

# Radio Propagation for Vehicular Communications

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**Abstract**—Radio propagation is affected by many factors including the frequency, distance, antenna heights, curvature of the earth, atmospheric conditions, and the presence of hills and buildings. The influence of each of these factors at frequencies above about 30 MHz is discussed with most of the quantitative data being presented in the series of nomograms. By means of three or four of these charts an estimate of the received power and the received field intensity for a given point-to-point radio transmission path ordinarily can be obtained in a minute or less. The theory of propagation over a smooth spherical earth is presented in a simplified form that is made possible by restricting the frequency range to above about 30 MHz where variations in the electrical constants of the earth have only a secondary effect. The empirical methods used in estimating the effects of hills and buildings and of atmospheric refraction are compared with experimental data on shadow losses and on fading ranges.

## I. INTRODUCTION

**METHODS** of calculating ground-wave propagation over a smooth spherical earth have been given by Burrows and Gray and by Norton for all values of distance, frequency, antenna height, and ground constants [1], [2]. These two methods are different in form but they give essentially the same results. Both methods are relatively simple to use at the lower frequencies where grounded antennas are in common use, but their complexity increases as the frequency increases. At frequencies above 30 or 40 MHz, elevated antennas are in common use and the radio path loss between two horizontal antennas tends to be equal to the loss between two vertical antennas. In addition, both types of transmission tend to be independent of the electrical constants of the ground so that considerable simplification is possible. This paper presents a series of nomograms that have been found useful in solving radio propagation problems in the very-high-frequency range and above. These charts are arranged so that radio transmission can be expressed in terms of either the received field intensity or the received power delivered to a matched receiver. The field-intensity concept may be more familiar, but the power-transfer concept becomes more convenient as the frequency is increased.

In addition to the smooth earth theory an approximate method is included for estimating the effects of hills and other obstructions in the radio path. The phenomena of atmospheric

Manuscript received January 3, 1977. This invited paper consists of selections from two earlier papers written by the author. Only minor changes have been made in keeping with uniformity and standardized methods of expressing parameters, e.g., MHz, dBW, and  $\mu\text{V/m}$ . The papers are "Radio Propagation at Frequencies Above 30 Megacycles," *PROCEEDINGS OF THE IRE*, vol. 35, no. 10, pp. 1122-1136, October 1947 (also presented at National Electronics Conference, October 1946), and "Radio Propagation Fundamentals," *Bell System Technical Journal*, vol. 36, no. 3, 593-626, May 1957. Reprinting is by permission of the IEEE, the *Bell System Technical Journal*, and the National Electronics Conference, an activity of National Engineering Consortium, Inc.

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refraction (bending away from straight-line propagation), atmospheric ducts (tropospheric propagation), and atmospheric absorption are discussed briefly, but the principal purpose is to provide simplified charts for predicting radio propagation under average weather conditions. It is expected that normally the nomograms will provide the desired answer directly without any additional computation except the addition of the dB values obtained from three or four individual charts. The basic formulas are presented as an aid to understanding the principles involved and as a more accurate method should one be required. This paper does not consider sky-wave propagation although ionospheric reflections may occur at frequencies above 30 MHz and may cause occasional long-distance interference between systems operating on the same frequency.

A convenient starting point for the theory of radio propagation is the condition of two antennas in free space, which is discussed in terms of both received field intensity and received power. Since most radio paths cannot be considered to be free-space paths the next step is to determine the effect of a perfectly flat earth, and this is followed by the effect of the curvature of the earth. After the basic smooth-earth theory is completed there is a discussion of the variations in received power caused by atmospheric conditions and by irregularities on the earth surface, but the methods used in predicting these factors are necessarily less exact than the data for a smooth spherical earth in a uniform atmosphere.

## II. FREE-SPACE FIELD

A free-space transmission path is a straight-line path in a vacuum or in an ideal atmosphere and sufficiently removed from all objects that might absorb or reflect radio energy. The free-space field intensity  $E_0$  at a distance  $d$  meters from the transmitting antenna is given by

$$E_0 = \frac{\sqrt{30g_1P_1}}{d} \text{ V/m} \quad (1)$$

where  $P_1$  is the radiated power in watts and  $g_1$  is the power-gain ratio of the transmitting antenna. The subscript 1 refers to the transmitter and the subscript 2 will refer to the receiver. For an ideal (isotropic) antenna that radiates power uniformly in all directions  $g = 1$ . For any balanced antenna in free space (or located more than a quarter-wavelength above the ground)  $g$  is the power-gain ratio of the antenna relative to the isotropic antenna. A small doublet or dipole whose overall physical length is short compared with a half-wavelength has a directivity gain of  $g = 1.5$  (1.76 dB), and a half-wave dipole has a gain of  $g = 1.64$  (2.15 dB) in the direction of maximum radiation. In other directions of transmission the field is reduced in accordance with the free-space antenna pattern obtained from

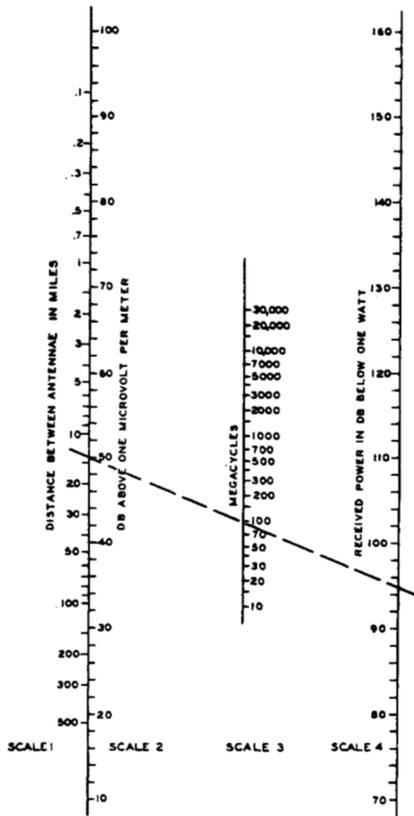


Fig. 1. Free-space field intensity and received power between half-wave dipoles, 1 W radiated.

theory or measurement. Consequently, the freespace field intensity in a direction perpendicular to a half-wave dipole is

$$E_0 = \sqrt{\frac{30 \times 1.64 P_1}{d}} \sim 7 \sqrt{\frac{P_1}{d}} \quad (2)$$

This field intensity in  $\mu\text{V}/\text{m}$  for 1 W of radiated power is shown on scale 2 of Fig. 1 as a function of the distance in miles shown on scale 1. For radiated power of  $P$  W the correction factor to apply to the field intensity or power is  $10 \log P$  (dB). For example, the free-space field intensity at 100 mi from a half-wave dipole radiating 1 W is 33 dB $\mu\text{V}/\text{m}$  (about 45  $\mu\text{V}/\text{m}$ ). When the radiated power is 50 W (17 dBW) the received field intensity is  $33 + 17 = 50$  dB $\mu\text{V}/\text{m}$  (about 315  $\mu\text{V}/\text{m}$ ). It will be noted that the field intensity is related to the energy density of the radio wave at the receiving antenna but is independent of the type of the receiving antenna.

The directivity gain of an array of  $n$  dipoles (sum of driven and parasitic elements) of optimum design is approximately equal to  $n$  times the gain of one dipole although some allowance should be made for antenna power losses. The theoretical power-gain ratio of a horn, paraboloid, or lens antenna whose aperture has an area of  $B$   $\text{m}^2$  is  $g = 4\pi B/\lambda^2$ ; however, the effective area is frequently taken as one-half to two-thirds of the actual area of the aperture to account for antenna inefficiencies.

### III. RELATION BETWEEN THE RECEIVED POWER AND THE RADIATED POWER

Before discussing the modifications in the free-space field that result from the presence of the earth, it is convenient to show the relation between the received field intensity (which is not necessarily equal to the free-space field intensity) and the power that is available to the receiver. The maximum useful power  $P_2$  that can be delivered to a matched receiver is given by

$$P_2 = \left( \frac{E\lambda}{2\pi} \right)^2 \frac{g_2}{120} \text{ W} \quad (3)$$

where

- $E$  received field intensity in volts per meter,
- $\lambda$  wavelength in meters =  $300/F$ ,
- $F$  frequency in megacycles,
- $g_2$  power-gain ratio of the receiving antenna.

This relation between received power and the received field intensity is shown by scales 2-4 in Fig. 1 for a half-wave dipole. For example, the maximum useful power at 100 MHz that can be picked up by a half-wave dipole in a field of 50 dB $\mu\text{V}/\text{m}$  is -95 dBW.

A general relation for the ratio of the received power to the radiated power obtained from (1) and (3) is

$$\frac{P_2}{P_1} = \left( \frac{\lambda}{4\pi d} \right)^2 g_1 g_2 \left( \frac{E}{E_0} \right)^2 \quad (4)$$

When both antennas are half-wave dipoles, the power transfer ratio is

$$\frac{P_2}{P_1} = \left( \frac{1.64\lambda}{4\pi d} \right)^2 \left( \frac{E}{E_0} \right)^2 = \left( \frac{0.13\lambda}{d} \right)^2 \left( \frac{E}{E_0} \right)^2 \quad (4a)$$

and is shown in Fig. 1 for free-space transmission ( $E/E_0 = 1$ ).

When the antennas are horns, paraboloids, or multi-element arrays, a more convenient expression for a ratio of the received power to the radiated power is given by

$$\frac{P_2}{P_1} = \frac{B_1 B_2}{(\lambda d)^2} \left( \frac{E}{E_0} \right)^2 \quad (4b)$$

where  $B_1$  and  $B_2$  are the effective areas of the transmitting and receiving antennas, respectively. This relation is obtained from (4) by substituting  $g = 4\pi B/\lambda^2$  and is shown in Fig. 2 for free-space transmission when  $B_1 = B_2$ . For example, the free-space loss at 4 GHz between two antennas of 10  $\text{ft}^2$  effective area is about 72 dB for a distance of 30 mi.

### IV. TRANSMISSION OVER PLANE EARTH

The presence of the ground modifies the generation and the propagation of the radio waves so that the received field inten-

sity is ordinarily less than would be expected in free space. The ground acts as a partial reflector and as a partial absorber, and both of these properties affect the distribution of energy in the region above the earth. The principal effect of plane earth on the propagation of radio waves is indicated by the following equation [4], [5]:

$$E = E_0 \left[ \underbrace{1}_{a)} + \underbrace{R e^{j\Delta}}_{b)} + \underbrace{(1-R)A e^{j\Delta}}_{c)} + \underbrace{\dots}_{d)} \right]. \quad (5)$$

where a) is the direct wave, b) is the reflected wave, c) is the surface wave, and d) is the induction field and secondary effects of the ground.  $R$  is the reflection coefficient of the ground and is approximately equal to  $-1$  when the angle  $\theta$  between the reflected ray and the ground is small. The commonly used concept of a *perfectly conducting* earth, for which the reflection coefficient for vertical polarization is  $+1$  for any angle of incidence, may cause some misunderstanding at this point. In practice the principal interest is in low angles, and as the angle  $\theta$  approaches zero the reflection coefficient approaches  $-1$  for any finite value for the conductivity of the earth, even if it were made of solid copper. The magnitude and phase of the reflection coefficient can be computed from the following equation<sup>1</sup>:

$$R = \frac{\sin \theta - z}{\sin \theta + z} \quad (6)$$

where

$$\begin{aligned} z &= \sqrt{\epsilon_0 - \cos^2 \theta} / \epsilon_0 \text{ for vertical polarization,} \\ z &= \sqrt{\epsilon_0 - \cos^2 \theta} \text{ for horizontal polarization,} \\ \epsilon_0 &= \epsilon - j60\sigma\lambda, \end{aligned}$$

<sup>1</sup> It will be noted that for vertical polarization this expression agrees with the data given by Burrows and subsequently included in [20], but for horizontal polarization it is the negative of that given in these references. This change was necessary in order to make (5) and (6) independent of polarization. The pseudo-Brewster angle frequently mentioned in the literature occurs when the reflection coefficient is a minimum and is approximately equal to the value of  $\theta$  for which  $\sin \theta = |z|$ ; this occurs with vertical polarization only, since  $z > 1$  for horizontal polarization. The reflection coefficient is sometimes modified by a divergence factor to give a first approximation of the effect of the curvature of the earth, but this additional complication does not seem essential here. The effect of the curvature of the earth is discussed in the next section, and for conditions of frequency and antenna height where some interpolation is required the possible variations due to atmospheric conditions are usually greater than the error introduced by the omission of the divergence factor. The measured data on the plane-earth reflection coefficient agrees reasonably well with the theoretical values at frequencies below about 1000 MHz. At higher frequencies the magnitude of the reflection coefficient is sometimes less than 1 presumably due to multiple reflections from the irregularities on the earth's surface. Measured values as low as  $-0.2$  at 10 000 MHz over rolling country have been reported by W. M. Sharpless. The low value of reflection coefficient is not expected to be important for ground-to-ground transmission, but it tends to smooth the lobes that occur in the high-angle radiation, hence, may be important in air-to-ground transmission.

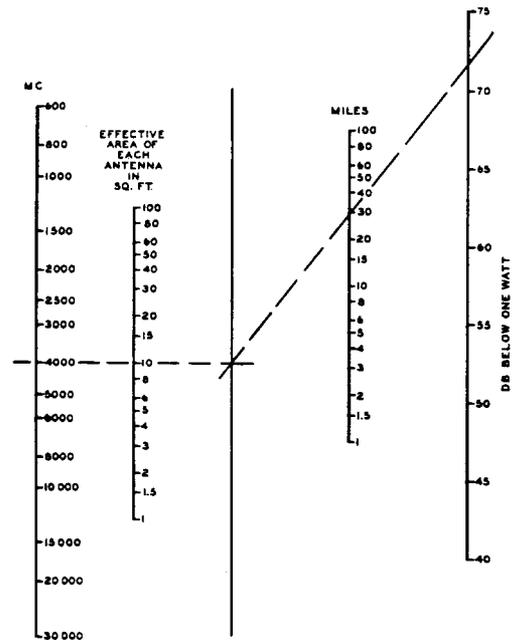


Fig. 2. Received power in free space between two antennas of equal effective areas, 1 W radiated.

- $\epsilon$  dielectric constant of the ground relative to unity in free space,
- $\sigma$  conductivity of the ground in mhos per meter,
- $\lambda$  wavelength in meters,
- $j = \sqrt{-1}$ ,
- $e^{j\Delta} = \cos \Delta + j \sin \Delta$ .

The surface-wave attenuation factor  $A$  depends upon the frequency, ground constants, and type of polarization. It is never greater than unity and decreases with increasing distance and frequency as indicated by the following approximate equation<sup>2</sup>:

$$A \cong \frac{-1}{1 + j \frac{2\pi d}{\lambda} (\sin \theta + z)^2} \quad (7)$$

The angle  $\Delta$  used in (5) is the phase difference in radians resulting from the difference in the length of the direct and reflected rays. It is equal to  $4\pi h_1 h_2 / \lambda d$  radians when the distance  $d$  between antennas is greater than about five times the sum of the two antenna heights  $h_1$  and  $h_2$ .

The effect of the ground shown in (5) indicates that ground-wave propagation may be considered to be the sum of three principal terms; namely, the direct wave, the reflected wave, and the surface wave. The first two types correspond to our common experience with visible light, but the surface

<sup>2</sup> This approximate expression is sufficiently accurate as long as  $A < 0.1$ , and it gives the magnitude of  $A$  within about 2 dB for all values of  $A$ . However, as  $A$  approaches unity, the error in phase approaches 180 deg. More accurate values are given by Norton, where in his nomenclature  $A = f(P, B)e^{i\phi}$ .

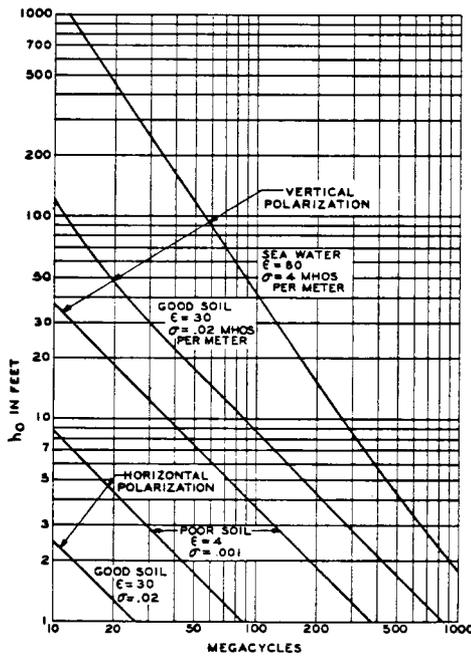


Fig. 3. Minimum effective height.

wave is less familiar. Since the earth is not a perfect reflector some energy is transmitted into the ground and is absorbed. As this energy enters the ground it sets up ground currents, which is another way of saying that the distribution of the electromagnetic field in the region near the surface of the earth is distorted relative to what it would have been over an ideal perfectly reflecting surface. The surface wave is defined as the vertical electric field for vertical polarization, or the horizontal electric field for horizontal polarization that is associated with these ground currents.<sup>3</sup>

The practical importance of the surface wave is limited to a region above the ground of about one wavelength over land or five to ten wavelengths over sea water since for greater heights the sum of the direct and reflected waves is larger in magnitude. Thus the surface wave is the principal component of the total ground wave at frequencies less than a few megahertz, but it is of secondary importance in the very high-frequency range (30 to 300 MHz) and it usually can be neglected at frequencies above 300 MHz.

A physical picture of the various components of the ground wave can be obtained from (5), but an equivalent expression which is more convenient for this discussion, is

$$\frac{E}{E_0} = 2 \sin \frac{\Delta}{2} + j[(1 + R) + (1 - R)A] e^{j(\Delta/2)}. \quad (8)$$

When the angle  $\Delta = 4\pi h_1 h_2 / \lambda d$  is greater than about 0.5 radian the terms inside the brackets (which include the surface

<sup>3</sup> Another component of the electric field associated with the ground currents is in the direction of propagation. It accounts for the success of the wave antenna at lower frequencies, but it is always smaller in magnitude than the surface wave as defined above. The components of the electric vector in three mutually perpendicular coordinates are given by Norton.

wave) are usually negligible, and a sufficiently accurate approximation is given by

$$\frac{E}{E_0} = 2 \sin \frac{2\pi h_1 h_2}{\lambda d}. \quad (8a)$$

In this case, the principal effect of the ground is to produce interference fringes or lobes so that the field intensity, at a given distance and for a given frequency, oscillates around the free-space field as either antenna height is increased.

When the angle  $\Delta$  is less than about 0.5 rad the receiving antenna is below the maximum of the first lobe, and the surface wave may be important. A sufficiently accurate approximation for this condition is<sup>4</sup>

$$\left| \frac{E}{E_0} \right| = \left| \frac{4\pi h_1' h_2'}{\lambda d} \right|. \quad (8b)$$

In this equation  $h' = h + jh_0$  where  $h$  is the actual antenna height and  $h_0 = \lambda/2\pi z$  has been designated as the minimum effective antenna height. The magnitude of the minimum effective height  $|h_0|$  is shown in Fig. 3 for sea water and for "good" and "poor" soil. "Good" soil corresponds roughly to clay, loam, marsh, or swamp while "poor" soil means rocky or sandy ground.

The surface wave is controlling for antenna heights less than the minimum effective height, and in this region the received field or power is not affected appreciably by changes in the antenna height. For antenna heights that are greater than the minimum effective height, the received field or power is increased approximately 6 dB every time the antenna height is doubled until free-space transmission is reached. It is ordinarily sufficiently accurate to assume that  $h'$  is equal to the actual antenna height or the minimum effective antenna height, whichever is the larger.

The ratio of the received power to the radiated power for transmission over plane earth is obtained by substituting (8b) into (4), resulting in

$$\frac{P_2}{P_1} = \left( \frac{\lambda}{4\pi d} \right)^2 g_1 g_2 \left( \frac{4\pi h_1' h_2'}{\lambda d} \right)^2 = \left( \frac{h_1' h_2'}{d^2} \right)^2 g_1 g_2. \quad (9)$$

This relation is independent of frequency and is shown in Figure 4 for half-wave dipoles ( $g = 1.64$ ). A line through the two

<sup>4</sup> This approximate expression is obtained from (8) by assuming

$$\sin \theta = \frac{h_1 + h_2}{d} \ll z \quad (a)$$

$$\sin \frac{2\pi h_1 h_2}{\lambda d} = \frac{2\pi h_1 h_2}{\lambda d} \quad (b)$$

$$A = \frac{\lambda}{j2\pi dz^2}. \quad (c)$$

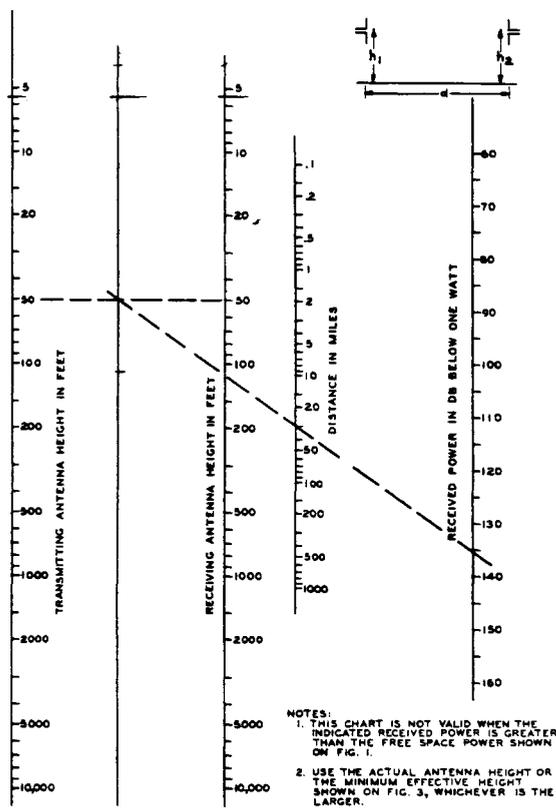


Fig. 4. Received power over plane earth between half-wave dipoles, 1 W radiated.

scales of antenna height determines the point on the unlabeled scale between them, and a second line through this point and the distance scale determines the received power for 1 W radiated. When the received field intensity is desired, the power indicated on Fig. 4 can be transferred to scale 4 of Fig. 1, and a line through the frequency on scale 3 indicates the received field intensity on scale 2. The results shown in Fig. 4 are valid as long as the value of received power indicated is lower than shown in Fig. 1 for free-space transmission. When this condition is not met, it means that the angle  $\Delta$  is too large for (8b) to be accurate, and that the received field intensity or power oscillates around the free-space value as indicated by (8a).

As an example, consider a 250-W 30-MHz transmitter with both transmitting and receiving dipoles mounted 50 ft above the ground and separated by a distance of 30 mi over plane earth. The transmission loss is shown in Fig. 4 to be 135.5 dB. Since 250W is 24 dBW, the received power is  $24 - 135.5 = -111.5$  dBW. (The free-space power transfer shown in Fig. 1 indicates a received power of  $24 - 91 = -67$  dBW, so Fig. 4 is controlling.) The received field intensity can be obtained from Fig. 1, which shows that a received power of  $-111.5$  dBW corresponds to a received field intensity of about 23 dBuV/m at a frequency of 30 MHz. Should one antenna be only 10 ft above "good" soil, rather than 50 ft, the minimum effective height of 30 ft shown in Fig. 3 should be used on one of the height scales in Fig. 4 in determining the transmission loss. It will be noted that this example assumes a perfectly flat earth. The curvature of the earth introduces an additional loss of about 4 dB, as discussed in the next section.

In addition to the effect of plane earth on the propagation of radio waves, the presence of the ground may affect the impedance of an antenna and thereby may have an effect on the generation and reception of radio waves. This effect usually can be neglected at frequencies above 30 MHz except where whip antennas are used. The impedance in the presence of the ground oscillates around the free-space value, but the variations are unimportant as long as the center of the antenna is more than a quarter-wavelength above the ground. A convenient method of showing the effect of the change in impedance of a balanced antenna near the ground is to replace the directivity gain  $g$  in the preceding formulas by the arbitrary factor of  $g' = g/r$  where  $r$  is the ratio of the input resistance in the presence of the ground to the input resistance of the same antenna in free space. This assumes an impedance match between the antenna and the transmitting equipment with proper tuning to balance out any reactance.

For horizontal dipoles less than a quarter-wavelength above the ground the ratio  $r$  is less than unity. It approaches zero as the antenna approaches a perfectly conducting earth, but in practice it does not reach zero at zero height because of the finite conductivity of the earth. The wave antenna and the top-loaded antenna frequently used at lower frequencies are sometimes called horizontal antennas, but since they are used to radiate or receive vertically polarized waves they are not horizontal antennas in the sense used here.

For vertical half-wave dipoles the factor  $r$  is approximately equal to unity since the height of the center of the antenna can never be less than a quarter-wavelength above the ground. For very short vertical dipoles, however, the ratio  $r$  is greater than unity, and it approaches a value of  $r = 2$  for antennas very near to the ground. This means that, whereas a short vertical dipole whose total length  $2l$  is small compared with the wavelength that has an input radiation resistance of  $80(\pi l/\lambda)^2 \Omega$  in free space, it has a resistance of  $160(\pi l/\lambda)^2 \Omega$  near the ground.

Correct results for a vertical whip antenna working against a perfectly counterpoise are obtained by using  $r = 2$ . This means that a vertical whip antenna of length  $l$  is 3 dB less efficient than a dipole of length  $2l$  (located more than a quarter-wavelength above the ground) for either transmitting or receiving. The poorer efficiency at the receiver is not important when external noise is controlling.

### V. FADING PHENOMENA

Variations in signal level with time are caused by changing atmospheric conditions. The severity of the fading usually increases as either the frequency or path length increases. Fading cannot be predicted accurately, but it is important to distinguish between two general types: (1) inverse bending and (2) multipath effects. The latter includes the fading caused by interference between direct and ground reflected waves as well as interference between two or more separate paths in the atmosphere. Ordinarily, fading is a temporary diversion of energy to some other than the desired location.

The path of a radio wave is not a straight line except for the ideal case of a uniform atmosphere. The transmission path

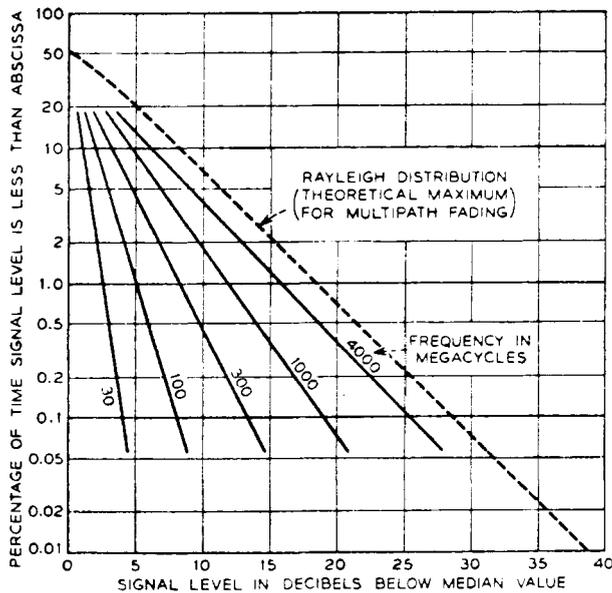


Fig. 5. Typical fading characteristics in the worst month on 30 to 40 mile line-of-sight paths with 50 to 100 ft clearance.

may be bent up or down depending on atmospheric conditions. This bending may either increase or decrease the effective path clearance and inverse bending may have the effect of transforming a line of sight path into an obstructed one. This type of fading may last for several hours. The frequency of its occurrence and its depth can be reduced by increasing the path clearance, particularly in the middle of the path.

Severe fading may occur over water or on other smooth paths because the phase difference between the direct and reflected rays varies with atmospheric conditions. The result is that the two rays sometimes add and sometimes tend to cancel. This type of fading can be minimized, if the terrain permits, by locating one end of the circuit high while the other end is very low. In this way the point of reflection is placed near the low antenna and the phase difference between direct and reflected rays is kept relatively steady.

Most of the fading that occurs on "rough" paths with adequate clearance is the result of interference between two or more rays traveling slightly different routes in the atmosphere. This multipath type of fading is relatively independent of path clearance, and its extreme condition approaches the Rayleigh distribution. In the Rayleigh distribution, the probability that the instantaneous value of the field is greater than the value  $R$  is  $\exp[-(R/R_0)^2]$  where  $R_0$  is the rms value.

Representative values of fading on a path with adequate clearance are shown in Fig. 5. After the multipath fading has reached the Rayleigh distribution, a further increase in either distance or frequency increases the number of fades of a given depth, but it decreases the duration so that the product is the constant indicated by the Rayleigh distribution.

## VI. TROPOSPHERIC TRANSMISSION BEYOND LINE OF SIGHT

A basic characteristic of electromagnetic waves is that the energy is propagated in a direction perpendicular to the sur-

face of uniform phase. Radio waves travel in a straight line only as long as the phase front is plane and is infinite in extent.

Energy can be transmitted beyond the horizon by three principal methods: reflection, refraction, and diffraction. Reflection and refraction are associated with either sudden or gradual changes in the direction of the phase front, while diffraction is an edge effect that occurs because the phase surface is not infinite. When the resulting phase front at the receiving antenna is irregular in either amplitude or position, the distinctions between reflection, refraction, and diffraction tend to break down. In this case the energy is said to be scattered. Scattering is frequently pictured as a result of irregular reflections although irregular refraction plus diffraction may be equally important.

The following paragraphs describe first the theories of refraction and of diffraction over a smooth sphere and a knife edge. This is followed by empirical data derived from experimental results on the transmission to points far beyond the horizon, on the effects of hills and trees, and on fading phenomena.

### Refraction

The dielectric constant of the atmosphere normally decreases gradually with increasing altitude. The result is that the velocity of transmission increases with the height above the ground, and on the average the radio energy is bent or refracted toward the earth. As long as the change in dielectric constant is linear with height, the net effect of refraction is the same as if the radio waves continued to travel in a straight line but over an earth whose modified radius is

$$ka = \frac{a}{1 + \frac{a}{2} \frac{d\epsilon}{dh}} \quad (10)$$

where

$$\begin{array}{ll} a & \text{true radius of earth,} \\ d\epsilon/dh & \text{rate of change of dielectric constant with height.} \end{array}$$

Under certain atmospheric conditions the dielectric constant may increase ( $0 < k < 1$ ) over a reasonable height, thereby causing the radio waves in this region to bend away from the earth. Since the earth's radius is about  $2.1 \times 10^7$  ft ( $6.4 \times 10^6$  m), a decrease in dielectric constant of only  $2.4 \times 10^{-8}$ /ft ( $7.9 \times 10^{-8}$ /m) of height results in a value of  $k = 4/3$ , which is commonly assumed to be a good average value [6]. When the dielectric constant decreases about four times as rapidly (or by about  $10^{-7}$  per foot ( $3.3 \times 10^{-7}$ /m) of height), the value of  $k = \infty$ . Under such a condition, as far as radio propagation is concerned, the earth can then be considered flat, since any ray that starts parallel to the earth will remain parallel.

When the dielectric constant decreases more rapidly than  $10^{-7}$ /ft ( $3.3 \times 10^{-7}$ /m) of height, radio waves that are radiated parallel to or at an angle above the earth's surface may be bent downward sufficiently to be reflected from the earth. After reflection the ray is again bent toward the earth, and the path of

a typical ray is similar to the path of a bouncing tennis ball. The radio energy appears to be trapped in a duct or waveguide between the earth and the maximum height of the radio path. This phenomenon is variously known as trapping, duct transmission, anomalous propagation, or guided propagation [7], [8]. It will be noted that in this case the path of a typical guided wave is similar in form to the path of sky waves, which are lower frequency waves trapped between the earth and the ionosphere. However, there is little or no similarity between the virtual heights, the critical frequencies, or the causes of refraction in the two cases.

Duct transmission is important because it can cause long distance interference with another station operating on the same frequency; however, it does not occur often enough nor can its occurrence be predicted with enough accuracy to make it useful for radio services requiring high reliability.

*Diffraction over a Smooth Spherical Earth and Ridges*

Radio waves are also transmitted around the earth by the phenomenon of diffraction. Diffractions is a fundamental property of wave motion, and in optics it is the correction to apply to geometrical optics (ray theory) to obtain the more accurate wave optics. In other words, all shadows are somewhat "fuzzy" on the edges, and the transition from "light" to "dark" areas is gradual rather than infinitely sharp. Our common experience is that light travels in straight lines and that shadows are sharp, but this is only because the diffraction effects for these very short wavelengths are too small to be noticed without the aid of special laboratory equipment. The order of magnitude of the diffraction at radio frequencies may be obtained by recalling that a 1000-MHz radio wave has about the same wavelength as a 1000-Hz sound wave in air so that these two types of waves may be expected to bend around absorbing obstacles with approximately equal facility.

The effect of diffraction around the earth's curvature is to make possible transmission beyond the line-of-sight. The magnitude of the loss caused by the obstruction increases as either the distance or the frequency is increased, and it depends to some extent on the antenna height [1]. The loss resulting from the curvature of the earth is indicated by Fig. 6 as long as neither antenna is higher than the limiting value shown at the top of the chart. This loss is in addition to the transmission loss over plane earth obtained from Fig. 4.

When either antenna is as much as twice as high as the limiting value shown in Fig. 6, this method of correcting for the curvature of the earth indicates a loss that is too great by about 2 dB with the error increasing as the antenna height increases. An alternate method of determining the effect of the earth's curvature is given by Fig. 7. The latter method is approximately correct for any antenna height, but it is theoretically limited in distance to points at or beyond the line-of-sight assuming that the curved earth is the only obstruction. Fig. 7 gives the loss relative to free-space transmission (and hence is used with Fig. 1) as a function of three distances:  $d_1$  is the distance to the horizon from the lower antenna,  $d_2$  is the distance to the horizon from the higher antenna, and  $d_3$  is the distance beyond the line-of-sight. In other words, the total distance between antennas is  $d = d_1 + d_2 + d_3$ . The dis-

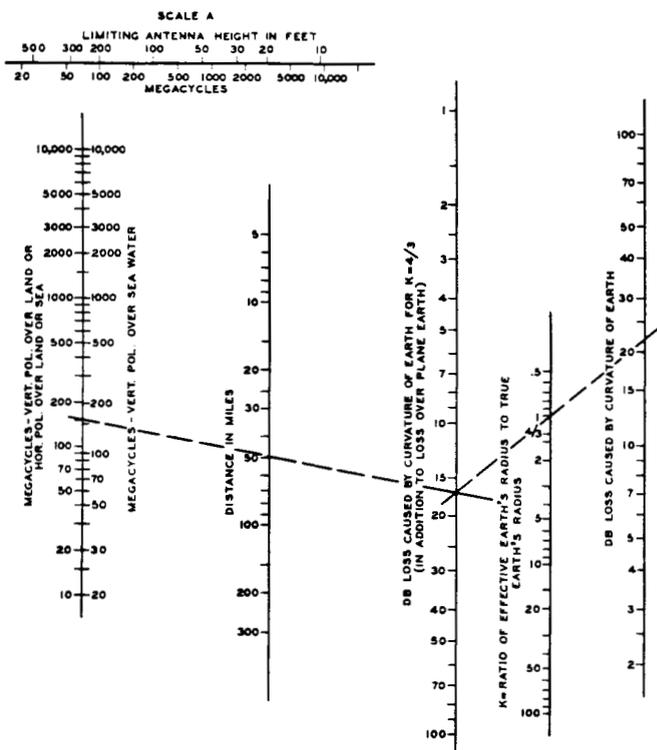


Fig. 6. Diffraction loss caused by curvature of the earth assuming neither antenna height is higher than shown on scale A.

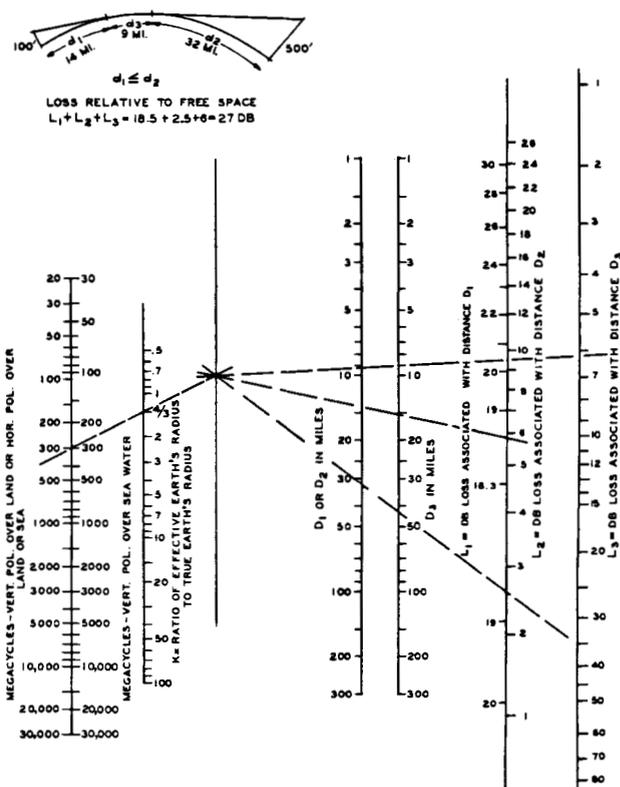


Fig. 7. Decibel loss relative to free-space transmission at points beyond line-of-sight over a smooth earth.

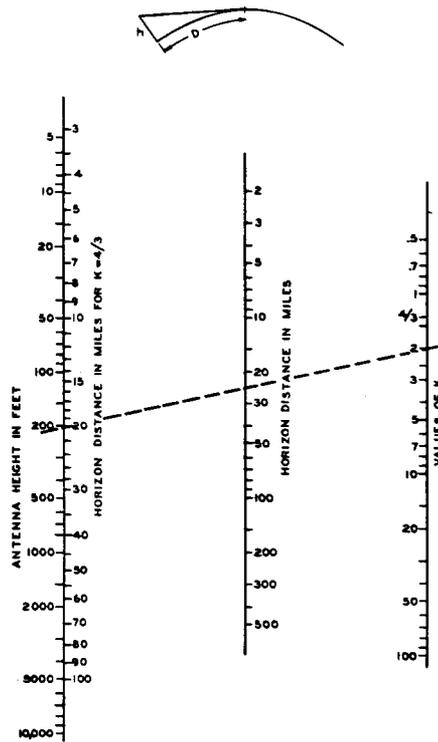


Fig. 8. Distance to horizon.

tance to the horizon over smooth earth is shown in Fig. 8 and is given by

$$d_{1,2} = \sqrt{2kah_{1,2}} \quad (11)$$

where  $h_{1,2}$  is the appropriate antenna height and  $ka$  is the effective earth's radius.

The preceding discussion assumes that the earth is a perfectly smooth sphere, and the results are critically dependent on a smooth surface and a uniform atmosphere. The modification in these results caused by the presence of hills, trees, and buildings is difficult or impossible to compute, but the order of magnitude of these effects may be obtained from a perfectly absorbing knife edge.

The diffraction of plane waves over a knife edge or screen causes a shadow loss whose magnitude is shown in Fig. 9. The height of the obstruction  $H$  is measured from the line joining the two antennas to the top of the ridge. It will be noted that the shadow loss approaches 6 dB as  $H$  approaches 0 (grazing incidence), and that it increases with increasing positive values of  $H$ . When the direct ray clears the obstruction  $H$  is negative, and the shadow loss approaches 0 dB in an oscillatory manner as the clearance is increased. In other words, a substantial clearance is required over line-of-sight paths in order to obtain "free-space" transmission. The knife edge diffraction calculation is substantially independent of polarization as long as the distance from the edge is more than a few wavelengths.

At grazing incidence the expected loss over a ridge is 6 dB, (Fig. 9) while over a smooth spherical earth Fig. 7 indicates a loss of about 20 dB. More accurate results in the vicinity of

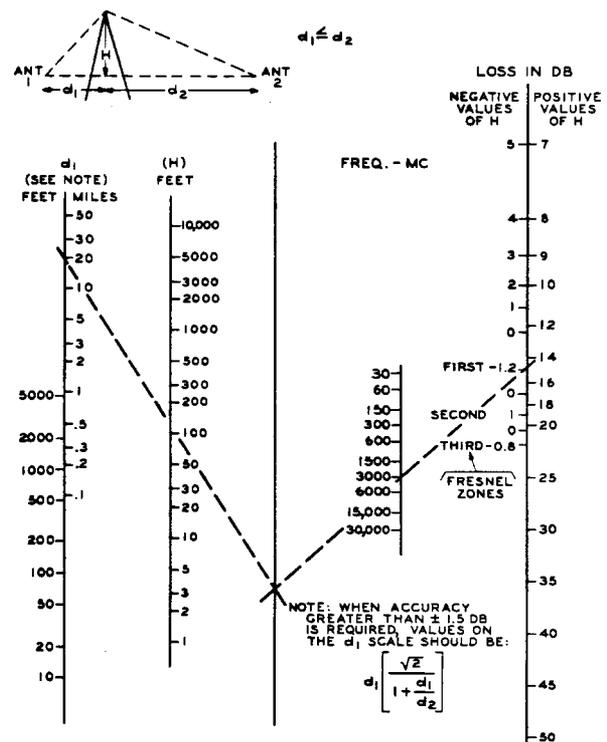


Fig. 9. Knife-edge diffraction loss relative to free space.

the horizon can be obtained by expressing radio transmission in terms of path clearance measured in Fresnel zones as shown in Fig. 10. In this representation the plane earth theory and the ridge diffraction can be represented by single lines, but the smooth sphere theory requires a family of curves with a parameter  $M$  that depends primarily on antenna heights and frequency. The big difference in the losses predicted by diffraction around a perfect sphere and by diffraction over a knife edge indicates that diffraction losses depend critically on the assumed type of profile. A suitable solution for the intermediate problem of diffraction over a rough earth has not yet been obtained.

The problem of two or more knife-edge obstructions between the transmitting and receiving antennas such as shown in Fig. 11(a) has not been solved rigorously. However, graphical integration indicates that the shadow loss for this case is equivalent within 2 or 3 dB to the shadow loss for the knife edge represented by the height of the triangle composed of a line joining the two antennas and a line from each antenna through the top of the peak that blocks the line-of-sight from that antenna.

Thus far it has been assumed that the transmission between the two antennas would be approximately the same as in free space if the obstacles could be removed. This assumption is usually valid only at centimeter wavelengths, and at lower frequencies it is necessary to include the effects of waves reflected from the ground. This results in four paths, namely,  $MOQ$ ,  $MOQ'$ ,  $M'OQ$ , and  $M'OQ'$  shown in Fig. 11(b) for a single obstruction. Each of these paths is similar in form to the single path illustrated by Figure 11(a). The sum of the field intensi-

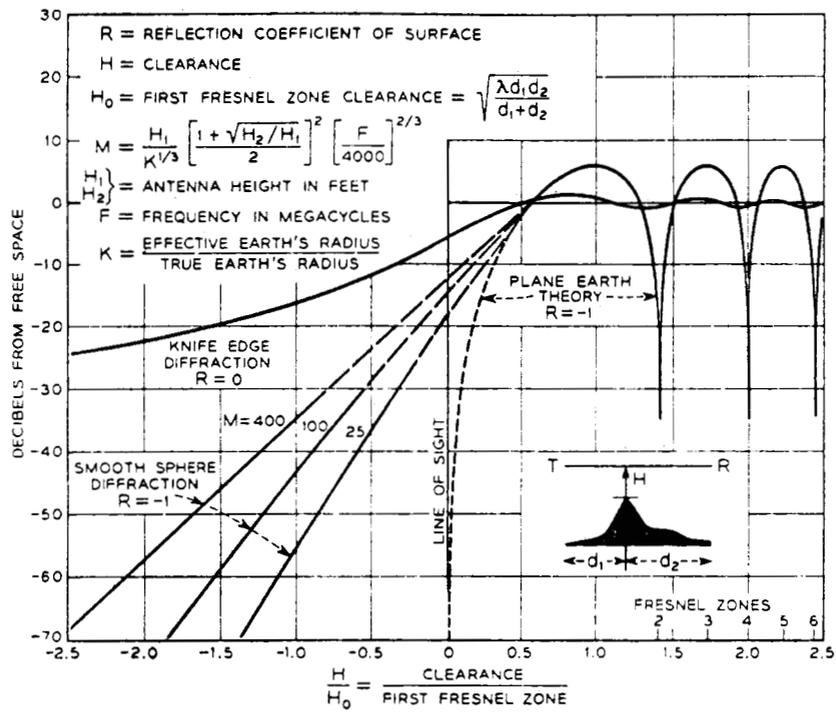


Fig. 10. Transmission loss versus clearance.

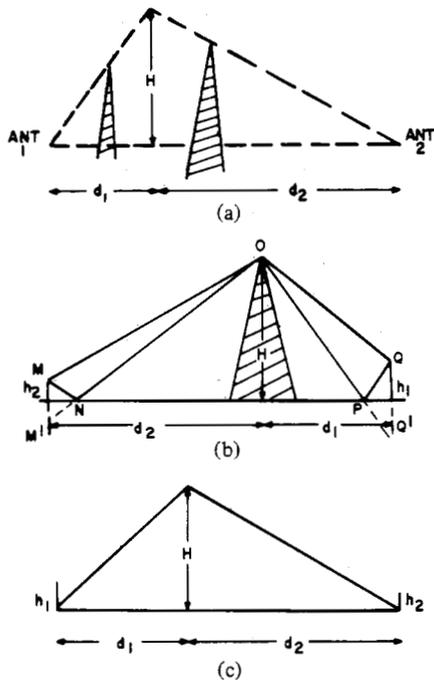


Fig. 11. Ideal profiles used in developing theory of diffraction over hills.

ties over these four paths, considering both magnitude and phase, is given by the following equation:

$$\left| \frac{E}{E_0} \right| = S_1 \left[ 1 - \frac{S_2}{S_1} e^{-j(\Delta+b)} - \frac{S_3}{S_1} e^{-j(\Delta+c)} + \frac{S_4}{S_1} e^{-j(b+c)} \right] \tag{12}$$

where

- $E$  received field intensity,
- $E_0$  free-space field intensity,
- $S_1$  magnitude of the shadow loss over path  $MOQ$ ,
- $S_2$  magnitude of the shadow loss over path  $MOQ'$ ,
- $S_3$  Magnitude of the shadow loss over path  $M'OQ$ ,
- $S_4$  Magnitude of the shadow loss over path  $M'OQ'$ ,
- $\Delta = 4\pi h_1 h_2 / \lambda(d_1 + d_2)$  radians,
- $b$  approximately equal to  $4\pi h_2 H / \lambda d_2$  rad,
- $c$  approximately equal to  $4\pi h_1 H / \lambda d_1$  rad.

This equation assumes that the reflection coefficient is  $-1$  and that the actual antenna heights are greater than the minimum effective antenna height  $h_0$ . This means that the surface wave is neglected, and the equation fails when either antenna height approaches zero. The angles  $b$  and  $c$  are phase angles associated with the diffraction phenomena, and the approximate values given above assume that  $H$  is greater than  $h_1$  or  $h_2$ . This assumption permits the shadow losses to be averaged so that  $S_1 = S_2 = S_3 = S_4 = S$ . After several algebraic manipulations (12) can be reduced to

$$\left| \frac{E}{E_0} \right| = 2(2S) \left| \sin \frac{\Delta}{2} \cos \frac{b-c}{2} + j \left( \sin^2 \frac{b+c}{4} - \sin^2 \frac{b-c}{4} \right) e^{j(\Delta/2)} \right| \tag{13}$$

where  $S$  is the average shadow loss for the four paths. This means that the shadow triangle should be drawn from a point

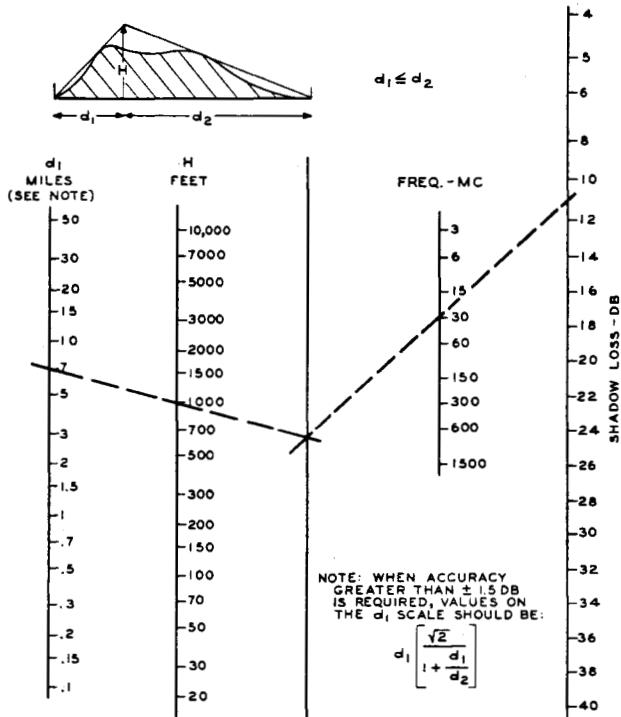


Fig. 12. Shadow loss relative to smooth earth.

midway between the location of the actual antenna and the image antenna as shown in Fig. 11(c). For small values of  $H$  this equation is approximately equal to

$$\left| \frac{E}{E_0} \right| = 2(2S) \sin \frac{\Delta}{2} \quad (14)$$

Since the field intensity over plane earth (assuming that the antenna heights are greater than the minimum effective height  $h_0$ ) is  $2E_0 \sin \Delta/2$ , the first-order effect of the hill is to add a loss of  $20 \log 2S$  which is shown by the nomogram in Fig. 12. The complete expression given in (12) indicates that, under favorable conditions, the field intensity behind sharp ridges may be greater than over plane earth. This result has been found experimentally in a few cases, but the correlation between theory and experiment is not complete. In general, the field intensity predicted by either (12) or (13) tends to be too high; that is, shadow losses rather than gains occur on most of the paths on which measured data are available. The less-approximate expression given in (14) agrees more closely with experimental data, is more conservative, and is easier to use. Consequently, it is usually assumed that the effect of obstructions to line-of-sight transmission (at least in the 30- to 150-MHz range) is to introduce the loss shown in Fig. 12 in addition to the normal loss over smooth earth for the antenna heights and distances involved. Measured results on a large number of paths in the 30- to 150-MHz range indicate that about 50 percent of the paths are within 5 or 6 dB of the values predicted on this basis. The correlation on 10 percent of the paths is no better than 10 to 12 dB, and an occasional path may differ by 20 dB.

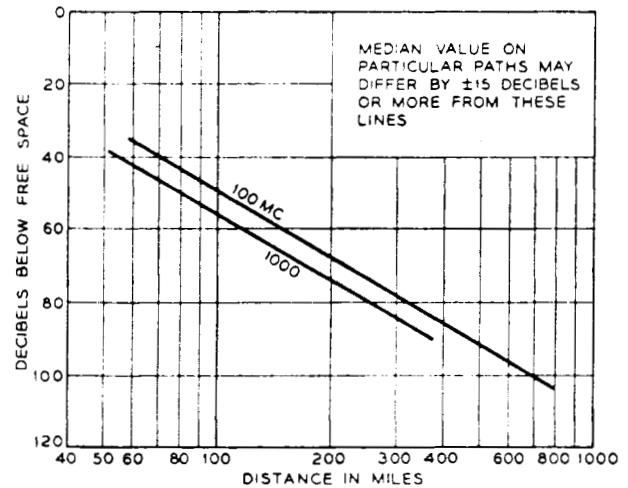


Fig. 13. Beyond-horizon transmission; median signal level versus distance.

## VII. EXPERIMENTAL DATA FAR BEYOND THE HORIZON

Most of the experimental data at points far beyond the horizon fall in between the theoretical curves for diffraction over a smooth sphere and for diffraction over a knife edge obstruction. Various theories have been advanced to explain these effects, but none has been reduced to a simple form for every day use [9]. The explanation most commonly accepted is that energy is reflected or scattered from turbulent air masses in the volume of air that is enclosed by the intersection of the beamwidths of the transmitting and receiving antennas [10].

The variation in the long term median signals with distance has been derived from experimental results and is shown in Fig. 13 for two frequencies [11]. The ordinate is in decibels below the signal that would have been expected at the same distance in free space with the same power and the same antennas. The strongest signals are obtained by pointing the antennas at the horizon along the great circle route. The values shown in Fig. 13 are essentially annual averages taken from a large number of paths, and substantial variations are to be expected with terrain, climate, and season as well as from day to day fading.

Antenna sites with sufficient clearance so that the horizon is several miles away will on the average provide a higher median signal (less loss) than shown in Fig. 13. Conversely, sites for which the antenna must be pointed upward to clear the horizon will ordinarily result in appreciably more loss than shown in Fig. 13. In many cases the effects of path length and angles to the horizon can be combined by plotting the experimental results as a function of the angle between the lines drawn tangent to the horizon from the transmitting and receiving sites [12].

When the path profile consists of a single sharp obstruction that can be seen from both terminals, the signal level may approach the value predicted by the knife edge diffraction theory [13]. While several interesting and unusual cases have been recorded, the knife edge or "obstacle gain" theory is not applicable to the typical but only to the exceptional paths.

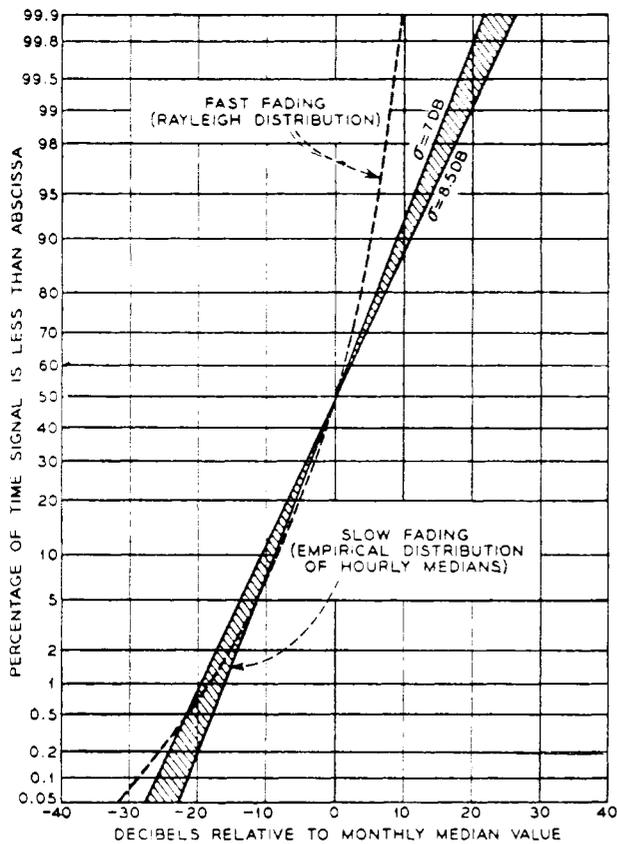


Fig. 14. Typical fading characteristics at points far beyond horizon.

As in the case of line-of-sight transmission the fading of radio signals beyond the horizon can be divided into fast fading and slow fading. The fast fading is caused by multipath transmission in the atmosphere, and for a given size antenna the rate of fading increases as either the frequency or the distance is increased. This type of fading is much faster than the maximum fast fading observed on line-of-sight paths, but the two are similar in principle. The magnitude of the fades is described by the Rayleigh distribution.

Slow fading means variations in average signal level over a period of hours or days, and it is greater on beyond horizon paths than on line-of-sight paths. This type of fading is almost independent of frequency and seems to be associated with changes in the average refraction of the atmosphere. At distances of 150 to 200 mi the variations in hourly median value around the annual median seem to follow a normal probability law in decibels with a standard deviation of about 8 dB. Typical fading distributions are shown in Fig. 14. The median signals levels are higher in warm humid climates than in cold dry climates, and seasonal variations of as much as  $\pm 10$  dB or more from the annual median have been observed [14].

Since the scattered signals arrive with considerable phase irregularities in the plane of the receiving antenna, narrow-beamed (high gain) antennas do not yield power outputs proportional to their theoretical area gains. This effect has sometimes been called loss in antenna gain but it is a propagation effect and not an antenna effect. On 150- to 200-mi paths this loss in received power may amount to 1 or 2 dB for a 40-dB

gain antenna, and perhaps 6 to 8 dB for a 50-dB antenna. These extra losses vary with time, but the variations seem to be uncorrelated with the actual signal level.

The bandwidth that can be used on a single radio carrier is frequently limited by the selective fading caused by multipath or echo effects. Echoes are not troublesome as long as the echo time delays are very short compared with one cycle of the highest baseband frequency. The probability of long delayed echoes can be reduced (and the rate of fast fading can be decreased) by the use of narrow beam antennas both within and beyond the horizon [13], [15]. Useful bandwidths of several megahertz appear to be feasible with the antennas that are needed to provide adequate signal-to-noise margins. Successful tests of television and of multichannel telephone transmission have been reported on a 188-mi path at 5 000 MHz [16].

The effects of fast fading can be reduced substantially by the use of either frequency or space diversity. The frequency or space separation required for diversity varies with time and with the degree of correlation that can be tolerated. A horizontal (or vertical) separation of about 100 wavelengths is ordinarily adequate for space diversity on 100- to 200-mi paths. The corresponding figure for the required frequency separation for adequate diversity seems likely to be more than 20 MHz.

### VIII. EFFECTS OF NEARBY HILLS—PARTICULARLY ON SHORT PATHS

The experimental results on the effects of hills indicate that the shadow losses increase with the frequency and with the roughness of the terrain [17]. An empirical summary of the available data is shown in Fig. 15. The roughness of the terrain is represented by the height  $H$  shown on the profile at the top of the chart. This height is the difference in elevation between the bottom of the valley and the elevation necessary to obtain line-of-sight from the transmitting antenna. The right-hand scale in Fig. 15 indicates the additional loss above the expected over plane earth. Both the median loss and the difference between the median and the 10 percent values are shown. For example, with variations in terrain of 500 ft, the estimated median shadow loss at 450 MHz is about 20 dB, and the shadow loss exceeded in only 10 percent of the possible locations between points  $A$  and  $B$  is about  $20 + 15 = 35$  dB. It will be recognized that this analysis is based on large-scale variations in field intensity and does not include the standing-wave effects which sometimes cause the field intensity to vary considerably within a few feet.

### IX. EFFECTS OF BUILDINGS AND TREES

The shadow losses resulting from buildings and trees follow somewhat different laws from those caused by hills. Buildings may be more transparent to radio waves than the solid earth, and there is ordinarily much more back scatter in the city than in the open country. Both of these factors tend to reduce the shadow losses caused by the buildings, but on the other hand

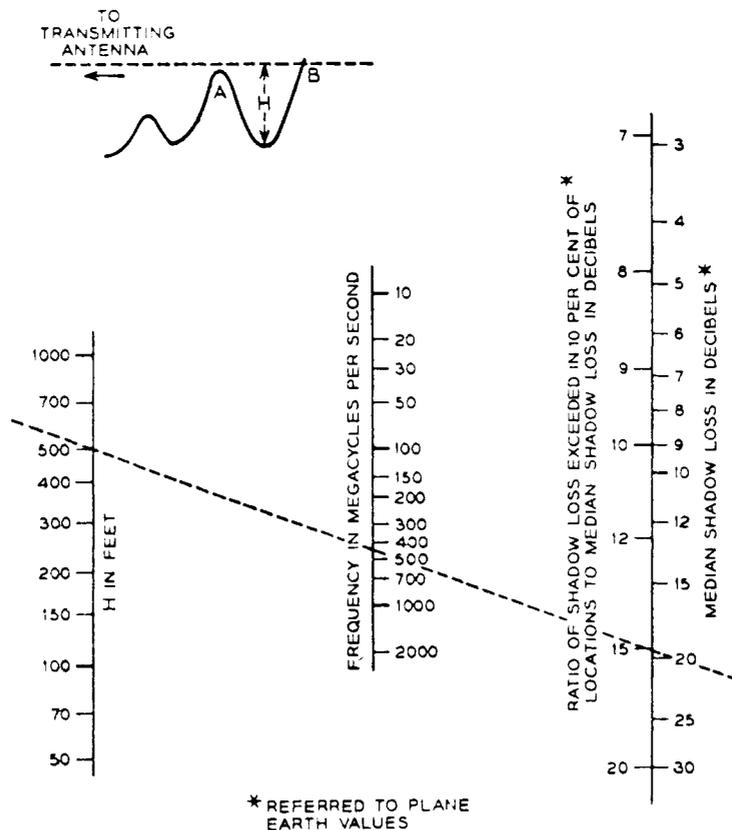


Fig. 15. Estimated distribution of shadow losses.

the angles of diffraction over or around the buildings are usually greater than for natural terrain. In other words, the artificial canyons caused by buildings are considerably narrower than natural valleys, and this factor tends to increase the loss resulting from the presence of buildings. The available quantitative data on the effects of buildings are confined primarily to New York City. These data indicate that in the range of 40 to 450 MHz there is no significant change with frequency, or at least the variation with frequency is somewhat less than that noted in the case of hills [18]. The median field intensity at street level for random locations in Manhattan (New York City) is about 25 dB below the corresponding plane earth value. The corresponding values for the 10 percent and 90 percent points are about 15 and 35 dB, respectively. Typical values of attenuation through a brick wall are from 2 to 5 dB at 30 MHz and 10 to 40 dB at 3000 MHz depending on whether the wall is dry or wet. Consequently, most buildings are rather opaque at frequencies of the order of thousands of megahertz.

When an antenna is surrounded by moderately thick trees and below tree-top level, the average loss at 30 MHz resulting from the trees is usually 2 or 3 dB for vertical polarization and is negligible with horizontal polarization. However, large and rapid variations in the received field intensity may exist within a small area, resulting from the standing-wave pattern set up by reflections from trees located at a distance of several wavelengths from the antenna. Consequently, several nearby locations should be investigated for best results. At 100 MHz the average loss from surrounding trees may be 5 to 10 dB for ver-

tical polarization and 2 or 3 dB for horizontal polarization. The tree losses continue to increase as the frequency increases, and above 300 to 500 MHz they tend to be independent of the type of polarization. Above 1000 MHz trees that are thick enough to block vision are roughly equivalent to a solid obstruction of the same overall size.

## X. MINIMUM ALLOWABLE INPUT POWER

The effective use of the preceding data for estimating the received power requires a knowledge of the power levels needed for satisfactory operation since the principal interest is in the signal-to-noise ratio. The signal level required at the input to the receiver depends on several factors including the noise introduced by the receiver (called first-circuit or set noise), the type and magnitude of any external noise, the type of modulation, and the desired signal-to-noise ratio. A complete discussion of these factors is beyond the scope of this paper, but the fundamental limitations are listed below in order to show the order of magnitude. The theoretical minimum noise level is that set by the thermal agitation of the electrons, and its root-mean-square power in dBW is  $-204$  plus  $10 \log$  (bandwidth) where (bandwidth) is approximately equal to twice the audio (or video) bandwidth [19]. The set noise of a typical receiver may be 5 to 15 dB higher than the theoretical minimum noise. The lower values in this range of "excess" noise are more likely to be met in the VHF range while the higher values are more probable in the SHF range. This means that the set noise in a 3000-Hz audio band

TABLE I  
FIGURES TO USE

Type of Terrain	Both Antennas Lower in Height than Shown on Fig. 6	One or Both Antenna Heights Higher than Shown on Fig. 6	
		Within Line-of-Sight	Beyond Line-of-Sight
Plane earth	Fig. 4 or 1	Figs. 4 or 1 or 2	Figs. 1 or 2 and 7
Smooth earth	Figs. 4 and 6	Figs. 4 or 1 or 2	Figs. 1 or 2
Irregular terrain	Figs. 4, 6, 7 and 12	Figs. 4 and 12 or 1 or 2 and 9	Figs. 1 or 2, 7 and 12

may be -151 to -161 dBW. Measured data indicate that the carrier power needs to be 12 to 20 dB higher than the noise power to provide an average signal-to-noise ratio that is sufficient for moderate intelligibility. This assumes that the modulation level has been adjusted so that most of the peaks of speech power can be transmitted without causing overmodulation in the transmitter. It follows that the required input power for a single-channel voice circuit is of the order of -140 dBW, which is roughly equivalent to 1 μV across a 70 Ω input resistance. This limiting input power is approximately correct (within 3 or 4 dB) for both amplitude and frequency modulation since the radio-frequency signal-to-noise ratio must be above that required for marginal operation before the use of frequency modulation can provide appreciable improvement in the audio signal-to-noise ratio.

The input power must be greater than -140 dBW when circuits of above marginal quality or greater bandwidth are desired and when external noise rather than set noise is controlling. Man-made noise is frequently controlling at 30 MHz, but is less serious at 150 MHz. Above 500 MHz set noise is almost always controlling. For circuits requiring a high degree of reliability a margin should also be included for the fading range to be expected during adverse weather conditions.

XI. SUMMARY AND EXAMPLES

In any given radio propagation problem some of the factors described above are important, while others can be neglected. Table I indicates the figures that apply to any given situation.

Whenever Fig. 4 is used, reference should be made to Figs. 1 and 3 as a check on its proper use. When Fig. 12 is used in determining the effects of hills, the profile is usually drawn on rectangular coordinates (neglecting the earth's curvature), and the shadow triangle is drawn to the base of the antenna half-way between the antenna and its image). Curved coordinates are sometimes used, but they are not necessary since the loss caused by the curvature of the earth is either negligible or has already been considered in Figs. 6 or 7. Fig. 9 is used for determining the first-zone clearance and for estimating the shadow losses when the transmission without the obstruction is expected to be the same as in free space. In the latter case, the shadow triangle is drawn from the actual antennas, and curved coordinates are useful since the curvature of the earth should be included in the profile. Various examples of the use of these figures have been given during the discussion of each individual chart, but further examples may help to illustrate the

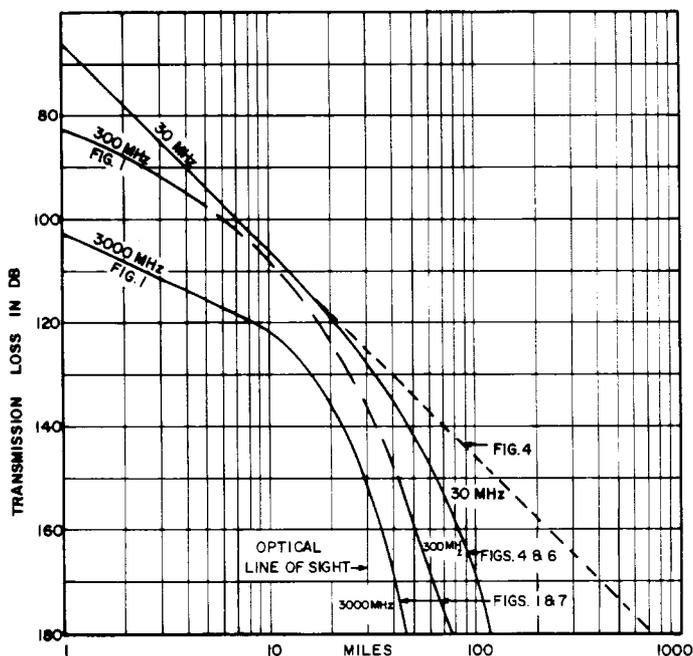


Fig. 16. Transmission over smooth earth at 30, 300, and 3000 MHz; half-wave dipoles at 250 and 30 ft.

relations between the various figures. Assume a transmitting dipole located 250 ft above the ground and a receiving dipole on a 30 ft mast. The estimated transmission losses at 30, 300, and 3000 MHz over smooth earth are shown in Fig. 16 for  $k = 4/3$  for various distances between these two dipoles. The solid lines indicate values obtained from the figures, and the dashed lines show the region where some interpolation is required.

The received power depends on the radiated power and the antenna-gain characteristics as well as on the transmission loss between dipoles. A typical 30-MHz transmitter may radiate 250 W so that at a distance of 30 mi the received power is  $10 \log 250 - 129 = -105$  dBW. (The value of 129 dB is obtained from Fig. 16.) Similarly, a 300-MHz transmitter may radiate 50 W from a 5-dB gain antenna, and when a 5-dB gain receiving antenna is used the estimated received power at 30 mi is  $10 \log 50 + 5 + 5 - 137 = -110$  dBW. At 3000 MHz the radiated power may be 0.1 W and antenna gains of 28 dB each are not uncommon, so the received power at 30 miles is  $10 \log 0.1 + 28 + 28 - 152 = -106$  dBW. (The values of radiated power used in this example are not the maximum continuous-wave powers that can be obtained but the down-

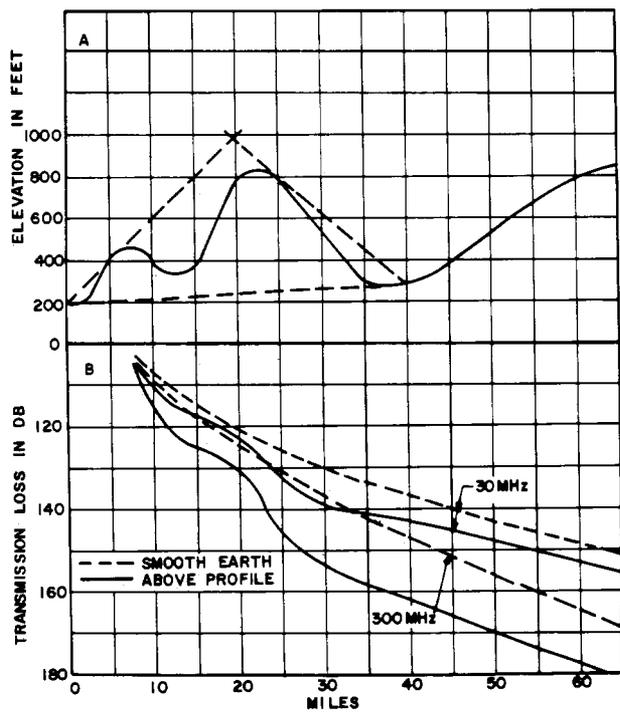


Fig. 17. Transmission loss over irregular terrain at 30 and 300 MHz; half-wave dipoles of 250 and 30 ft.

TABLE II  
SHADOW-LOSS COMPUTATIONS

Miles from Transmitter	H (feet)	d <sub>1</sub> (miles)	Shadow Loss in dB Obtained from Fig. 12	
			30 MHz	300 MHz
12.5	200	4.5	3.5	9
20	70	6.5	2	4
25	310	4.5	5.5	13
35	750	14.5	7	16
40	760	20	6	14
50	610	18	5	12
60	490	16.5	4.5	11

ward trend with increasing frequency is a characteristic of the available tubes.)

Over irregular terrain it is assumed that the shadow loss based on knife-edge diffraction theory is to be added to the transmission loss obtained from smooth-earth theory. The computation of shadow losses for the profile shown in Fig. 17 (a) is given in Table II.

The estimated transmission loss for 30 and 3000 MHz including the shadow loss from Table II is shown by the solid lines on Fig. 17(b), while the dashed lines are the corresponding values over smooth earth taken from Fig. 17. Super-

imposed on these average values will be unpredictable variations of  $\pm 6$  to 10 dB resulting from the effects of trees and buildings and from profile irregularities that were smoothed out in drawing the profile shown in Fig. 17(a).

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