incident power.

A.2.14 Reflection Loss

$$\text{Reflection loss} = -10 \ \log(1 - R^2) \ge 0 \tag{A.38}$$

Decibel value of incident power loss due to reflected power.

A.2.15 Return Loss

$$\text{Return loss} = -20 \ \log \ R \ge 0 \tag{A.39}$$

Decibel value of incident power divided by reflected power.

For the above formulas the following definitions apply:

$$R = \text{Absolute value} \left[\frac{(Z_L - Z_G)}{(Z_L + Z_G)} \right] \tag{A.40}$$

Z_L = load (termination) impedance (ohms);
Z_G = generator (source) impedance (ohms).

A.2.16 *Q* (Quality) Factor (Figure of Merit for Resonant Circuits or Cavities)

$$Q = \frac{f_0}{(f_U - f_L)} \tag{A.41}$$

f_0 = circuit resonant frequency;
f_U = circuit upper half power frequency*;
f_L = circuit lower half power frequency*.

*Frequency at which output power is 3 dB less than that at f_0.

A.2.17 *Q* (Quality) Factor (Figure of Merit for Optical Receivers)

$$Q = \text{optical signal-to-noise ratio for a given BER and transmission rate} \tag{A.42}$$

A.2.18 Typical Long-Term Interference Objectives

$$\text{Same system interference} = \text{Radio front end noise} - 6\ \text{dB} \tag{A.43}$$

$$\text{Foreign system interference} = \text{radio front end noise} - 10\ \text{dB} \tag{A.44}$$

A.2.19 Frequency Planning Carrier-to-Interference Ratio (*C/I*)

$$C/I(\text{dB}) = P(\text{dB}) + G(\text{dB}) + L(\text{dB}) + D(\text{dB}) \tag{A.45}$$

$$P(\text{dB}) = \text{transmitter power differential}$$

$$= P_C(\text{dBm}) - L_C(\text{dB}) - P_I(\text{dBm}) + L_I(\text{dB}) \tag{A.46}$$

$$G(\text{dB}) = \text{antenna gain differential}$$

$$= G_C(\text{dB}) - G_I(\text{dB}) \tag{A.47}$$

$$L(\text{dB}) = \text{free space loss differential}$$

$$= 20\ \log\left(\frac{d_I}{d_C}\right) \tag{A.48}$$

$$D(\text{dB}) = \text{antenna discrimination}$$

$$= D_C(\text{dB}) + D_I(\text{dB}) \tag{A.49}$$

P_C(dBm) = transmitter power of desired signal;
P_I(dBm) = transmitter power of undesired signal;
L_C(dB) = power loss of desired signal between transmitter and transmit antenna;
L_I(dB) = power loss of undesired signal between transmitter and transmit antenna;
G_C(dB) = gain of transmit antenna at site A toward site B;
G_I(dB) = gain of transmit antenna at site C toward site D;
D_C(dB) = discrimination (relative to main lob power) of receive antenna at site B toward site C;
D_I(dB) = discrimination (relative to main lobe power) of transmit antenna at site C toward site B;
d_C = distance from site A to site B;
d_I = distance from site C to site B;
Site A = transmit location of desired signal;
Site B = receive location of desired signal;
Site C = transmit location of interfering signal;
Site D = intentional receive location of interfering signal.

A.2.20 Noise Figure, Noise Factor, Noise Temperature, and Front End Noise

The minimum noise of an ideal amplifier perfectly impedance matched to its receive antenna is the noise introduced by a (hypothetical) resistor of the interface impedance (typically 50 ohms) operating at operating temperature T (usually assumed to be 290 K = 17 °C = 63 °F). In general, the noise P delivered to a matched device by the noise source resistor at temperature T may be shown to be the following:

n = noise produced by a matched resistor operating at temperature T;
= KTb (W);

K = Boltzmann's constant = 1.38×10^{-23} (joules/degree kelvin);
T = noise temperature of the resistor (degrees kelvin = degrees celsius + 273);
b = noise bandwidth of the device (Hz).

If the amplifier adds noise to the received signal, that noise is characterized by adding another noise temperature to characterize the added noise. The relationship to amplifier signal to noise ratio is the following:

$$\begin{aligned} \text{nf} &= \text{noise factor} \\ &= 1 + \left(\frac{T_e}{T_o}\right) \\ &= \frac{\text{s/n}_\text{I}}{\text{s/n}_\text{O}} \end{aligned} \tag{A.50}$$

T_o = amplifier operating ("room") temperature (nominally 290 degrees K);
T_e = amplifier additional ("excess") noise temperature (degrees K);
= device "noise temperature";
= $T_o(\text{nf} - 1)$;
s/n_I = signal-to-noise power ratio at input to amplifier;
s/n_O = signal-to-noise power ratio at output of amplifier.

$$\begin{aligned} \text{NF (dB)} &= \text{noise figure} \\ &= 10\ \log(\text{nf}) \\ &= \text{S/N}_\text{I} - \text{S/N}_\text{O} \end{aligned}$$

nf = $10^{\text{NF}/10}$;
S/N_I = signal-to-noise ratio at input to amplifier (dB);
= $10\ \log(\text{s/n}_\text{I})$;
S/N_O = signal-to-noise ratio at output of amplifier (dB);
= $10\ \log(\text{s/n}_\text{O})$.

For cascaded (series) active amplifiers:

$$\begin{aligned} \text{nf} &= \text{overall noise factor of the cascaded amplifiers} \\ &= \text{nf}_1 + \frac{(\text{nf}_2 - 1)}{g_1} + \frac{(\text{nf}_3 - 1)}{g_2} + \cdots + \frac{(\text{nf}_n - 1)}{g_{(n-1)}} \end{aligned} \tag{A.51}$$

nf_1 = noise factor of the first device;
nf_2 = noise factor of the second device;
nf_3 = noise factor of the third device;
g_1 = gain (power ratio) of first device;
g_2 = gain (power ratio) of second device;
g_n = gain (power ratio) of nth device.

The implied assumption is all devices are matched impedances and bandwidth shrinkage of cascaded devices is insignificant.

The noise figure of an attenuator is simply the attenuation (dB, > 0) of the attenuator.

The noise figure of a cascaded attenuator and an amplifier is the sum of the two (dB values).

The “front end” noise produced by an amplifier may be calculated as follows:

$$
\begin{aligned}
n &= \text{noise produced by a matched “internal” resistor} \\
&= K(T_o + T_e)B\ 10^6\ (\text{W}) \\
&= 1.38 \times 10^{-17} T_o \left[1 + \left(\frac{T_e}{T_o}\right)\right] B \\
&= 4.00 \times 10^{-15}\ \text{nf}\ B
\end{aligned}
\tag{A.52}
$$

B = noise bandwidth of the device (MHz);
N = front end noise = 10 log(n).

$$
\begin{aligned}
N(\text{dBW}) &= 10\ \log(n) \\
&= -144 + \text{NF(dB)} + 10\ \log(B)
\end{aligned}
\tag{A.53}
$$

$$
\begin{aligned}
N(\text{dBm}) &= N(\text{dBW}) + 30 \\
&= -114 + \text{NF(dB)} + 10\ \log(B)
\end{aligned}
\tag{A.54}
$$

$$N(\text{dBW/MHz}) = -144 + \text{NF} \tag{A.55}$$

$$\text{N(dBW/4 kHz)} = -168 + \text{NF} \tag{A.56}$$

A common problem is to determine the signal associated with a known radio threshold signal-to-noise ratio. Assume that the radio receiver is limited by front end noise.

$$
\begin{aligned}
S(\text{dBm}) &= \text{S/N(dB)} + N(\text{dBm}) \\
&= \text{S/N(dB)} - 114 + \text{NF(dB)} + 10\ \log(B)
\end{aligned}
\tag{A.57}
$$

S = received signal power level (dBm) at threshold;
S/N = receiver threshold signal-to-noise ratio (dB);
NF = receiver noise figure (dB);
B = receiver bandwidth (MHz).

Remember that the receiver noise figure is the noise figure of the front end amplifier plus the loss (dB) between the amplifier and the measurement location. The typical amplifier noise figure for low frequency microwave radios is about 2 dB. The typical waveguide and receiver filter loss in front of a receiver is about 2 dB. Therefore, the typical microwave radio receiver noise figure is 4 dB.

A.2.21 Shannon’s Formula for Theoretical Limit to Transmission Channel Capacity

$$
\begin{aligned}
C &\le W\ \log_2[1 + (\text{S/N})] \\
&\le 3.322\ W\ \log_{10}[1 + (\text{S/N})] \\
&\le\approx 3.322\ W\ \log_{10}(\text{S/N}) \\
&\le 0.3322\ W[\text{S/N(dB)}]
\end{aligned}
\tag{A.58}
$$

C = channel capacity (Mb/s);
W = channel bandwidth (MHz);
S/N = channel signal-to-noise power ratio.

In the above equations, replacing the power ratio (S/N + 1) with S/N introduces less than 1% dB error for all S/Ns $\geq$10 dB.

Shannon's limit may be rewritten to define the minimum S/N required to achieve a given spectral efficiency:

$$\text{S/N(dB)} \geq 3\ C/W \tag{A.59}$$

$$\text{S/N(dB)} \geq 3[\text{spectral efficiency (bits/s/Hz)}] \tag{A.60}$$

Assumptions are filtering is rectangular ("brick wall"), noise is Gaussian and the transmitted signal spectrum fills the transmission channel bandwidth.

A.3 ANTENNAS (FAR FIELD)

See Chapter 8 for near field considerations.

A.3.1 General Microwave Aperture Antenna (Far Field) Gain (dBi)

$$\begin{aligned} &11.1 + 20\ \log\ F(\text{GHz}) + 10\ \log A(\text{ft}^2) + 10\ \log\left(\frac{E}{100}\right) + 20\ \log\ \cos\left(\frac{C}{2}\right) \\ &21.5 + 20\ \log\ F(\text{GHz}) + 10\ \log\ A(\text{m}^2) + 10\ \log\left(\frac{E}{100}\right) + 20\ \log\ \cos\left(\frac{C}{2}\right) \end{aligned} \tag{A.61}$$

F = frequency of radio wave;
A = antenna physical area;
C = angle between incoming and outgoing radio signal paths;
E = antenna power transmission efficiency expressed as a percentage.

A.3.2 General Microwave Antenna (Far Field) Relative Gain (dBi)

$$G_{\text{ref}} + 20\ \log\left(\frac{F}{F_{\text{ref}}}\right) \tag{A.62}$$

G_{ref} = antenna gain (dBi) at F_{ref} (GHz);
F_{ref} = reference frequency (GHz);
F = frequency of interest (GHz).

This formula is typically used to interpolate antenna catalog data. If F_{ref} is the mid-band frequency, accuracy is typically within 0.1 dB of measured values at band edges.

A.3.3 Parabolic (Circular) Microwave Antenna (Far Field) Gain (dBi)

$$\begin{aligned} &10.1 + 20\ \log\ F + 20\ \log\ D(\text{ft}) + 10\ \log\ \left(\frac{E}{100}\right) \\ &20.5 + 20\ \log\ F + 20\ \log\ D(\text{m}) + 10\ \log\ \left(\frac{E}{100}\right) \end{aligned} \tag{A.63}$$

F = frequency of radio wave (GHz);
D = antenna diameter;
E = antenna power transmission efficiency expressed as a percentage, $40 \leq E \leq 60$, $E \approx 55$ typically.

A.3.4 Parabolic (Circular) Microwave Antenna Illumination Efficiency

$$10\ \log\left(\frac{E}{100}\right) = -\ 10.1 + G - 20\ \log\left(F\right) - 20\ \log[D(\text{ft})]$$
$$= -\ 20.4 + G - 20\ \log\left(F\right) - 20\ \log[D(\text{m})] \quad (A.64)$$
$$\frac{E}{100} = 10^{[10\ \log\{E/100\}]/10} \quad (A.65)$$

G = antenna far-field gain (dBi);
F = frequency of radio wave (GHz);
D = antenna diameter;
E = antenna power transmission efficiency (%).

A.3.5 Panel (Square) Microwave Antenna (Far Field) Gain (dBi)

$$11.1 + 20\ \log\ F(\text{GHz}) + 20\ \log\ S(\text{ft}) + 10\ \log\left(\frac{E}{100}\right)$$
$$21.5 + 20\ \log\ F(\text{GHz}) + 20\ \log\ S(\text{m}) + 10\ \log\left(\frac{E}{100}\right) \quad (A.66)$$

F = frequency of radio wave;
S = length of side of square;
E = antenna power transmission efficiency expressed as a percentage, typically $E \approx 100$.

A.3.6 Panel (Square) Microwave Antenna Illumination Efficiency

$$\frac{E}{100} = \frac{(N0 + N1\beta + N2\beta^2 + N3\beta^3 + N4\beta^4)}{(1 + D1\beta + D2\beta^2 + D3\beta^3 + D4\beta^4)} \quad (A.67)$$
$$\beta = \left\{\left(\frac{W}{\lambda}\right)\ \sin\left(\frac{\phi_{3\text{dB}}}{2}\right)\right\} \quad (A.68)$$
$$0.447 \le \beta \le 1.49$$

E = antenna power transmission efficiency (%)

$$10 \le E \le 100$$

W = width of the square antenna (measured along the edge)
λ = radio free space wavelength
λ (ft) = 0.98357/f (GHz)
λ (m) = 0.29980/f (GHz)
$\phi_{3\text{dB}}$ = antenna 3-dB beam width
= angle measured between the two −3 dB power values (referenced to the boresight power)
$N0 = -0.4468979109577574$
$N1 = 2.705347403057084$
$N2 = -5.689139811168476$
$N3 = 5.017375871680245$

$N4 = -1.037085334383484$
$D1 = -7.914244751535077$
$D2 = 24.1096714821637$
$D3 = -33.58979930453501$
$D4 = 18.85685129777957$

A.3.7 Angle Between Incoming and Outgoing Radio Signal Paths, C, for a Passive Reflector

Refer to Figure A.1

A = total horizontal included angle (measured in the horizontal plane) formed by the incoming and outgoing radio paths converging at the reflector or antenna;
= positive angle measured (on a map) between paths from transmitter and receiver antenna to the reflector or antenna (ignoring relative height of sites);
θ_1 = smaller vertical path angle formed by the horizontal plane and a line between the reflector or antenna and the transmitter or receiver antenna;
θ_2 = larger vertical path angle formed by the horizontal plane and a line between the reflector or antenna and the transmitter or receiver antenna;

$$B = \text{reflector bearing angle correction}$$

$$= \arctan \frac{\left[\tan\left(\frac{A}{2}\right)\right][\cos\ \theta_1 - \cos\ \theta_2]}{(\cos\ \theta_1 + \cos\ \theta_2)} \tag{A.69}$$

$$\theta_3 = \text{reflector tilt relative to perpendicular to horizontal plane} \tag{A.70}$$

$$\theta_3 = \arctan\left[\frac{\{(\cos\ B)(\sin\ \theta_1 + \sin\ \theta_2)\}}{\left\{\cos\ \left(\frac{A}{2}\right)(\cos\ \theta_1 + \cos\ \theta_2)\right\}}\right] \tag{A.71}$$

$$C = \text{angle between incoming and outgoing radio signal path}$$

$$= 2\ \arccos\left\{\frac{(\sin\ \theta_1 + \sin\ \theta_2)}{[2(\sin\ \theta_3)]}\right\} \tag{A.72}$$

$$= A \quad \text{when } \theta_1 \text{ and } \theta_2 \text{ are} < 20^\circ (< 0.1\ \text{dB gain error})$$

For the above all cosines and tangents are positive for angles between 0° and 90°. They are negative for angles between 90° and 180°. Sines are positive for paths going down from the repeater or antenna

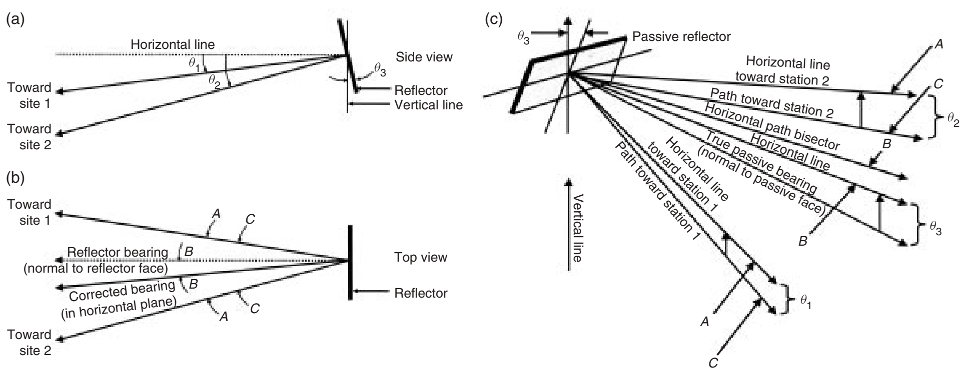

Figure A.1 Reflector geometry measured (a) using horizontal plane projections of the paths, (b) using a horizontal plane projection of reflector bearing, and (c) between actual paths towards the sites.

(toward the transmitter or receiver) or negative for path going up (toward the transmitter or receiver). *B* always rotates the passive bearing toward the path with the least vertical angle θ_1.

A.3.8 Signal Polarization Rotation Through a Passive Reflector, $\Delta\phi$

The definitions in the previous paragraph can be used.

$$\begin{aligned}\Delta\phi &= \text{rotation of both signal polarizations for signal passing through a} \\ &\qquad \text{passive reflector relative to signal in horizontal plane} \\ &= \phi_2 + \phi_2 - \pi (\text{or } 180^\circ) \qquad \text{(A.73)} \\ \phi_1 &= \text{arc cos } \{[\sin \theta_1 - (\sin \theta_2 \cos C)]/(\cos \theta_2 \sin C)\}; \\ \phi_2 &= \text{arc cos}\{[\sin \theta_3 - (\sin\theta_1 \cos(C/2))]/[\cos \theta_1 \sin(C/2)]\}.\end{aligned}$$

$\Delta\phi$ is negative when counterclockwise as viewed from the right-hand path when facing the reflector. $\Delta\phi$ is negative when clockwise as viewed from the left-hand path when facing the reflector. These are true regardless of which paths are assigned to θ_1 and θ_2 above.

A.3.9 Signal Effects of Polarization Rotation

The definitions in the previous two paragraphs can be used.

$$\text{Received signal loss (dB)} = 10 \log (\cos^2 \Delta\phi) \qquad \text{(A.74)}$$

$$\text{Cross-polarization discrimination (XPD, dB)} = -10 (\log \sin^2 \Delta\phi) \qquad \text{(A.75)}$$

A.3.10 Passive Reflector (Far Field) Two-Way (Reception and Retransmission) Gain (dBi)

$$22.2 + 40 \log F(\text{GHz}) + 20 \log A(\text{ft}^2) + 20 \log \cos\left(\frac{C}{2}\right)$$

$$42.9 + 40 \log F(\text{GHz}) + 20 \log A(\text{m}^2) + 20 \log \cos\left(\frac{C}{2}\right) \qquad \text{(A.76)}$$

F = frequency of radio wave;
A = reflector area;
C = angle between incoming and outgoing radio signal paths.

A.3.11 Rectangular Passive Reflector 3-dB Beamwidth (Degrees, in Horizontal Plane)

Square projection onto both paths

One edge parallel to Earth (zero rotation)

$$\frac{49.8}{F[(\text{GHz}) \times W (\text{ft}) \cos(C/2)]}$$

$$\frac{15.2}{F [(\text{GHz}) \times W (\text{m}) \cos(C/2)]}$$

F = frequency of radio wave;
W = reflector width;
C = angle between incoming and outgoing radio signal paths.

Square projection onto both paths

Diamond shape (square with 45° rotation)

W = width of unrotated square

$$\frac{50.8}{\{F(\text{GHz})\ W\ (\text{ft})\cos(C/2)\}}$$
$$\frac{15.5}{\{F(\text{GHz})\ W\ (\text{m})\cos(C/2)\}}$$

F = frequency of radio wave;
W = reflector width;
C = angle between incoming and outgoing radio signal paths.

A.3.12 Elliptical Passive Reflector 3-dB Beamwidth (Degrees)

$$\frac{57.9}{\{F(\text{GHz})D\ (\text{ft})\}}$$
$$\frac{17.7}{\{F(\text{GHz})D\ (\text{m})\}}$$

F = frequency of radio wave;
D = smaller reflector diameter.

It is assumed that the reflector has a 45° angle to the paths of propagation so that the projection of the reflector shape onto the path is circular.

A.3.13 Circular Parabolic Antenna 3-dB Beamwidth (Degrees)

$$\frac{(57.9\ \text{NBW})}{[F(\text{GHz})D(\text{ft})]} \approx \frac{88.0}{[F(\text{GHz})D(\text{ft})]}$$
$$\frac{(17.7\ \text{NBW})}{[F(\text{GHz})D(\text{m})]} \approx \frac{26.8}{[F(\text{GHz})D(\text{m})]}$$

$$E(\text{dB}) = -10.1 + G(\text{dB}) - 20\ \log[F(\text{GHz})] - 20\ \log[D(\text{ft})]$$
$$= -20.4 + G(\text{dB}) - 20\ \log[F(\text{GHz})] - 20\ \log[D(\text{m})] \tag{A.77}$$
$$X = E(\text{PR}) = 10^{E(\text{dB})/10} \tag{A.78}$$
$$\text{NBW} = \frac{\text{NBWn}}{\text{NBWd}} \tag{A.79}$$
$$\text{NBWn} = (C1 + C2X + C3X^2 + C4X^3 + C5X^4) \tag{A.80}$$
$$\text{NBWd} = 1 + C6X + C7X^2 + C8X^3 + C9X^4 + C10X^5 \tag{A.81}$$

$C1 = 11.65806521303201$
$C2 = 96.51565982372452$
$C3 = -152.1423956242208$
$C4 = -4.217102204230871$
$C5 = 48.37581221497451$
$C6 = 49.10391941747248$
$C7 = 32.82051399322144$

$C8 = -201.8020843245464$
$C9 = 130.4562878319081$
$C10 = -11.38859718015969$

$E(\text{dB})$ = 10 log (antenna efficiency, power ratio);
$E(\text{PR})$ = antenna efficiency (power ratio), $0.1 \leq E \leq 1.0$;
NBW = antenna-normalized 3.01-dB bandwidth;
F = frequency of radio wave;
D = antenna diameter;
G = antenna isotropic gain (dBi).

A.3.14 Passive Reflector Far Field Radiation Pattern Envelopes

Normalized to 0 dB for $\theta = 0^\circ$;

d = width of square or diameter of circle projected onto path;
= physical width or diameter $\times \cos(C/2)$;
C = angle between incoming and outgoing radio signal paths;
λ = free space wavelength of radio wave;
θ = azimuth of measurement point relative to path of maximum transmission (bore sight);
$X = (d/\lambda)|\sin\theta| \geq 0$;
$P(\text{dB})$ = far-field radiation pattern envelope relative power intensity.

A.3.14.1 Rectangular Reflector

Square projection onto both paths;
One edge parallel to Earth (zero rotation).

$$\text{For } X \leq 0.50 \ : \ P = 20 \ \log\left[\frac{\sin(\pi X)}{(\pi X)}\right] \tag{A.82}$$

$$\text{For } X > 0.50 \ : \ P = -20 \ \log(\pi X) \tag{A.83}$$

A.3.14.2 Diamond Reflector

Square projection onto both paths;
Diamond shape (square with 45° rotation);
d = width of unrotated square.

$$\text{For } X \leq 0.70 \ : \ P = 40 \ \log\left[\frac{\sin(2.221X)}{(2.221X)}\right] \tag{A.84}$$

$$\text{For } X > 0.70 \ : \ P = 6.02 - 40 \ \log(\pi X) \tag{A.85}$$

A.3.14.3 Elliptical Reflector

Circular projection onto both paths

$$\text{For } X \leq 0.775 \ : \ P = 20 \ \log\left[\frac{\sin(2.680X)}{(2.680X)}\right] \tag{A.86}$$

$$\text{For } X > 0.775 \ : \ P = 4.06 - 30 \ \log(\pi X) \tag{A.87}$$

A.3.15 Inner Radius for the Antenna Far-Field Region

This is sometimes called *outer radius for the antenna near-field region*.

$$d_{FF} = \text{radial distance from the antenna to the far field edge}$$
$$= \frac{2D^2}{\lambda} \tag{A.88}$$

$\lambda =$ wavelength of the radio wave in the same dimensions as D;
$d_{FF}\,(\text{ft}) = 2.033D^2\,(\text{ft})\ \ F(\text{GHz})$;
$d_{FF}(\text{m}) = 6.671D^2(\text{m})\ \ F(\text{GHz})$;
$F =$ frequency of radio wave;
$D =$ larger linear dimension of the antenna in a plane of projection orthogonal to the line of wave propagation;
$=$ parabolic antenna diameter;
$=$ passive reflector {[width $\times$ cos(α_h)] or [height $\times$ cos(α_v)], which ever is larger};
$\alpha_h =$ angle between path direction and perpendicular to face of reflector measured in a horizontal plan;
$\alpha_v =$ angle between path direction and perpendicular to face of reflector measured in a vertical plan.

As noted in Chapter 8, a more accurate version of this formula is to take into consideration the illumination efficiency of the antenna:

$$d_{FF} = \text{radial distance from the antenna to the far field edge}$$
$$= \frac{2\eta D^2}{\lambda} \tag{A.89}$$

$\eta =$ illumination power efficiency (0 to 1).

The above formulas have no specified accuracy. They are rough rules of thumb. If the far-field transition is defined as the point at which the antenna boresight gain is 1 dB less than the far-field gain for that distance, the following formula may be used to estimate that limit for a circular antenna:

$$\Delta_{dB}(1\ \text{dB far-field transition distance}) = -10.46 + 8.730\eta - 4.116\eta^2 - \frac{0.4638}{\eta} \tag{A.90}$$

For a square antenna, the 1-dB transition point is given by the following:

$$\Delta_{dB}(\text{far-field transition distance}) = -8.544 + 6.188\eta - 1.954\eta^2 - \frac{0.5349}{\eta} \tag{A.91}$$

$$\Delta_{dB} = 10\ \log\left[\frac{d}{\left(\frac{2D^2}{\lambda}\right)}\right]$$

$d =$ radial distance from the antenna in the same units as D;
$\eta =$ illumination efficiency (0–1).

A.4 NEAR-FIELD POWER DENSITY

A.4.1 Circular Antennas

$$S(\Delta) = \text{near-field power density(mW/cm}^2\text{)}$$
$$= 10^{(\psi/10)}$$
$$\psi = 10\ \log_{10}[S(\Delta] = 10\ \log_{10}\left[\frac{S(\Delta)}{S(\Delta=1)}\right] + 10\ \log_{10}\ [S(\Delta=1)]$$

$$10\ \log_{10}\left[\frac{S(\Delta)}{S(\Delta=1)}\right] = -2\ \Delta_{\text{dB}} \quad \text{(A.92)}$$

or

$$10\ \log_{10}\left[\frac{S(\Delta)}{S(\Delta=1)}\right] = A + B\eta + \frac{C}{\eta} + D\eta^2 + \frac{E}{\eta^2} + F\eta^3 \quad \text{(A.93)}$$

$A =$ 40.430453;
$B = -61.480406$;
$C = -0.46691971$;
$D =$ 55.376708;
$E = 0.04791274$;
$F = -19.805638$,
whichever is smaller.

$S(\Delta = 1) = (\pi p\eta)/(16D^2)$;
$\Delta = d/(2D^2/\lambda) =$ normalized distance parameter for circular antenna
$= 0.49179d\text{(ft)}/[D\text{(ft)}^2 f\text{(GHz)}] = 0.14990d\text{(m)}/[D\text{(m)}^2 f\text{(GHz)}]$;
$\Delta_{\text{dB}} = 10\ \log(\Delta)$;
$\Delta = 1(\Delta_{\text{dB}} = 0)$ normalized distance at nominal far field crossover point;
$\eta =$ antenna efficiency $(0 \le \eta \le 1) = E/100$;
$E =$ antenna efficiency (%), see earlier far-field equations
$p =$ transmitter power (mW);
$D =$ aperture diameter (cm) = 30.48 diameter (ft);
f = operating frequency;
d = perpendicular distance from the antenna aperture.

A.4.2 Square Antennas

$$S(\Delta) = \text{near-field power density(mW/cm}^2\text{)}$$
$$= 10^{(\psi 10)}$$
$$\psi = 10\ \log_{10}[S(\Delta] = 10\ \log_{10}\left[\frac{S(\Delta)}{S(\Delta=1)}\right] + 10\ \log_{10}[S(\Delta=1)]$$

$$10\ \log_{10}\left[\frac{S(\Delta)}{S(\Delta=1)}\right] = -2\ \Delta\text{dB} \quad \text{(A.94)}$$

or

$$10\ \log_{10}\left[\frac{S(\Delta)}{S(\Delta=1)}\right] = A + B\eta + \frac{C}{\eta} + D\eta^2 + \frac{E}{\eta^2} + F\eta^3 \quad \text{(A.95)}$$

$A = 34.223061$;
$B = -58.288613$;
$C = 0.51017224$;

$D = 64.124471;$
$E = -0.013593334;$
$F = -29.354905,$
whichever is smaller.

$S(\Delta = 1) = (p\eta)/(4W^2);$
$\Delta = d/(2W^2/\lambda)$ = normalized distance parameter for square antenna;
$= 0.49179d(\text{ft})/[W(\text{ft})^2 f(\text{GHz})] = 0.14990d(\text{m})/[W(\text{m})^2 f(\text{GHz})];$
$\Delta_{dB} = 10\ \log(\Delta);$
$\Delta = 1$ ($\Delta_{dB} = 0$) normalized distance at nominal far-field crossover point;
η = antenna efficiency ($0 \le \eta \le 1$) $= E/100$;
E = antenna efficiency (%), see earlier far-field equations;
p = transmitter power (mW);
W = antenna width (cm) = 30.48 width (ft);
f = operating frequency;
d = perpendicular distance from the antenna aperture.

A.5 ANTENNAS (CLOSE COUPLED)

A.5.1 Coupling Loss L_{NF} (dB) Between Two Antennas in the Near Field

The following formulas estimate near-field coupling loss between antennas:
$D = \Delta_{dB}$ = normalized distance between antennas;
$-10 \le D = \Delta_{dB} \le 0;$
$\Delta_{dB} = 10\ \log(\Delta);$
$\Delta = d/(2D^2/\lambda)$ = normalized distance parameter for the larger circular antenna;
$\Delta = d/(2W^2/\lambda)$ = normalized distance parameter for the larger square antenna;
R = ratio of smaller antenna width/larger antenna width;
$0 \le R \le 1;$
$N = \eta$ = illumination efficiency;
$0.25 \le \eta = 1.0;$
L_{NF} = near-field antenna to antenna coupling loss (dB);
= value to be added to far-field free space loss.

A.5.2 Coupling Loss L_{NF}(dB) Between Identical Antennas

$$L_{NF} = C1 + C2 \times D + C3 \times N + C4 \times D^2 + C5 \times N^2 + C6 \times D \times N + C7 \times D^3 + C8 \times N^3 + C9 \times D \times N^2 + C10 \times D^2 \times N \quad \text{(A.96)}$$

Circular (Parabolic) Antennas	Square (Aligned) Antennas
$C1 = 0.5688330523739922$	$C1 = 0.246159837605786$
$C2 = -0.2843725447552475$	$C2 = -0.8316031494312162$
$C3 = -5.913339006420615$	$C3 = -6.285507130745876$
$C4 = -0.005544709076135127$	$C4 = -0.07920675867559671$
$C5 = 12.46763586356988$	$C5 = 15.99365443163881$
$C6 = 0.2328735832473644$	$C6 = 1.544481166675684$
$C7 = 0.00194088480963481$	$C7 = -0.0002143259518259517$
$C8 = -7.561944604459402$	$C8 = -10.80646533812489$
$C9 = 0.299596446278667$	$C9 = -0.1245258794169019$
$C10 = -0.0905373781148429$	$C10 = -0.02108477297350535$

A.5.3 Coupling Loss L_{NF} (dB) Between Different-Sized Circular Antennas

$$L_{NF} = C1 + C2 \times D + C3 \times R + C4 \times D^2 + C5 \times R^2 + C6 \times D \times R + C7 \times D^3 + C8 \times R^3 + C9 \times D \times R^2 + C10 \times D^2 \times R \quad \text{(A.97)}$$

$N = \eta = 1.00$	$N = \eta = 0.75$
$C1 = -0.7499780289155289$	$C1 = -0.2714292776667776$
$C2 = -0.5133628527953528$	$C2 = 0.02152531477781479$
$C3 = 3.911939824296074$	$C3 = 3.675637157518407$
$C4 = -0.07520881757131757$	$C4 = 0.0321249222999223$
$C5 = -6.863575712481961$	$C5 = -10.12064281204906$
$C6 = 0.6943731306193806$	$C6 = -0.1223080099067599$
$C7 = 0.002120276482776482$	$C7 = 0.006847157472157472$
$C8 = 3.416898148148148$	$C8 = 6.74849537037037$
$C9 = 0.08539001623376626$	$C9 = 0.334229301948052$
$C10 = -0.0209452047952048$	$C10 = -0.0458997668997669$

$N = \eta = 0.50$	$N = \eta = 0.25$
$C1 = 0.1013892010767011$	$C1 = 0.1093747294372294$
$C2 = 0.2001590773115773$	$C2 = 0.142861466033966$
$C3 = -0.6023601345413846$	$C3 = -0.5683255501443001$
$C4 = 0.05432371378621378$	$C4 = 0.03395236985236985$
$C5 = 0.4445470328282829$	$C5 = 0.6277401244588743$
$C6 = -0.4858658146020646$	$C6 = -0.3156246699134199$
$C7 = 0.005887286324786324$	$C7 = 0.002853418803418803$
$C8 = -0.01127946127946136$	$C8 = -0.192550505050505$
$C9 = 0.3789476461038961$	$C9 = 0.1949728084415584$
$C10 = -0.05209527139527139$	$C10 = -0.02924848484848485$

A.5.4 Coupling Loss L_{NF} (dB) Between Different-Sized Square Antennas (Both Antennas Aligned)

$$L_{NF} = C1 + C2 \times D + C3 \times R + C4 \times D^2 + C5 \times R^2 + C6 \times D \times R + C7 \times D^3 + C8 \times R^3 + C9 \times D \times R^2 + C10 \times D^2 \times R \quad \text{(A.98)}$$

$N = \eta = 1.00$	$N = \eta = 0.75$
$C1 = -0.9729305264180264$	$C1 = -0.3371434274059274$
$C2 = -0.8812381138306138$	$C2 = -0.1846516780441781$
$C3 = 4.742492976699226$	$C3 = 0.887764439033189$
$C4 = -0.2101820207570208$	$C4 = -0.01862155622155622$
$C5 = -8.833426902958152$	$C5 = -1.338570752164502$
$C6 = 1.227099754828505$	$C6 = 0.1831897564935065$
$C7 = -0.005192715617715618$	$C7 = 0.006001243201243201$
$C8 = 4.532586279461279$	$C8 = 0.6045138888888888$
$C9 = 0.1829265422077922$	$C9 = 0.3801099837662338$
$C10 = 0.0529536297036297$	$C10 = -0.004622077922077921$

$N = \eta = 0.50$	$N = \eta = 0.25$
$C1 = -0.0139240342990343$	$C1 = 0.1224828477078477$
$C2 = 0.1031087360787361$	$C2 = 0.169524861989862$
$C3 = -0.2748664266289267$	$C3 = -0.7053111198986198$
$C4 = 0.03496463952713952$	$C4 = 0.04124757742257742$
$C5 = 0.08828057359307374$	$C5 = 0.8015615981240979$
$C6 = -0.343841555944056$	$C6 = -0.3901483508158508$
$C7 = 0.006170593758093758$	$C7 = 0.003830542605542605$
$C8 = 0.08970959595959589$	$C8 = -0.2554187710437709$
$C9 = 0.4123693181818182$	$C9 = 0.2612280844155844$
$C10 = -0.0415508991008991$	$C10 = -0.03715820845820846$

For values of η between the values above, calculate the values for $\eta = 1.00$, 0.75, 0.50, and 0.25 and use two-dimensional cubic interpolation (A.10.1) to determine the desired value.

A.5.5 Coupling Loss L_{NF} (dB) for Antenna and Square Reflector in the Near Field

The following formulas estimate near field coupling loss between an antenna and a reflector:

$D = \Delta_{dB}$ = normalized distance between antenna and reflector;
$-8 \leq D = \Delta_{dB} \leq 0$;
$\Delta_{dB} = 10\ \log(\Delta)$;
$\Delta = d/(2W^2/\lambda)$ = normalized distance parameter for the reflector width;
R = ratio of antenna width/reflector width;
$0 \leq R \leq 1$;
$N = \eta$ = illumination efficiency;
$0.25 \leq \eta \leq 1.0$;
L_{NF} = near field antenna to reflector coupling loss (dB);
= value to be added to far-field free space loss.

A.5.6 Coupling Loss L_{NF} (dB) for Circular Antenna and Square Reflector

$$L_{NF} = C1 + C2 \times D + C3 \times R + C4 \times D^2 + C5 \times R^2 + C6 \times D \times R + C7 \times D^3 + C8 \times R^3 + C9 \times D \times R^2 + C10 \times D^2 \times R \tag{A.99}$$

$N = \eta = 1.00$	$N = \eta = 0.75$
$C1 = -0.4566947811447811$	$C1 = -0.2431097426647427$
$C2 = -0.2020482792780412$	$C2 = 0.06178604780801208$
$C3 = 2.191165239698573$	$C3 = 1.058045280984448$
$C4 = -0.02878061052703909$	$C4 = 0.03989009482580911$
$C5 = -4.37696097883598$	$C5 = -2.486101521164021$
$C6 = 0.3237634547000618$	$C6 = -0.03516842648423002$
$C7 = 0.007908487654320989$	$C7 = 0.01286877104377104$
$C8 = 2.162461419753087$	$C8 = 1.312422839506173$
$C9 = 0.3109449404761905$	$C9 = 0.3230811011904762$
$C10 = -0.03379225417439703$	$C10 = -0.04815576685219542$

$N = \eta = 0.50$	$N = \eta = 0.25$
$C1 = -0.131068519320186$	$C1 = -0.05018120811287484$
$C2 = 0.1926641022755904$	$C2 = 0.2815940879028379$
$C3 = 0.4079589168136392$	$C3 = -0.09241433875794949$
$C4 = 0.07328302583659727$	$C4 = 0.09614733044733044$
$C5 = -1.326173390652558$	$C5 = -0.2993532848324522$
$C6 = -0.2101255144557823$	$C6 = -0.2734471239177489$
$C7 = 0.01526261223344557$	$C7 = 0.01694427609427609$
$C8 = 0.7796810699588479$	$C8 = 0.2686085390946506$
$C9 = 0.3159635416666667$	$C9 = 0.2477760416666666$
$C10 = -0.05155578231292517$	$C10 = -0.04260757575757575$

For values of η between the values above, calculate the values for $\eta = 1.00$, 0.75, 0.50, and 0.25 and use two-dimensional cubic interpolation (A.10.1) to determine the desired value.

A.5.7 Coupling Loss L_{NF} (dB) for Square Antenna and Square Reflector (Both Aligned)

$$L_{NF} = C1 + C2 \times D + C3 \times R + C4 \times D^2 + C5 \times R^2 + C6 \times D \times R + C7 \times D^3 + C8 \times R^3 + C9 \times D \times R^2 + C10 \times D^2 \times R \quad (A.100)$$

$N = \eta = 1.00$	$N = \eta = 0.75$
$C1 = -0.6466395109828443$	$C1 = -0.3507770017636684$
$C2 = -0.4242813704505371$	$C2 = -0.06543854400009162$
$C3 = 3.395700560098338$	$C3 = 1.723228713590936$
$C4 = -0.08242216209716208$	$C4 = 0.008616482855768571$
$C5 = -6.342335537918872$	$C5 = -3.605354938271605$
$C6 = 0.8051073917748917$	$C6 = 0.206446064471243$
$C7 = 0.004274172278338946$	$C7 = 0.01070726711560045$
$C8 = 2.923791152263375$	$C8 = 1.771116255144033$
$C9 = 0.1849970238095238$	$C9 = 0.2753497023809524$
$C10 = -0.006447186147186151$	$C10 = -0.03658038033395176$

$N = \eta = 0.5$	$N = \eta = 0.25$
$C1 = -0.1980865496232163$	$C1 = -0.07070867564534233$
$C2 = 0.1141195494056208$	$C2 = 0.2586083355092878$
$C3 = 0.8217887665009888$	$C3 = 0.03945184971407202$
$C4 = 0.05379672232529374$	$C4 = 0.09023423177351747$
$C5 = -2.054483906525573$	$C5 = -0.5848569223985893$
$C6 = -0.07965355287569573$	$C6 = -0.2614715762213976$
$C7 = 0.01389718013468013$	$C7 = 0.01650486812570146$
$C8 = 1.094843106995885$	$C8 = 0.4112782921810701$
$C9 = 0.3029285714285714$	$C9 = 0.2718556547619048$
$C10 = -0.04733208101422387$	$C10 = -0.04616696042053185$

For values of η between the values above, calculate the values for $\eta = 1.00$, 0.75, 0.50, and 0.25 and use two-dimensional cubic interpolation (A.10.1) to determine the desired value.

A.5.8 Two Back-to-Back Square Reflectors Combined Gain

The following factor Y (dB) accounts for the combined gain of two flat square reflectors relative to the gain of a single reflector the size of the smaller reflector.

The formulas are valid for $R_W \geq 1$ and for all X.

d = separation between the two reflectors
a^2 = projected area of smaller reflector
b^2 = projected area of larger reflector
projected area = physical area × [cosine (1/2 path included angle at reflector)]
a = width of the smaller reflector assuming reflector projection is square
b = width of the larger reflector assuming reflector projection is square
λ = free space wavelength of radio wave
a, b, d, and λ are in the same linear units
$X_p = d/(2a^2/\lambda)$
$R_W = b/a$
$K = (1/2)$ sqrt $(1/X_p)$, sqrt is the square root function
$p = K(R_W - 1)$
$q = K(R_W + 1)$

Y(dB) = combined gain of both reflectors relative to gain of single smaller reflector

$$= 20 \ \log[(U^2 + V^2)(2X_p)] \tag{A.101}$$

$$U = q\ C(q) - p\ C(p) + (1/\pi)[\sin(\pi p^2/2) - \sin(\pi q^2/2)]$$
$$V = q\ S(q) - p\ S(p) + (1/\pi)[\cos(\pi q^2/2) - \cos(\pi p^2/2)]$$
$$C(z) = 0.5 + f(z)\sin(\pi z^2/2) - g(z)\cos(\pi z^2/2)$$
$$S(z) = 0.5 - f(z)\cos(\pi z^2/2) - g(z)\sin(\pi z^2/2)$$
$$f(z) = (1 + 0.926\ z)/(2 + 1.792\ z + 3.104\ z^2)$$
$$g(z) = 1/(2 + 4.142\ z + 3.492\ z^2 + 6.670\ z^3)$$

For $X \gg 1$, $Y(\text{dB}) \cong 6 + 40 \ \log \ R_W + 40 \ \log \ K$
For $X \ll 1$, $Y(\text{dB}) \cong 0$.

If one of the transmit or receive parabolic antennas is in the near field of the combined reflectors, the addition of the parabolic and rectangular reflector near-field correction may be needed.

A.6 PATH GEOMETRY

A.6.1 Horizons (Normal Refractivity over Spherical Earth)

d = distance to horizon from antenna;
h = antenna height above spherical Earth;
K = equivalent earth radius factor.

Geometric horizon

$$d(\text{miles}) = 1.225 \ \text{sqrt}[h(\text{ft})]$$
$$d(\text{km}) = 3.570 \ \text{sqrt}[h(\text{m})] \tag{A.102}$$

Electromagnetic wave horizon

$$d(\text{miles}) = 1.225 \ \text{sqrt}[Kh(\text{ft})]$$
$$d(\text{km}) = 3.570 \ \text{sqrt}[Kh(\text{m})] \tag{A.103}$$

Radio horizon (typical $K = 4/3$)

$$d(\text{miles}) = 1.414\ \text{sqrt}[h(\text{ft})]$$
$$d(\text{km}) = 4.122\ \text{sqrt}[h(\text{m})] \quad (A.104)$$

Optical horizon (typical noonday $K = 7/6$)

$$d(\text{miles}) = 1.323\ \text{sqrt}[h(\text{ft})]$$
$$d(\text{km}) = 3.856\ \text{sqrt}[h(\text{m})] \quad (A.105)$$

A.6.2 Earth Curvature (Height Adjustment Used on Path Profiles)

$$h(\text{ft}) = \frac{[d_1(\text{miles}) \times d_2(\text{miles})]}{(1.500 \times K)}$$
$$h(\text{m}) = \frac{[\text{d}_1(\text{km}) \times d_2(\text{km})]}{(12.75 \times K)} \quad (A.106)$$

d_1 = distance from one end of the path to the location of interest;
d_2 = distance from other end of the path to the location of interest;
K = equivalent earth radius factor.

A.6.3 Reflection Point

Figure A.2 is an illustration for the smooth-earth case.

h_1 = physical height of the antenna at one end of the path *above the reflection point physical height*;
h_2 = physical height of the antenna at the other end of the path *above the reflection point physical height*;
$h_1 \geq h_2$;
d_1 = distance from one end of the path to the reflection point;
d_2 = distance from the other end of the path to the reflection point;
D = distance from one end of the path to the other end;
= total path distance;
$= d_1 + d_2$;
K = equivalent earth radius factor ≥ 0;
a = physical earth radius $\cong$ 6367 km $\cong$ 3957 (statute) miles;
$C = (h_1 - h_2/h_1 + h_2) \geq 0$;
$M = D^2/[4Ka(h_1 + h_2)]$.

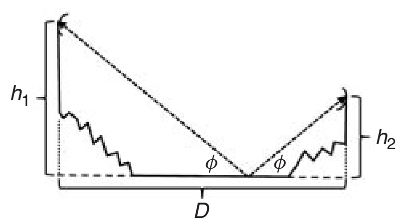

Figure A.2 Smooth-earth surface path profile reflection geometry.

$$d_1 = \left(\frac{D}{2}\right)(1 + B) \tag{A.107}$$

$$d_2 = D - d_1 \tag{A.108}$$

$$B = 2\sqrt{\frac{M+1}{3M}}\cos\left[\left(\frac{\pi}{3}\right) + \left(\frac{1}{3}\arccos\left[\frac{3C}{2}\sqrt{\frac{3M}{(M+1)^3}}\right]\right)\right]$$

$$\phi\,(\text{rad}) = \left[\frac{(h_1 + h_2)}{D}\right][1 - M(1 + B^2)]$$

All of these equations are in the same units of distance.
The following relationships also apply to the reflection point:
For d_1, d_2, and D in miles and h_1 and h_2 in feet:

$$\text{For } K = \frac{2}{3} \ : \ \left(\frac{h_1}{d_1}\right) - d_1 = \left(\frac{h_2}{d_2}\right) - d_2 \tag{A.109}$$

$$\text{For } K = \frac{4}{3} \ : \ \left(\frac{h_1}{d_1}\right) - \left(\frac{d_1}{2}\right) = \left(\frac{h_2}{d_2}\right) - \left(\frac{d_2}{2}\right) \tag{A.110}$$

$$\text{For } K = \infty \ : \ d_1 = \left[\frac{h_1}{(h_1 + h_2)}\right]D, \quad d_2 = D - d_1 \tag{A.111}$$

For d_1, d_2, and D in kilometers and h_1 and h_2 in meters:

$$\text{For } K = \frac{2}{3} \ : \ \left(\frac{h_1}{d_1}\right) - \left(\frac{d_1}{8.5}\right) = \left(\frac{h_2}{d_2}\right) - \left(\frac{d_2}{8.5}\right) \tag{A.112}$$

$$\text{For } K = \frac{4}{3} \ : \ \left(\frac{h_1}{d_1}\right) - \left(\frac{d_1}{17}\right) = \left(\frac{h_2}{d_2}\right) - \left(\frac{d_2}{17}\right) \tag{A.113}$$

$$\text{For } K = \infty \ : \ d_1 = \left[\frac{h_1}{(h_1 + h_2)}\right]D, \quad d_2 = D - d_1 \tag{A.114}$$

Figure A.3 is an illustration for the smooth inclined earth case.
All the above-mentioned definitions apply with the following exceptions:

h_1 = physical height of the antenna at one end of the path *above the extended flat terrain physical height*;
h_2 = physical height of the antenna at the other end of the path *above the extended flat terrain physical height*.

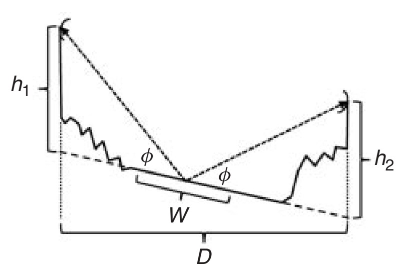

Figure A.3 Smooth inclined earth path profile reflection geometry.

The minimum width W of the flat terrain is approximately [first Fresnel zone radius (F_1)]/sin ϕ.

A.6.4 Fresnel Zone Radius (Perpendicular to the Radio Path)

$$F_n(\text{ft}) = 72.1 \text{ sqrt}\left\{\frac{[n \times d_1\ (\text{miles}) \times d_2(\text{miles})]}{[F(\text{GHz}) \times D\ (\text{miles})]}\right\}$$

$$F_n(\text{m}) = 17.3 \text{ sqrt}\left\{\frac{[n \times d_1\ (\text{km}) \times d_2(\text{km})]}{[F(\text{GHz}) \times D(\text{km})]}\right\} \tag{A.115}$$

n = Fresnel zone number (an integer);
d_1 = distance from one end of the path to the reflection;
d_2 = distance from the other end of the path to the reflection;
D = total path distance = $d_1 + d_2$;
F = frequency of radio wave.

A.6.5 Fresnel Zone Projected onto the Earth's Surface

$$\text{Ln} = \frac{d\sqrt{1+\left[\frac{(4h_1h_2)}{(n\lambda d)}\right]}}{1+\left[\frac{(h_1+h_2)^2}{(n\lambda d)}\right]} \tag{A.116}$$

Ln = projected length of Fresnel zone projected onto the smooth earth in the direction of the radio path (twice the projected semi-major axis).

$$\text{Wn} = \frac{\sqrt{1+\left[\frac{(4h_1h_2)}{(n\lambda d)}\right]}}{\sqrt{1+\left[\frac{(h_1+h_2)^2}{(n\lambda d)}\right]}} \tag{A.117}$$

Wn = projected width of Fresnel zone projected onto the smooth earth in the direction of the radio path (twice the projected semi-minor axis);
n = Fresnel zone integer designation (1 for first Fresnel zone);
d = path length between transmitter and receiver;
h_1 = transmit antenna height above reflection point height;
h_2 = receive antenna height above reflection point height;
λ = radio wavelength;
λ (ft) = 0.98357/F (GHz);
λ (m) = 0.29980/F (GHz);
F = radio frequency;

D, h_1, h_2, and λ are all in the same distance units.

A.6.6 Reflection Path Additional Distance

$$d(\text{ft}) = \frac{\left(\frac{h}{F_1}\right)^2}{[2.033F(\text{GHz})]}$$

$$d(\text{m}) = \frac{\left(\frac{h}{F_1}\right)^2}{[6.671F(\text{GHz})]} \tag{A.118}$$

$$= \text{reflection path distance} - \text{direct path distance}$$

h = (radial)distance of reflection point below main path;
F_1 = first Fresnel zone radius;
F = Frequency of radio wave.

A.6.7 Reflection Path Additional Delay

$$T(\text{ns}) = \frac{1000}{\Delta F(\text{MHz})} \tag{A.119}$$

ΔF = frequency difference between consecutive peaks on spectrum analyzer display.

$$d(\text{ft}) = \frac{T(\text{ns})}{1.017}$$

$$d(\text{m}) = \frac{T(\text{ns})}{3.336} \tag{A.120}$$

$$= \text{reflection path distance} - \text{direct path distance}$$

A.6.8 Reflection Path Relative Amplitude

$$A_{\text{R}}(\text{dB}) = 20 \ \log\left\{\frac{\left[\left(10^{A_{\text{PK-NL}}/20}\right) - 1\right]}{\left[\left(10^{A_{\text{PK-NL}}/20}\right) + 1\right]}\right\} \tag{A.121}$$

$A_{\text{PK-NL}}(\text{dB})$ = peak to null variation on received spectrum power (dB difference between consecutive power maximum and power minimum points on spectrum)

A.6.9 Antenna Launch Angle

$$\theta = \text{antenna launch angle}$$

$$= \arctan\phi_1 - \arcsin\phi_2 \tag{A.122}$$

$$\phi_1 = \left(\frac{d}{D}\right)$$

$$= 1.894 \times 10^{-4}\,\frac{d(\text{ft})}{D(\text{miles})}$$

$$= 1.000 \times 10^{-3}\,\frac{d(\text{m})}{D(\text{km})}$$

$$\phi_2 = \frac{[\mathrm{sqrt}(d^2 + D^2)]}{(2Ka)}$$
$$= \frac{\{\mathrm{sqrt}[D^2(\mathrm{miles}) + 3.587 \times 10^{-8} d^2(\mathrm{ft})]\}}{7913\ K}$$
$$= \frac{\{\mathrm{sqrt}[D^2(\mathrm{km}) + 1.000 \times 10^{-6} d^2(\mathrm{m})]\}}{12,735\ K}$$

$d = h_{\mathrm{F}} - h_{\mathrm{N}}$;
h_{F} = height of the far-end antenna above mean sea level;
h_{N} = height of the near-end antenna above mean sea level;
D = distance between near-and far-end antennas;
K = equivalent earth radius factor;
a = earth radius ≅ 3957 (statute) miles ≅ 6367 km.

For d much smaller than D (nearly horizontal paths such that [d (ft)/D (miles)] <900 or [d (m)/D (km)] <170) the following has less than 1% error:

$$\theta(\mathrm{rad}) = \frac{\{d - [D^2/(2Ka)]\}}{D}$$
$$\theta(\mathrm{degrees}) = \frac{\{[d(\mathrm{ft})/92.15] - [D^2(\mathrm{miles})/(138.1K)]\}}{D(\mathrm{miles})} \tag{A.123}$$
$$\theta(\mathrm{degrees}) = \frac{\{[d(\mathrm{m})/17.45] - [D^2(\mathrm{km})/(222.3K)]\}}{D(\mathrm{km})}$$

Angles are positive if above the horizon and negative if below.

Note that this formula can be used to determine the angle of arrival of a reflected signal after the reflection point has been determined. It can also be used to estimate beam angle movement (to estimate power fading due to antenna pattern) over an expected K factor range.

A.6.10 Antenna Height Difference

$$[h_{\mathrm{F}}(\mathrm{ft}) - h_{\mathrm{N}}(\mathrm{ft})] = 46.08\ D(\mathrm{miles})(\theta_{\mathrm{N}} - \theta_{\mathrm{F}})$$
$$[h_{\mathrm{F}}(\mathrm{m}) - h_{\mathrm{N}}(\mathrm{m})] = 8.727\ D(\mathrm{km})(\theta_{\mathrm{N}} - \theta_{\mathrm{F}}) \tag{A.124}$$

h_{F} = height of the far-end antenna above mean sea level;
h_{N} = height of the near-end antenna above mean sea level;
D = distance between near-and far-end antennas;
θ_{N}(degrees) = launch angle of near end;
θ_{F}(degrees) = launch angle of far end.

Angles are positive if above the horizon and negative if below.

It is assumed launch angles are small (nearly horizontal).

A.6.11 *K* Factor (From Launch Angles)

$$K = -\frac{D(\mathrm{miles})}{[69.12(\theta_{\mathrm{N}} + \theta_{\mathrm{F}})]}$$
$$= -\frac{D(\mathrm{km})}{[111.2(\theta_N + \theta_F)]} \tag{A.125}$$

D = distance between near-and far-end antennas;
θ_N(degrees) = launch angle of near end;
θ_F(degrees) = launch angle of far end.

Angles are positive if above the horizon and negative if below.
It is assumed launch angles are small (nearly horizontal).

A.6.12 Refractive Index and K Factor (From Atmospheric Values)

$$n = \text{atmospheric index of refraction}$$
$$N = 1,000,000(n - 1)$$
$$a = \text{earth radius}$$
$$\begin{aligned} K &= \text{effective earth radius factor} \\ &= \frac{1}{\left[1 + a\left(\frac{dn}{dh}\right)\right]} \\ &= \frac{253}{\{253 + [dN/dh(\text{N units per mile})]\}} \\ &= \frac{157}{\{157 + [dN/dh(\text{N units per km})]\}} \end{aligned} \tag{A.126}$$

K(light) = typically 6/5(average) to 7/5(midday);
K(radio wave) = typically 4/3(average);
N(light) = N_1;
N(radio wave) = $N_1 + N_2$;
$N_1 = (77.6\ p)/(273 + T)$;
$N_2 = (373,000\ \mathrm{W})/(273 + T)^2$;
p = atmospheric pressure (mbar);
= 33.9(atmospheric pressure in inches of mercury);
= 1.33(atmospheric pressure in millimeters of mercury);
T = atmospheric temperature in degree centigrade;
W = water vapor pressure (mbar);
= $H_R E_S$;
H_R = atmospheric relative humidity(%)/100;
E_S = atmospheric saturation vapor pressure(mbar);
= $6.108 \times 10^{[(7.500T)/(237.3+T]}$.

A.7 OBSTRUCTION LOSS

A.7.1 Knife-Edge Obstruction Loss

$$\begin{aligned} L_{KE} &= \text{knife-edge loss (dB)} \\ &= -10\ \log_{10}[0.25 + 0.5(\mathrm{SFI} + \mathrm{CFI}) + 0.5(\mathrm{SFI}^2 + \mathrm{CFI}^2)], \quad X > 0 \\ &= -10\ \log_{10}[0.25 - 0.5(\mathrm{SFI} + \mathrm{CFI}) + 0.5(\mathrm{SFI}^2 + \mathrm{CFI}^2)], \quad X \le 0 \\ &= \frac{\text{received signal power without obstruction}}{\text{received signal power with obstruction}} \end{aligned} \tag{A.127}$$

$$X = h/F_1$$

h = perpendicular distance from main beam to obstruction;
F_1 = first Fresnel zone distance;
$A = = 2^{1/2}|X|$;
$F = (1 + 0.9260A)/(2 + 1.792A + 3.104A^2)$;
$G = 1/(2 + 4.142A + 3.492A^2 + 6.670A^3)$;
$S = \sin(\pi A^2/2)$;
$C = \cos(\pi A^2/2)$.

$$\text{CFI} = 0.5 + (FS) - (GC)$$

$$\text{SFI} = 0.5 - (FC) - (GS)$$

A curved edge can be considered a knife edge if the following condition applies:

$$|\phi| \le \frac{\lambda}{4r}$$

ϕ = angle (rad) formed by a horizontal plane passing through the obstruction's edge and the ray that hits the obstruction's edge;
λ = wavelength of the radio wave;
r = radius of curvature of the obstruction's edge.

A.7.2 Rounded-Edge Obstruction Path Loss

$$\begin{aligned} L_{\text{RE}} &= \text{rounded-edge loss(dB)} \\ &= 10\ \log_{10}\left[\frac{\text{received signal power without rounded-edge obstruction}}{\text{received signal power with rounded-edge obstruction}}\right] \quad \text{(A.128)} \\ &= L_{\text{KE}} + L_{\text{RO}} \end{aligned}$$

$$L_{\text{KE}} = \text{knife-edge loss (dB)as calculated above} \quad \text{(A.129)}$$

$$\begin{aligned} L_{\text{RO}} &= \text{additional rounded obstruction loss (dB)} \\ &\ge\ 0 \\ &= -6 - 20\ \log_{10}(mn) + 7.2\ m^{1/2} - (2 - 17n)m + 3.6\ m^{3/2} - 0.8\ m^2 \quad \text{if } mn > 4 \\ &= +7.2\ m^{1/2} - (2 - 12.5\ n)m + 3.6\ m^{3/2} - 0.8\ m^2 \quad \text{if } mn \le 4 \quad \text{(A.130)} \end{aligned}$$

$m = r[(d_1 + d_2)/(d_1 d_2)]/(\pi r/\lambda)^{1/3}$;
$n = h(\pi r/\lambda)^{2/3}/r$;
h = perpendicular distance from main beam to obstruction;
≥ 0 (path is obstructed or grazing);
d_1 = distance from transmit antenna to path ray intersection above obstruction;
d_2 = distance from receive antenna to path ray intersection above obstruction;
r = obstruction radius of curvature;
l = radio free space wavelength;
$= 0.98357/F$(GHz)(ft);
$= 0.29980/F$(GHz)(m);
F = radio operating frequency(GHz).

A.7.3 Smooth-Earth Obstruction Loss

$$X = \frac{h}{F_1} \leq 0.75$$

h = perpendicular distance from main beam to obstruction;
F_1 = first Fresnel zone distance;
P = received signal power with obstruction/received signal power without obstruction.

$$\text{For } X \leq 0, \quad P(\text{dB}) = -10 + 20X \tag{A.131}$$

$$\text{For } 0 \leq X \leq 0.75, \quad P(\text{dB}) = -10 + 20X - 6.665X^2 \tag{A.132}$$

A.7.4 Infinite Flat Reflective Plane Obstruction Loss

$$X = \frac{h}{F_1} \geq 0$$

h = perpendicular distance from main beam to obstruction;
F_1 = first Fresnel zone distance;
P = received signal power with obstruction/received signal power without obstruction.

$$P\,(\text{dB}) = 10 \log \left\{1 + C_{\text{comp}} \cos\left[\pi\left(1 + X^2\right)\right]\right\}^2 \tag{A.133}$$

C_{comp} = composite earth reflection coefficient = $10^{CS\,(\text{dB})/20}10^{CD\,(\text{dB})/20}10^{R(\text{dB})/20}$;
$C_{\text{S}}(\text{dB})$ = reflection (earth roughness scattering) coefficient;
$C_{\text{D}}(\text{dB})$ = divergence coefficient (if the earth is flat);
$R(\text{dB})$ = reflection coefficient (see Chapter 13).

A.7.5 Reflection (Earth Roughness Scattering) Coefficient

It is the relative magnitude of signal reflected from a rough earth's surface ignoring polarization, divergence and earth's dielectric constant.

$\Delta = \Delta h \sin \phi/\lambda \cong 0.01745\ \Delta h\ \phi$ (degrees)/λ for small grazing angle ϕ (since $\sin \phi \cong \phi$ (rad) within 10% for ≤ 0.785 (45°) or 1% for ≤ 0.262 (15°)).

ϕ (degrees) = grazing angle;
λ = radio wave free space wavelength.

A.7.5.1 Gaussian Model

$C_{\text{S}}(\text{dB})$ = reflection coefficient (dB) = $10\ \log[e^{-16\pi^2\Delta^2}] = -685.810\Delta^2$

Δh = standard deviation of the normal distribution of reflecting surface heights (A.134)

A.7.5.2 Uniform Model

$$C_{\text{S}}\,(\text{dB}) = \text{reflection coefficient (dB)} = 10\ \log\left[\frac{\sin^2(2\pi\Delta)}{(2\pi\Delta)^2}\right]$$

Δh = maximum difference of uniformly distributed reflecting surface heights (A.135)

$$\text{Reflection coefficient envelope (dB)} = 20\ \log\left[\frac{\sin(2\pi\Delta)}{(2\pi\Delta)}\right], \quad \Delta_{\mathrm{dB}} \leq -5.8$$

$$\text{Reflection coefficient envelope (dB)} = -16.0 - 2\Delta_{\mathrm{dB}}, \quad \Delta_{\mathrm{dB}} > -5.8$$

$\Delta_{\mathrm{dB}} = 10\ \log(\Delta)$.

Notice that Δh is based on difference in heights, not absolute height.

A.7.5.3 *Empirical Models*

R = reflection coefficient;
= (voltage) amplitude of the reflected signal relative to the amplitude of the incident signal;
$20\ \log_{10}(R)$ = reflected signal power/incident signal power (dB).

Exponential

$$R = \frac{1}{\left[1 + \left(\frac{P^2}{2}\right)\right]} \tag{A.136}$$

Pseudoexponential

$$R = \frac{1}{\left[1 + \left(\frac{2P^2}{3}\right)\right]^{(3/4)}} \tag{A.137}$$

Normal

$$R = \exp\left(-\frac{P^2}{2}\right) \tag{A.138}$$

$$\exp(x) = e^x$$

Longley–Rice Empirical

$$R = \exp\left(-\frac{P}{2}\right) \tag{A.139}$$

P = Rayleigh roughness parameter

= effective terrain roughness

$$= 4\pi\left(\frac{\sigma}{\lambda}\right)\ \sin\theta \tag{A.140}$$

σ = root mean square(RMS)surface height(measured from crest to trough);
λ = radio wave free space wavelength;
θ = grazing angle of incident signal relative to the mean surface plane.

A.7.6 Divergence Coefficient from Earth

It is the relative magnitude of signal reflected from a curved earth ignoring polarization, roughness and the earth's dielectric constant.

Typically, this factor is not significant except for very shallow grazing angle paths.

$$\begin{aligned} C_\mathrm{D} &= \text{reflection coefficient} \\ &= \text{abs}\left(\frac{\text{reflected signal magnitude}}{\text{incident signal magnitude}}\right) \\ &= \text{sqrt}\left\{1 + \left[\frac{(2d_1 d_2)}{(Krd\ \ \sin(\phi))}\right]\right\} \end{aligned} \tag{A.141}$$

ϕ(rad) = grazing angle at reflection = angle of incidence = angle of reflection;
d_1 = distance from one end of the path to the location of interest;
d_2 = distance from the other end of the path to the location of interest;
$d = d_1 + d_2$;
R_e = Earth's equatorial radius ≅ 6378 km ≅ 3963 miles;
R_p = Earth's polar radius ≅ 6357 km ≅ 3950 miles;
E^2 = eccentricity^2 = $(R_\mathrm{e}{}^2 - R_\mathrm{p}{}^2)/R_\mathrm{e}{}^2$;
ψ = latitude at reflection;
r = Earth's radius at reflection = R_p/sqrt$[1 - (E^2 \cos\psi)]$;
K = equivalent earth radius factor;
C_D(dB) = 20 log(C_D);
ϕ(degrees) = $(180/\pi)\,\phi$(rad).

A.7.7 Divergence Factor for a Cylinder

$$\text{Cylinder divergence factor (dB)} = -12 - 10\ \log(\text{Rn}) \tag{A.142}$$

Rn = cylinder's radius/λ (wavelength);
= 1.017 cylinder's radius (ft) f (GHz);
= 3.336 cylinder's radius (meters) f (GHz);
f = operating frequency.

A.7.8 Divergence Factor for a Sphere

$$\text{Sphere divergence factor (dB)} = -32 - 40\ \log\ (\text{Rn}) \tag{A.143}$$

Rn = sphere's radius/λ (wavelength);
= 1.017 cylinder's radius (ft) f (GHz);
= 3.336 cylinder's radius (m) f (GHz);
f = operating frequency.

A.7.9 Signal Reflected from Flat Earth

See Chapter 13 for details.

A.7.10 Ducting

F_{min} (GHz) = lowest frequency propagated by a atmospheric duct;
dh = thickness of duct (atmospheric gradient);
dN (N units) = refractive index change across the duct;

For dh in feet:

$$F_{\min}(\text{GHz}) = \frac{393}{[\mathrm{d}h(\mathrm{d}N - 0.0479\mathrm{d}n)^{1/2}]} \cong \frac{2150}{\mathrm{d}h^{3/2}} \tag{A.144}$$

For dh in meters:

$$F_{\min}(\text{GHz}) = \frac{120}{[\mathrm{d}h(\mathrm{d}N - 0.157\mathrm{d}n)^{1/2}]} \cong \frac{362}{\mathrm{d}h^{3/2}}$$

A.8 MAPPING

A.8.1 Path Length and Bearing

Consider two locations, A and B. Neither location can be at an earth pole or on opposite sides of the Earth. In addition, the great circle path must not cross the ±180° longitude line (International Date Line).

θ_A = latitude of A (degrees);
θ_B = latitude of B (degrees);
Φ_A = longitude of A (degrees);
Φ_B = longitude of B (degrees);
North latitudes and east longitudes are taken as positive;
South latitudes and west longitudes are taken as negative.

The angles must be converted to decimal notation.
The angle Φ is assumed initially to be composed of Φ_d degrees, Φ_m minutes, and Φ_s seconds.

$$\Phi(\text{decimal degrees}) = \Phi_d + \left(\frac{\Phi_m}{60}\right) + \left(\frac{\Phi_s}{3600}\right) \tag{A.145}$$

$$\Phi(\text{decimal radians}) = 0.017453292 \times \Phi(\text{decimal degrees}) \tag{A.146}$$

$$\Phi(\text{decimal degrees}) = 57.295780 \times \Phi(\text{decimal radians}) \tag{A.147}$$

$$Z(\text{degrees}) = \arccos\{(\sin\theta_A \sin\theta_B) + [\cos\theta_A \cos\theta_B \cos(\phi_A - \phi_B)]\}$$
$$= \text{angular difference between the two sites measured at the center of the Earth} \tag{A.148}$$

The above formula is derived from the spherical law of cosines and is commonly suggested for great circle path distance calculations. However, the following ("haversine") formula has significantly less round-off error:

$$Z(\text{degrees}) = 2 \times \arcsin\left\{\text{sqrt}\left\{\left(\sin\left[\frac{(\theta_A - \theta_B)}{2}\right]\right)^2 + \left[\cos\theta_A \cos\theta_B \left(\sin\left[\frac{(\phi_A - \phi_B)}{2}\right]\right)^2\right]\right\}\right\} \tag{A.149}$$

The haversine equation is preferred for all path distance calculations (except for the rare case of sites at opposite sides of the Earth).

D = great circle distance from A to B

R = radius of the Earth

$D = 0.017453292\ R\,Z$, Z in degrees, D and R both in kilometers or both in miles (A.150)

For most modern map coordinate systems (NAD83, WGS84, WGS72, WGS66, GRS80, GRS67, and IAU68), the following are the primary earth radiuses:

R_e = Earth′s equatorial radius ≅ 6378.1 km ≅ 3963.3 miles;
R_p = Earth’s polar radius ≅ 6356.8 km ≅ 3950.0 miles;
E^2 = eccentricity^2 = $(R_e{}^2 - R_p{}^2)/R_e{}^2$;
R = Earth’s radius on path = R_p/sqrt $[1 - (E^2 \cos\ \theta_{AVG})]$;
$\theta_{AVG} = (\theta_A + \theta_B)/2$.

If three significant figures are adequate, the following apply:
R ≅ 6367 km ≅ 3957 (statute) miles.

If site A latitude and longitude are in cells C2 and D2, respectively, and site B latitude and longitude are in cells E2 and F2, respectively, the following formula may be placed in an Excel spread sheet cell to calculate the above simplified reduced round-off error formula for path great circle distance in miles:

```
= IF(AND(ISNUMBER(C2),ISNUMBER(D2),ISNUMBER(E2),ISNUMBER(F2)),
ROUND(ABS(MAX(0.0001,(2*ASIN((((SIN(((((D2*PI()/180)–
(F2*PI()/180))/2)))ˆ2)*COS((C2*PI()/180))*COS((E2*PI()/180))+
(SIN((((C2*PI()/180)–
(E2*PI()/180)))/2))ˆ2)ˆ0.5))*180/PI()*0.017453292*3957*5280)),8),CHAR(45))
```

In the above formula, it is assumed that the longitude and latitude are in decimal degrees. Longitude is positive if East (0 to 180°) and negative if West (0° to −180°). Latitude is positive if North (0 to 90°) and negative if South (0° to −90°).

Bearing is the horizontal angle (measured clockwise from true north) that points from the site of interest toward the other site. North is 0°, East is 90°, South is 180°, and West is 270°.

B_A = bearing at A toward B;
B_B = bearing at B toward A.

$$\alpha(\text{degrees}) = \text{arc}\ \cos\left\{\frac{\left[\sin\theta_B - \left(\sin\theta_A\ \cos Z\right)\right]}{(\cos\theta_A \sin Z)}\right\} \quad \text{(A.151)}$$

$$\beta(\text{degrees}) = \text{arc}\ \cos\left\{\frac{\left[\sin\theta_A - \left(\sin\theta_B\ \cos Z\right)\right]}{(\cos\theta_B\ \sin Z)}\right\} \quad \text{(A.152)}$$

$$0^\circ \le \alpha \le +180^\circ$$

$$0^\circ \le \beta \le +180^\circ$$

If $\phi_A \le \phi_B$,

$$B_A = \alpha$$

$$B_B = 360 - \beta$$

If $\phi_A > \phi_B$,

$$B_A = 360 - \alpha$$

$$B_B = \beta$$

Sometimes an angle must be converted from decimal notation to degrees, minutes, and seconds format:

Φ (decimal degrees)

$$\Phi_d(\text{degrees}) = \text{Int}\,[\Phi] \qquad (A.153)$$

$$\Phi_m(\text{min}) = \text{Int}[60(\Phi - \Phi_d] \qquad (A.154)$$

$$\Phi_s(\text{s}) = 3600\,[\Phi - \Phi_d - (\Phi_m\, 60)] \qquad (A.155)$$

Int [x] rounds x down to the largest integer less than or equal to x.

$$\Phi(\text{degrees, min, s}) = \Phi_d, \Phi_m, \Phi_s \qquad (A.156)$$

A.9 TOWERS

A.9.1 Three-Point Guyed Towers

Refer to Figure A.4.

A.9.1.1 Minimum Land Area (Tower Orientation may Limit Antenna Placement)

$$A = \{D \times [1 + \sin\,(30^\circ)]\} + E + F + \text{margin} \qquad (A.157)$$

$$B = [2 \times D \times \cos(30^\circ)] + (2 \times F) + \text{margin} \qquad (A.158)$$

margin = additional distance to allow for unforseen circumstances.

A.9.1.2 Any Tower Orientation

$$C = 2 \times (D + E) + \text{margin} \qquad (A.159)$$

$$D = \text{tower height} \times \text{factor}$$

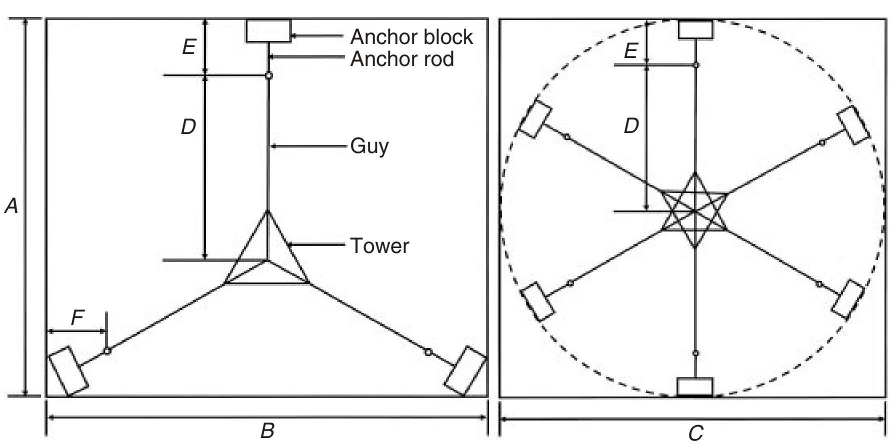

Figure A.4 Tower land area.

factor = percentage of tower height expressed as a fraction

= 0.8 typically(range is from 1.0 to 0.4 but cost increases as factor gets smaller)

E = 15 ft (5 m) typically

F = 20 ft (6 m) typically (A.160)

A.9.2 Three-Leg Self-Supporting Tower

The distances in Table A.23 are the lengths of the sides of the rectangle enclosing the tower leg pads.

A.9.3 Four-Leg Self-Supporting Tower

TABLE A.23 Three-Leg Self-Supporting Tower

Tower Height, ft	Land Width 1, ft	Land Width 2, ft
50	26.6 + margin	23.4 + margin
75	30.2 + margin	26.6 + margin
100	33.5 + margin	29.3 + margin
125	37.2 + margin	32.8 + margin
150	42.0 + margin	37.1 + margin
175	46.8 + margin	41.3 + margin
200	50.8 + margin	44.6 + margin
225	54.5 + margin	48.0 + margin
250	58.7 + margin	51.2 + margin
275	62.1 + margin	54.8 + margin
300	66.8 + margin	59.0 + margin
325	70.0 + margin	61.6 + margin
350	73.1 + margin	64.4 + margin

The distances in Table A.24 are the lengths of one side of the square enclosing the tower leg pads.

TABLE A.24 Four-Leg Self-Supporting Tower

Tower Height, ft	Land Width, ft
50	21.8 + margin
75	25.3 + margin
100	28.9 + margin
125	32.2 + margin
150	35.9 + margin
175	39.5 + margin
200	43.3 + margin
225	46.9 + margin
250	50.6 + margin
275	53.8 + margin
300	57.6 + margin
325	60.0 + margin
350	64.0 + margin

Land width includes the space necessary for the guy anchors or tower leg pads. Margin is accommodation for ditches, fences, and easements (typically 40 ft). *Source*: The land widths were adapted from White, R. F., *Engineering Considerations for Microwave Communications Systems*, San Carlos: Lenkurt Electric, p. 86, 1970.

A.10 INTERPOLATION

For all the following equations, double precision arithmetic is strongly recommended. Also the convention that multiplication occurs before addition is assumed.

A.10.1 Two-Dimensional Interpolation

Sometimes it is necessary to interpolate between tabular data points. The following methods create linear ($Y = A + BX$), quadratic ($Y = A + BX + CX^2$), or cubic ($Y = A + BX + CX^2 + DX^3$) polynomials. If the data is known to have a nonlinear relationship to X, (such as squared or logarithmic components), first transform the input X data. For example, make the substitution $Z = X^2$ or $Z = \log(X)$, respectively. Replace X below with Z. The results will be the appropriate polynomials (e.g., $Y = A + BZ + CZ^2 + DZ^3$, which are equivalent to $Y = A + B(X^2) + C(X^4) + D(X^6)$ or $Y = A + B\ \log(X) + B\ \log^2(X) + B\ \log^3(X)$).

The following discussion uses Figure A.5.

A.10.1.1 Lagrangian Interpolation This method uses a limited number of data points to produce a polynomial that exactly reproduces the input data.

A.10.1.2 Linear Interpolation Given a set of two X,Y values, (X_1, Y_1) and (X_2, Y_2) with $X_1 < X_2$, interpolate a value of Y for X such that $X_1 < X < X_2$:

$$Y = Y_1 + \left\{ \left(\frac{\text{CC}}{\text{CA}} \right) \times \text{CB} \right\} \tag{A.161}$$

$\text{CA} = X_2 - X_1$;
$\text{CB} = X - X_1$;
$\text{CC} = Y_2 - Y_1$.

Special Cases: Linear interpolation is best when it interpolates a function that is nearly linear in X and Y. When graphed, this type of function is a straight line if the X and Y graph coordinates are both linear or both logarithmic.

If the graphed function is a straight line when the Y coordinates are linear but the X coordinates are logarithmic, the following modified linear interpolation formula works well:

$$Y = Y1 + \left[(\mathrm{Y}_2 - \mathrm{Y}_1) \ \frac{\log_{10}\left(\dfrac{X}{X_1}\right)}{\log_{10}\left(\dfrac{X_2}{X_1}\right)} \right] \tag{A.162}$$

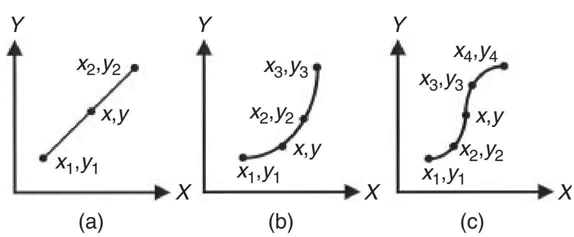

Figure A.5 Two-dimensional interpolation: (a) linear, (b) quadratic, and (c) cubic.

If the graphed function is a straight line when the Y coordinates are logarithmic but the X coordinates are linear, the following modified linear interpolation formula works well:

$$\log Y = \log_{10}(Y_1) + \left[(X - X_1) \frac{\log_{10}\left(\frac{Y_2}{Y_1}\right)}{(X_2 - X_1)} \right] \tag{A.163}$$

$$Y = 10^{\log Y}$$

When the best approach is not clear, testing all three cases with data samples is often helpful. In the above cases, the common logarithm $\log_{10}(\,)$ and base 10 were used. The formulas are applicable for any logarithm and base (e.g., ln( ) and e).

A.10.1.3 Quadratic Interpolation Given a set of three X, Y values, (X_1, Y_1), (X_2, Y_2), and (X_3, Y_3) with $X_1 < X_2 < X_3$, interpolate a value of Y for X such that $X_1 < X < X_3$ (The X values do not need to be equally spaced.):

$$Y = A + (B \times X) + (C \times X^2) \tag{A.164}$$

$$\begin{aligned}
C21 &= X_2 - X_1;\\
C31 &= X_3 - X_1;\\
C32 &= X_3 - X_2;\\
K1 &= C21 \times C31;\\
K2 &= C21 \times C32;\\
K3 &= C31 \times C32;\\
\mathrm{CA1} &= (X_2 \times X_3)/K1;\\
\mathrm{CA2} &= -(X_1 \times X_3)/K2;\\
\mathrm{CA3} &= (X_1 \times X_2)/K3;\\
\mathrm{CB1} &= -(X_2 + X_3)/K1;\\
\mathrm{CB2} &= (X_1 + X_3)/K2;\\
\mathrm{CB3} &= -(X_1 + X_2)/K3;\\
\mathrm{CC1} &= 1/\mathrm{K1};\\
\mathrm{CC2} &= -1/\mathrm{K2};\\
\mathrm{CC3} &= 1/K3;\\
A &= \mathrm{CA1} \times Y_1 + \mathrm{CA2} \times Y_2 + \mathrm{CA3} \times Y_3;\\
B &= \mathrm{CB1} \times Y_1 + \mathrm{CB2} \times Y_2 + \mathrm{CB3} \times Y_3;\\
C &= \mathrm{CC1} \times Y_1 + \mathrm{CC2} \times Y_2 + \mathrm{CC3} \times Y_3.
\end{aligned}$$

A.10.1.4 Cubic Interpolation Given a set of four X, Y values, (X_1, Y_1), (X_2, Y_2), (X_3, Y_3), and (X_4, Y_4) with $X_1 < X_2 < X_3 < X_4$, interpolate a value of Y for X such that $X_1 < X < X_4$ (The X values do not need to be equally spaced.) This method will automatically perform linear or quadratic interpolation if the data is linear or quadratic:

$$Y = A + (B \times X) + (C \times X^2) + (D \times X^3) \tag{A.165}$$

$$\begin{aligned}
C21 &= X_2 - X_1;\\
C31 &= X_3 - X_1;\\
C32 &= X_3 - X_2;\\
C41 &= X_4 - X_1;\\
C42 &= X_4 - X_2;\\
C43 &= X_4 - X_3;\\
K1 &= C21 \times C31 \times C41;\\
K2 &= C21 \times C32 \times C42;
\end{aligned}$$

$K3 = C31 \times C32 \times C43;$
$K4 = C41 \times C42 \times C43;$
$\mathrm{CA1} = (X_2 \times X_3 \times X_4)/K1;$
$\mathrm{CA2} = -(X_1 \times X_3 \times X_4)/K2;$
$\mathrm{CA3} = (X_1 \times X_2 \times X_4)/K3;$
$\mathrm{CA4} = -(X_1 \times X_2 \times X_3)/K4;$
$\mathrm{CB1} = -(X_2 \times X_3 + X_2 \times X_4 + X_3 \times X_4)/K1;$
$\mathrm{CB2} = (X_1 \times X_3 + X_1 \times X_4 + X_3 \times X_4)/K2;$
$\mathrm{CB3} = -(X_1 \times X_2 + X_1 \times X_4 + X_2 \times X_4)/K3;$
$\mathrm{CB4} = (X_1 \times X_2 + X_1 \times X_3 + X_2 \times X_3)/K4;$
$\mathrm{CC1} = (X_2 + X_3 + X_4)/K1;$
$\mathrm{CC2} = -(X_1 + X_3 + X_4)/K2;$
$\mathrm{CC3} = (X_1 + X_2 + X_4)/K3;$
$\mathrm{CC4} = -(X_1 + X_2 + X_3)/K4;$
$\mathrm{CD1} = -1/K1;$
$\mathrm{CD2} = 1/K2;$
$\mathrm{CD3} = -1/K3;$
$\mathrm{CD4} = 1/K4;$
$A = \mathrm{CA1} \times Y_1 + \mathrm{CA2} \times Y_2 + \mathrm{CA3} \times Y_3 + \mathrm{CA4} \times Y_4;$
$B = \mathrm{CB1} \times Y_1 + \mathrm{CB2} \times Y_2 + \mathrm{CB3} \times Y_3 + \mathrm{CB4} \times Y_4;$
$C = \mathrm{CC1} \times Y_1 + \mathrm{CC2} \times Y_2 + \mathrm{CC3} \times Y_3 + \mathrm{CC4} \times Y_4;$
$D = \mathrm{CD1} \times Y_1 + \mathrm{CD2} \times Y_2 + \mathrm{CD3} \times Y_3 + \mathrm{CD4} \times Y_4.$

A.10.1.5 Least Squared Error Interpolation This method uses a user-defined number of data points to produce a polynomial that approximates the input data with least squared error. If it uses the same data points as those used in LaGrangian interpolation, it achieves the same result.

Given a set of $n + 1$ (X, Y) values, $(X_1, Y_1), (X_2, Y_2) \ldots (X_{n+1}, Y_{n+1})$ with $X_1 < X_2 < \cdots < X_{n+1}$, interpolate a value of Y for X such that $X_1 < X < X_{n+1}$. (The X values do not need to be equally spaced.)

$\mathrm{Ca1} = n + 1;$
$\mathrm{Ca2} = X_1 + X_2 + X_3 + \cdots + X_{n+1};$
$\mathrm{Ca3} = {X_1}^2 + {X_2}^2 + {X_3}^2 + \cdots + {X_{n+1}}^2;$
$\mathrm{Ca4} = {X_1}^3 + {X_2}^3 + {X_3}^3 + \cdots + {X_{n+1}}^3;$
$\mathrm{Ca5} = {X_1}^4 + {X_2}^4 + {X_3}^4 + \cdots + {X_{n+1}}^4;$
$\mathrm{Ca6} = {X_1}^5 + {X_2}^5 + {X_3}^5 + \cdots + {X_{n+1}}^5;$
$\mathrm{Ca7} = {X_1}^6 + {X_2}^6 + {X_3}^6 + \cdots + {X_{n+1}}^6;$
$\mathrm{Cb1} = -(Y_1 + Y_2 + Y_3 + \cdots + Y_{n+1});$
$\mathrm{Cb2} = -(X_1 \times Y_1 + X_2 \times Y_2 + X_3 \times Y_3 + \cdots + X_{n+1} \times Y_{n+1});$
$\mathrm{Cb3} = -({X_1}^2 \times Y_1 + {X_2}^2 \times Y_2 + {X_3}^2 \times Y_3 + \cdots + {X_{n+1}}^2 \times Y_{n+1});$
$\mathrm{Cb4} = -({X_1}^3 \times Y_1 + {X_2}^3 \times Y_2 + {X_3}^3 \times Y_3 + \cdots + {X_{n+1}}^3 \times Y_{n+1});$
$\mathrm{Cc1} = \mathrm{Ca1} \times \mathrm{Ca3} - \mathrm{Ca2}^2;$
$\mathrm{Cc2} = \mathrm{Ca1} \times \mathrm{Ca4} - \mathrm{Ca2} \times \mathrm{Ca3};$
$\mathrm{Cc3} = \mathrm{Ca1} \times \mathrm{Ca5} - \mathrm{Ca2} \times \mathrm{Ca4};$
$\mathrm{Cc4} = \mathrm{Ca1} \times \mathrm{Ca5} - \mathrm{Ca3}^2;$
$\mathrm{Cc5} = \mathrm{Ca1} \times \mathrm{Ca6} - \mathrm{Ca3} \times \mathrm{Ca4};$
$\mathrm{Cc6} = \mathrm{Ca1} \times \mathrm{Ca7} - \mathrm{Ca4}^2;$
$\mathrm{Cc7} = \mathrm{Ca1} \times \mathrm{Cb2} - \mathrm{Ca2} \times \mathrm{Cb1};$
$\mathrm{Cc8} = \mathrm{Ca1} \times \mathrm{Cb3} - \mathrm{Ca3} \times \mathrm{Cb1};$
$\mathrm{Cc9} = \mathrm{Ca1} \times \mathrm{Cb4} - \mathrm{Ca4} \times \mathrm{Cb1};$
$\mathrm{Cd1} = \mathrm{Cc1} \times \mathrm{Cc4} - \mathrm{Cc2} \times \mathrm{Cc2};$
$\mathrm{Cd2} = \mathrm{Cc1} \times \mathrm{Cc5} - \mathrm{Cc2} \times \mathrm{Cc3};$
$\mathrm{Cd3} = \mathrm{Cc1} \times \mathrm{Cc5} - \mathrm{Cc3} \times \mathrm{Cc2};$
$\mathrm{Cd4} = \mathrm{Cc1} \times \mathrm{Cc6} - \mathrm{Cc3} \times \mathrm{Cc3};$
$\mathrm{Cd5} = \mathrm{Cc1} \times \mathrm{Cc8} - \mathrm{Cc2} \times \mathrm{Cc7};$
$\mathrm{Cd6} = \mathrm{Cc1} \times \mathrm{Cc9} - \mathrm{Cc3} \times \mathrm{Cc7};$

Cd7 = Cd1 × Cd4 − Cd3 × Cd2;
Cd8 = Cd1 × Cd6 − Cd3 × Cd5.

A.10.1.6 Linear Interpolation

$$Y = A + (B \times X) \tag{A.166}$$

$B = -$Cc7/Cc1;
$A = -$(Ca2 × B + Cb1)/Ca1.

A.10.1.7 Quadratic Interpolation

$$Y = A + (B \times X) + (C \times X^2) \tag{A.167}$$

$C = -$Cd5/Cd1;
$B = -$(Cc2 × C + Cc7)/Cc1;
$A = -$(Ca2 × B + Ca3 × C + Cb1)/Ca1.

A.10.1.8 Cubic Interpolation

$$Y = A + (B \times X) + (C \times X^2) + (D \times X^3) \tag{A.168}$$

$D = -$Cd8/Cd7;
$C = -$(Cd2 × D + Cd5)/Cd1;
$B = -$(Cc2 × C + Cc3 × D + Cc7)/Cc1;
$A = -$(Ca2 × B + Ca3 × C + Ca4 × D + Cb1)/Ca1.

A.10.2 Three-Dimensional Interpolation

A.10.2.1 All Data on a Rectangular Grid The following descriptions rely on Figure A.6.

A.10.2.2 Linear Interpolation Given four sets of x, y, z values located on a rectangular X,Y grid, interpolate a value of z for a set of x,y coordinates within the grid of X,Y values.

$$\begin{aligned} z(x, y) = {} & z(x_1, y_1)[(1 - \Delta X)(1 - \Delta Y)] + z(x_1, y_2)[(1 - \Delta X)\Delta Y] \\ & + z(x_2, y_1)[\Delta X(1 - \Delta Y)] + z(x_2, y_2)[\Delta X \Delta Y] \end{aligned} \tag{A.169}$$

$\Delta X = (x - x_1/x_2 - x_1)$;
$\Delta Y = (y - y_1/y_2 - y_1)$.

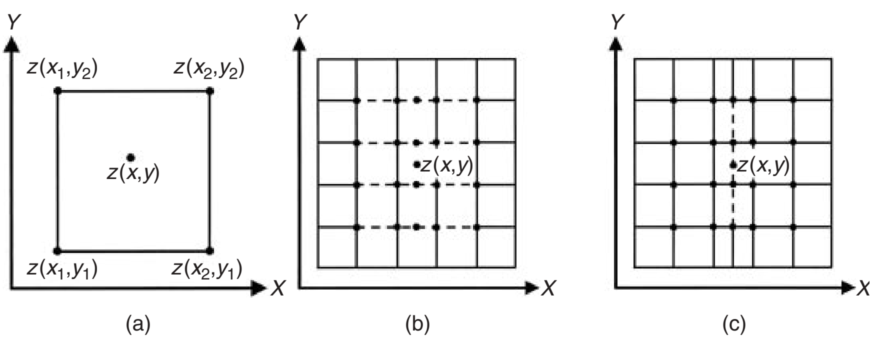

Figure A.6 Rectangular three-dimensional interpolation: (a) linear and (b) first step, and (c) second step of higher order.

A.10.2.3 Higher Order Interpolation Using two-dimensional interpolation, interpolate a set of z values for the desired x value and multiple y values (Figure A.6b). Next, again using two-dimensional interpolation, interpolate the desired z value for the desired x and y values (Figure A.6C). Linear, quadratic, or cubic interpolation may be used for either of these two-dimensional interpolations. This approach is helpful in filling out a sparse data set so linear interpolation may be used.

A.10.2.4 Data Scattered If the data is scattered over the x–y plane, the problem is much more challenging. There is no known optimal solution to this problem. Many approaches (Kriging and methods by Shepard, Lam, Watson and Philip, Cooke and Mostaghimi, Akima, Montefusco, and Casciola and Briggs) are available but all have their trade-offs. Stability and loss of data integrity are common problems (especially for sparse data). The simplest approach is to treat the data set as a group of triangles in the x–y plane and interpolate the z values based on a plane (flat surface) defined by the three-dimensional coordinates at the vertices of the triangle enclosing the x–y value of interest. The following discussion relates to Figure A.7.

Data is assumed to lie on a rectangular grid. The data points (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), and (X_3, Y_3, Z_3) enclose the desired data point (X, Y) in the x–y plane. The value Z at (X, Y) is desired.

$$Z = \frac{([X \times B_1] + [Y \times B_2] + B_3)}{B_4} \tag{A.170}$$

$A_1 = X_2 - X_3;$
$A_2 = Y_2 - Y_3;$
$A_3 = Z_2 - Z_3;$
$A_4 = (X_2 \times Y_3) - (X_3 \times Y_2);$
$A_5 = (X_2 \times Z_3) - (X_3 \times Z_2);$
$A_6 = (Y_2 \times Z_3) - (Y_3 \times Z_2);$
$B_1 = -(A_3 \times Y_1) + (A_2 \times Z_1) - A_6;$
$B_2 = (A_3 \times X_1) - (A_1 \times Z_1) + A_5;$
$B_3 = (A_6 \times X_1) - (A_5 \times Y_1) + (A_4 \times Z_1);$
$B_4 = (A_2 \times X_1) - (A_1 \times Y_1) + A_4.$

The data points (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), and (X_3, Y_3, Z_3) must not lie in a straight line. They must be picked in such a way that they enclose the data location of interest. One method of doing this is to pick four x, y, z points in each of the four quadrants that lie closest to X, Y.

From the database of known values, determine the following four three-dimensional points:

X_A, Y_A, Z_A such that $X_A \le X$ and $Y_A \ge Y$ (upper left quadrant).
X_B, Y_B, Z_B such that $X_B \ge X$ and $Y_B \ge Y$ (upper right quadrant).

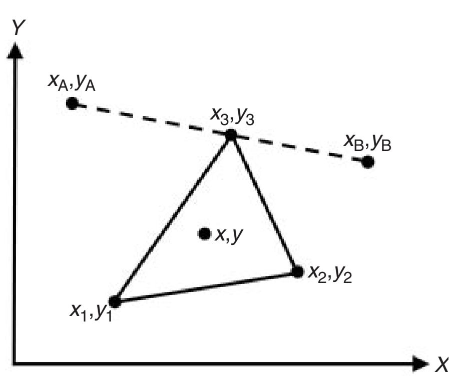

Figure A.7 Triangular three-dimensional interpolation.

X_1, Y_1, Z_1 such that $X_1 \leq X$ and $Y_1 \leq Y$ (lower left quadrant).
X_2, Y_2, Z_2 such that $X_2 \geq X$ and $Y_2 \leq Y$ (upper right quadrant).

The = portion of $\leq$ or $\geq$ is necessary to deal with the edges of a rectangular data grid. If $X_A = X_B = X_1 = X_2$ and $Y_A = Y_B = Y_1 = Y_2$ then the desired three-dimensional point is found and $Z = Z_1$. Otherwise, interpolation to determine Z is required.

A.10.2.5 Initial Linear Interpolation Perform linear interpolation to determine X_3, Y_3, and Z_3, given X_A, Y_A, Z_A and X_B, Y_B, Z_B using one of the following distinct cases:

Case L_1: If $X_A = X_B$ and $Y_A = Y_B$ then the two points are the same.

$$X_3 = X_A$$
$$Y_3 = Y_A$$
$$Z_3 = Z_A$$

Case L_2: If $X_A = X_B$ then the two points form a vertical line.

$$X_3 = X_A$$
$$Y_3 = Y_A$$
$$Z_3 = Z_A$$

Case L_3: If $Y_A = Y_B$ then the two points form a horizontal line.

$$Z_3 = Z_A + \left(\mathrm{CC1} \times \frac{\mathrm{CA2}}{\mathrm{CA1}}\right)$$

$X_3 = X$;
$Y_3 = Y_A$;
$\mathrm{CA1} = X_B - X_A$;
$\mathrm{CA2} = X_3 - X_A$;
$\mathrm{CC1} = Z_B - Z_A$.

Case L_4: If none of the above three cases occur, perform normal linear interpolation.

$$X_3 = X$$
$$Y_3 = Y_A + \left(\mathrm{CB1} \times \frac{\mathrm{CA2}}{\mathrm{CA1}}\right)$$

$\mathrm{CA1} = X_B - X_A$;
$\mathrm{CB1} = Y_B - Y_A$;
$\mathrm{CC1} = Z_B - Z_A$;
$\mathrm{CA2} = X_3 - X_A$.

$$\mathrm{Z}_3 = \mathrm{Z}_A + (\mathrm{CC1} \times \mathrm{CC3}/\mathrm{CC2})$$

$\mathrm{CB2} = \mathrm{Y}_3 - \mathrm{Y}_A$;
$\mathrm{CC2} = \mathrm{sqrt}(\mathrm{CA1} \times \mathrm{CA1} + \mathrm{CB1} \times \mathrm{CB1})$;
$\mathrm{CC3} = \mathrm{sqrt}(\mathrm{CA2} \times \mathrm{CA2} + \mathrm{CB2} \times \mathrm{CB2})$;
$\mathrm{sqrt}(X)$ = square root of X.

A.10.2.6 Final Triangular Interpolation After determining X_3, Y_3, and Z_3, use triangular interpolation to determine Z, given X and Y plus X_1, Y_1, Z_1 and X_2, Y_2 and Z_2 using one of the following distinct cases:

Case T_1: If $X_1 = X_2$ and $Y_1 = Y_2$ and $X_1 = X_3$ and $Y_1 = Y_3$ then all points are the same.

$$Z = Z_1 \tag{A.171}$$

Case T_2: If $X_1 = X_2$ and $Y_1 = Y_2$ then the two lower points are the same.

Subcase T_{2a}: If $X_1 = X_3$ then the two distinct points form a vertical line

$$Z = Z_1 + \left(\mathrm{CC1} \times \frac{\mathrm{CA2}}{\mathrm{CA1}}\right) \tag{A.172}$$

$\mathrm{CA1} = Y_3 - Y1$;
$\mathrm{CA2} = Y - Y_1$;
$\mathrm{CC1} = Z_3 - Z_1$.

Subcase T_{2b}: If Subcase T_{2a} does not apply, the two distinct points form a diagonal line.

$$Z = Z_1 + \left(\mathrm{CC1} \times \frac{\mathrm{CC3}}{\mathrm{CC2}}\right) \tag{A.173}$$

$\mathrm{CA1} = X_3 - X_1$;
$\mathrm{CB1} = Y_3 - Y_1$;
$\mathrm{CC1} = Z_3 - Z_1$;
$\mathrm{CA2} = X - X_1$;
$\mathrm{CB2} = Y - Y_1$;
$\mathrm{CC2} = \mathrm{sqrt}(\mathrm{CA1} \times \mathrm{CA1} + \mathrm{CB1} \times \mathrm{CB1})$;
$\mathrm{CC3} = \mathrm{sqrt}(\mathrm{CA2} \times \mathrm{CA2} + \mathrm{CB2} \times \mathrm{CB2})$.

Case T_3: If none of the above cases apply, perform normal triangular interpolation.

$$Z = \frac{([X \times B_1] + [Y \times B_2] + B_3)}{B_4} \tag{A.174}$$

$A_1 = X_2 - X_3$;
$A_2 = Y_2 - Y_3$;
$A_3 = Z_2 - Z_3$;
$A_4 = (X_2 \times Y_3) - (X_3 \times Y_2)$;
$A_5 = (X_2 \times Z_3) - (X_3 \times Z_2)$;
$A_6 = (Y_2 \times Z_3) - (Y_3 \times Z_2)$;
$B_1 = -(A_3 \times Y_1) + (A_2 \times Z_1) - A_6$;
$B_2 = (A_3 \times X_1) - (A_1 \times Z_1) + A_5$;
$B_3 = (A_6 \times X_1) - (A_5 \times Y_1) + (A_4 \times Z_1)$;
$B_4 = (A_2 \times X_1) - (A_1 \times Y_1) + A_4$.

Defined X, Y, Z values at the four grid corners are necessary. In addition, data representing different domains (e.g., land or sea data that may have a different range of values) must be sufficiently “fenced” with known data points separating the domains so that the triangular interpolation does not extend between domains.