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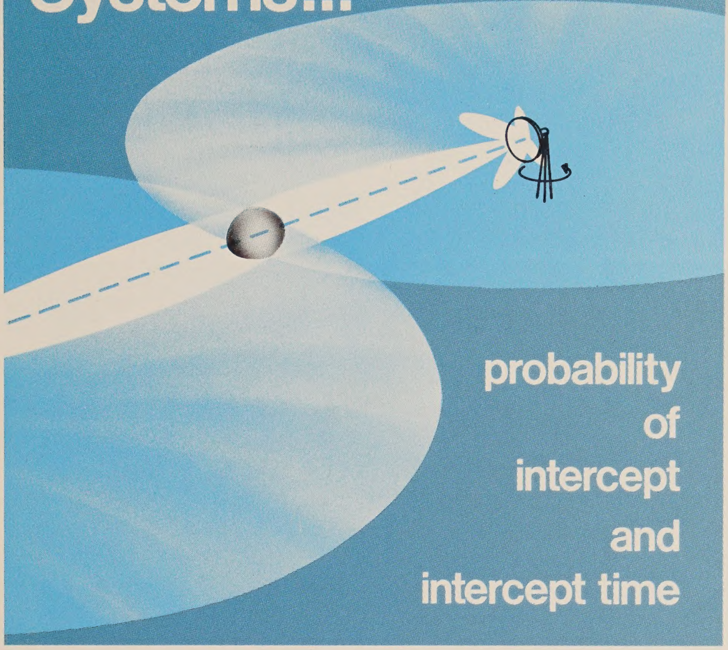
EW Acquisition Systems...



probability of intercept and intercept time

Successful deployment of reconnaissance and surveillance systems demand the interception and detection of target emissions. The system, as the term is used here, includes the target as well as the receiver, and generally contains functions or parameters which are in some sense intermittent. Scanning antennas, sweeping or stepping receivers, frequency hopping emitters and blinking jammers are examples of systems exhibiting intermittent functions. The major impact of these intermittencies is on the time necessary for reconnaissance or surveillance receivers to intercept an emitter. If the system has more than a single intermittent function, the resulting interception becomes one of probability, rather than being uniquely defined by the system parameters.

This article deals with solutions to the problems of intercept probability and time of intercept for surveillance and reconnaissance systems. First, the probability that an intercept between an emitter and acquisition receiver is derived which gives a solution in terms of the system's operational parameters, such as, antenna rotation rate and beamwidth, and receiver bandwidth and pass-band. Secondly, the probable time an acquisition receiver must wait to achieve an intercept is found in terms of the system's operational parameters, even if the emitter parameters are not know exactly.

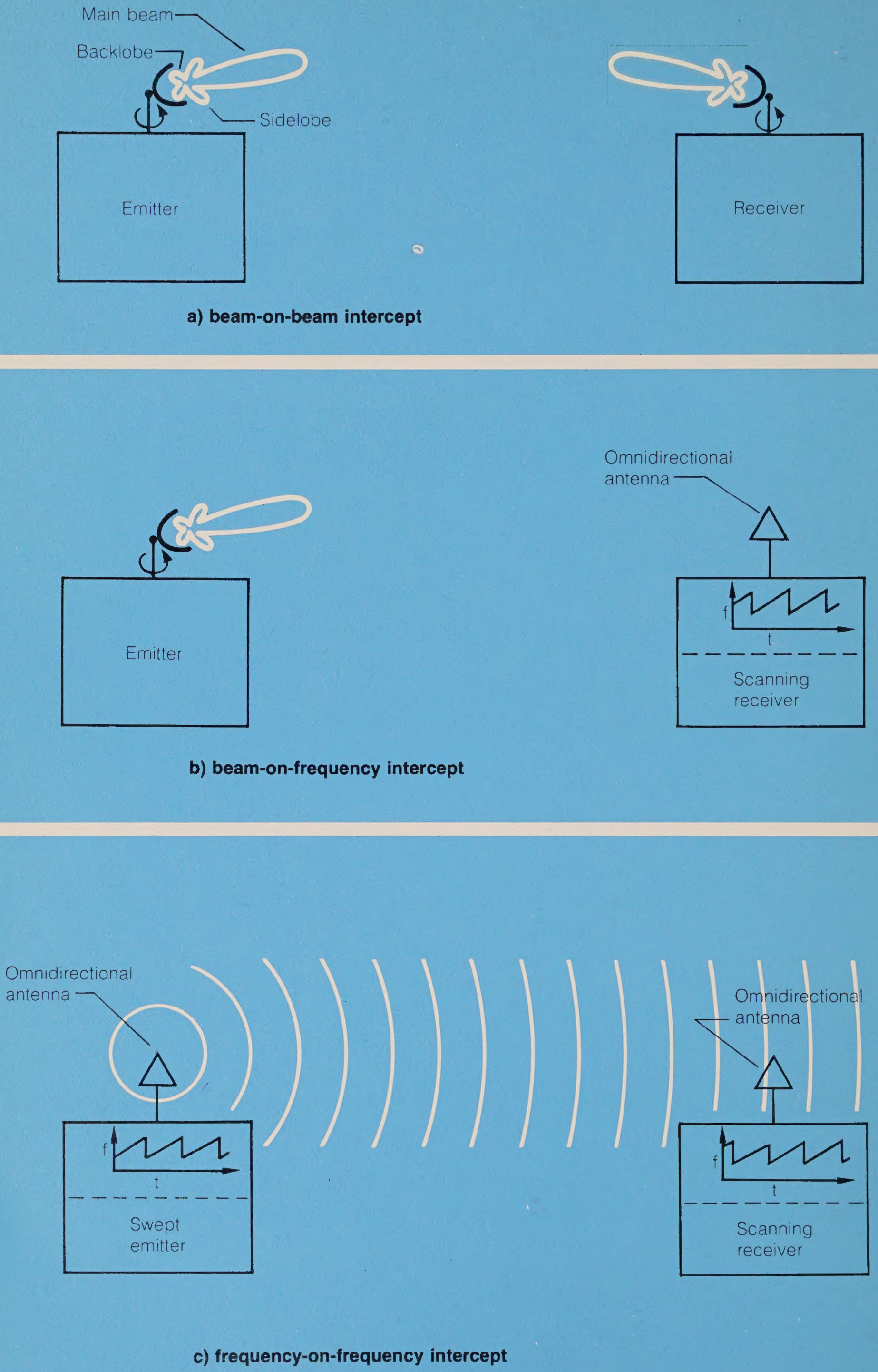


Figure 1. Three different emitter-receiver intercept possibilities.

Interception and Time of Intercept

Interception is a distinct process from detection. Detection is a function of signal level, noise level and threshold of an acquired activity, whereas, interception requires the time-coincidence of two or more activities. Before detection may occur, interception must have occurred.

Questions pertaining to interception and the time required to intercept are usually framed in one of two equivalent ways:

1. What is the probability an intercept will occur within a specified time after initiation of a particular activity?
2. What is the observation time required after initiation of a particular activity to be assured a specific probability of intercept will be attained?

Like the distinction between the processes of interception and detection, a distinction is also made between the intercept and observation time associated with an intercept. Intercept time is normally referred to as the duration of time from a fixed reference (for example, the start of an activity) to the coincidence of two or more activities. This time is not specific, but depends on the occurrence of the intercept. Observation time also refers to the duration of time from a fixed reference to a coincidence, except that it implies a specific probability of intercept. It is the time required of an observation in order to achieve a preselected probability of intercept.

Intercept Probability and Intercept Time Applications

To achieve an optimum probability of intercept, it is necessary to consider those system parameters, such as antenna beamwidth and rotation rate, which have the greatest influence on the time required to achieve an intercept. For example, increasing the receiving antenna's rotation rate may reduce the time to an intercept, but at a decreasing rate to the point where it becomes uneconomical. A trade-off between beamwidth and rotation rate may also be made to achieve a specific probability of intercept or a specified intercept time. However, an emitter's beamwidth and rotation rate may not be under the designer's control, thus restricting most predictions to statistical relationships between intercept probability and intercept time as functions of the system parameters.

A few of the more common applications that illustrate the problems involved are shown in Figure 1, and include:

- Beam-on-Beam Intercept (Fig. 1a). Both the emitter and acquisition receiver are using rotating, directional antennas. An intercept occurs only when the main beams of both antennas are pointing at one another.
- Beam-on-Frequency Intercept (Fig. 1b). Either the emitter or acquisition receiver is using a rotating, directional antenna, and the receiver is scanning in frequency. An intercept occurs when the receiving antenna is aligned along the emitter-receiver direction and the receiver pass-band encompasses the frequency of the emitter.
- Frequency-on-Frequency Intercept (Fig. 1c). The emitter is radiating continuously, but is sweeping or jumping in frequency; or is fixed in frequency, but pulsed. The acquisition receiver also is sweeping or stepping in frequency. An intercept occurs when the emitter is "on" and its frequency is within the receiver's pass-band.

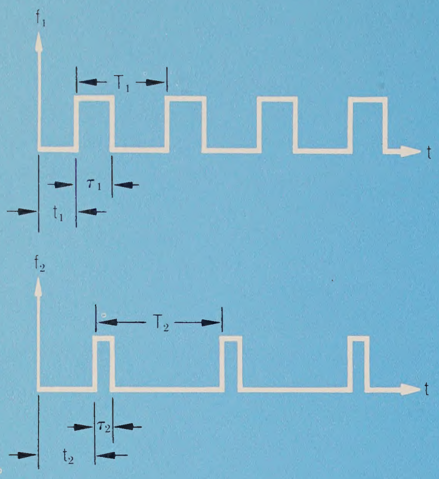


Figure 2. Two independent window functions having different widths (τ), periods (T) and starting times (t) may be used to represent either the emitter or acquisition receiver.

General Results to Determine Probability of Intercept

To answer the question, "What is the probability of an intercept during a specified time?", consider two, independent "window functions" as illustrated in Figure 2. These two functions may represent the activities of an emitter or acquisition receiver in the previous intercept examples, and may have different widths (τ), periods (T) and starting times (t).

The parameters (τ, T, t) are time-related to the operational parameters used to describe the rotating antenna or scanning receiver. For the rotating antenna, the time required of the antenna to rotate one revolution (period) is:

$$T\ (\text{seconds}) = \frac{360^\circ}{S}$$

where S is the antenna rotational rate (degrees per second). The time the antenna's main beam points in a particular direction is:

$$\tau\ (\text{seconds}) = \frac{\theta}{S}$$

where θ is the antenna beamwidth (degrees).

For the scanning receiver, T is the time required of the receiver to scan across a frequency band (ΔHz), and the time that a particular frequency remains in the receiver pass-band is:

$$\tau\ (\text{seconds}) = \frac{BT}{\Delta}$$

where B (Hz) is the bandwidth of the receiver pass-band.

An intercept occurs when the window functions overlap, or for the beam-on-beam intercept example, when the main beams of both antennas are in line. An intercept occurs in the first period when

$$-\tau_1 \leq t_1 - t_2 \leq \tau_2.$$

The times τ_1 and τ_2 are the length of time the window functions are open, or duration of activity for the emitter or acquisition receiver. If the starting times are equal ($t_1 - t_2 = 0$), the two window functions are time-coincident, and an intercept will occur immediately during the first period when both durations overlap. If the above condition is met, at least a partial overlap will occur within the first period of the more rapidly occurring function; otherwise a coincidence will not occur. This may be shown by noting that if the activity τ_2 in the observation period T_2 occurs before the activity τ_1 stops, then an intercept will occur. Also, if the activity τ_2 in the observation period T_2 ends after the activity τ_1 has begun, then an intercept results.

$T_1 \le T_2$			$T_2 \le T_1$		
$P_{12}(T_1)$			$P_{12}(T_2)$		
For	$\tau_2 \le T_1$	$T_1 \le \tau_2$	$\tau_1 \le T_2$	$T_2 \le \tau_1$	For
$\tau_1 \le T_2 - T_1$	$\frac{1}{T_2}\left[\tau_1 + \tau_2\left(1 - \frac{\tau_2}{2T_1}\right)\right]$	$\frac{1}{T_2}\left(\tau_1 + \frac{T_1}{2}\right)$	$\frac{1}{T_1}\left[\tau_2 + \tau_1\left(1 - \frac{\tau_1}{2T_2}\right)\right]$	$\frac{1}{T_1}\left(\tau_2 + \frac{T_2}{2}\right)$	$\tau_2 \le T_1 - T_2$
$T_2 - T_1 \le \tau_1$	$1 - \frac{(T_1-\tau_2)^2 + (T_2-\tau_1)^2}{2T_1T_2}$	$1 - \frac{(T_2-\tau_1)^2}{2T_1T_2}$	$1 - \frac{(T_2-\tau_1)^2 + (T_1-\tau_2)^2}{2T_1T_2}$	$1 - \frac{(T_1-\tau_2)^2}{2T_1T_2}$	$T_1 - T_2 \le \tau_2$
Probability of intercept for a time T $P_{12}(T) = 1 - [1 - P_{12}(T_1)]^{T/T_1}$			Probability of intercept for a time T $P_{12}(T) = 1 - [1 - P_{12}(T_2)]^{T/T_2}$		
Observation time for a desired probability of intercept (P_{oi}) $T_o = T_1 \cdot \frac{\ln(1 - P_{oi})}{\ln[1 - P_{12}(T_1)]}$			Observation time for a desired probability of intercept (P_{oi}) $T_o = T_2 \cdot \frac{\ln(1 - P_{oi})}{\ln[1 - P_{12}(T_2)]}$		

Table 1. Probability of intercept (P_{12}) and observation time (T_o) in terms of the window function parameters τ, T and t.

Since the two starting times are assumed independent of each other, the probability of a coincidence during the first period of the window function with the shorter period can be found. (An outline of this derivation is given in the Appendix, page 10.) Table 1 expresses this probability, P_{12}, in terms of the window function parameters for the cases $T_1 \le T_2$ and $T_2 \le T_1$. Either the emitter or acquisition system may be designated as function (1), the other being denoted as function (2).

Once the value of $P_{12}(T_1)$ or $P_{12}(T_2)$, whichever is appropriate, is determined from the conditions, the probability that at least one intercept will occur in a time T is given by the equation:

$$P_{12}(T) = 1 - \left[1 - P_{12}(T_1)\right]^{T/T_1}$$

if T_1 is the more rapidly occurring activity ($T_1 \le T_2$), or by the equation:

$$P_{12}(T) = 1 - \left[1 - P_{12}(T_2)\right]^{T/T_2}$$

if T_2 is the more rapidly occurring activity ($T_2 \le T_1$).

These two equations answer the question, "What is the probability of intercept within a specified time T?"

Observation Time

The answer to the second question, "What is the observation time required to assure a specific probability of intercept?" is given by the equation:

$$T_o = T_1 \cdot \frac{\ln(1 - P_{oi})}{\ln\left[1 - P_{12}(T_1)\right]}$$

if T_1 is the more rapidly occurring activity ($T_1 \le T_2$), or by the equation:

$$T_o = T_2 \cdot \frac{\ln(1 - P_{oi})}{\ln\left[1 - P_{12}(T_2)\right]}$$

if T_2 is the more rapidly occurring activity ($T_2 \le T_1$). In both equations, P_{oi} is the pre-specified probability of intercept, as a decimal fraction.

Solutions to System Applications

The relationships for probability of intercept and observation time do not, per se, answer the questions of "How fast to rotate an antenna?" or "What receiver scan rate to use?" to achieve a probability of intercept or observation time, but if values for rotation and scan rates are used, the probability of intercept and observation time can be obtained from which trade-offs may be made. For a given set of system parameters, the data obtained from Table 1 are readily used to calculate probability or observation time as illustrated in the following two examples.

- Beam-on-Beam Intercept Example. A receiving antenna has a beamwidth of 5° and is rotating at 60 rpm (360 degrees per second). The emitting antenna's beamwidth and rotation rate under observation are not exactly known. Determine the observation time (T_o) required to achieve a 50% probability of intercept for variations in the emitting antenna's beamwidth and rotation rate.

Since the emitter's operational parameters are unknown, the observation time can be found only by assuming a range of values for the emitting antenna's beamwidth and rotation rate and examining the results. Assuming the receiving antenna period occurs more rapidly, then $T_a \leq T_e$, where the subscript (a) denotes the receiving system and (e) denotes the emitter. For $T_1 \leq T_2$ (i.e., $T_a \leq T_e$ in this example), the antenna period is:

$$T_a = \frac{360°}{360°/\text{Sec.}} = 1 \text{ Sec}$$

and

$$\tau_a = \frac{5°}{360°/\text{Sec.}} = .014 \text{ Sec.}$$

Substituting values for θ_e and S_e into emitter relationships

$$T_e = \frac{360°}{S_e}$$

and

$$\tau_e = \frac{\theta_e}{S_e}$$

results in a corresponding value for the probability of $P_{12}(T_a)$ in Table 1 ($T_1 \leq T_2$).

Curves of the observation time for variations in emitter beamwidth and rotation rate are shown in Figure 3. The observation time required for a 50% probability of intercept increases for narrower emitting antenna beamwidths and slower emitting antenna rotation rates. Similarly, for a given set of emitter parameters, the observation time increases for narrower acquisition antenna beamwidths and slower rotation rates.

- Beam-on-Frequency Intercept Example. A surveillance reciever scans a 2000 MHz band with a 20 MHz pass-band. Determine the time required to achieve a 50% probabil-

Acquisition antenna	Emitting antenna	
$T_a = 1$ Sec $\tau_a = .014$ Sec	$T_e = \frac{360°}{S_e}$ $\tau_e = \frac{\theta_e}{S_e}$	$T_o = T_a \cdot \frac{\ln(1 - P_{oi})}{\ln[1 - P_{12}(T_a)]}$

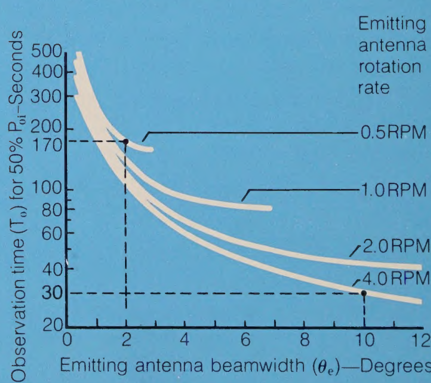


Figure 3. Curves of the observation time (T_o) for a 50% probability of intercept (P_{oi}) between an emitter and a receiving antenna which operates at a beamwidth of 5° (θ_a) and rotation rate of 360° per second (S_a).

ity of intercept for variations in an emitter beamwidth and rotation rate if the time required for the receiver to scan the 2000 MHz band is 2 seconds, and again if the time is .25 seconds.

If the receiver scan time (T_a) is assumed to be shorter, and has either the value 2 seconds or .25 seconds, then the time that a particular emitter frequency remains in the receiver pass-band is:

$$\tau_a = \frac{20\text{ MHz}}{2000\text{ MHz}} \cdot 2 = .02\text{ Sec}$$

or

$$\tau_a = \frac{20\text{ MHz}}{2000\text{ MHz}} \cdot .25 = .0025\text{ Sec.}$$

The emitting antenna times T_e and τ_e again depend on the values chosen for emitter beamwidth and rotation rate. For an emitter operating CW, or with a pulse repetition frequency (PRF) sufficient to cause at least one emitter pulse to occur during the time the receiver remains at a particular frequency (.02 or .0025 seconds), the observation time for a 50% probability of intercept is as shown in the curves of Figure 4.

Nomograph Aids to Simplify Results

Once the observation time is calculated for a specific probability of intercept, the observation times required for different probability of intercepts may be determined directly from the nomograph shown in Figure 5. A straight line from the original probability of intercept to the desired probability of intercept intersects the scale of observation time ratios. For the example intercept line shown in Figure 5, it takes approximately 6.5 times longer to achieve a desired 90% probability of intercept than it does to achieve the original 30% probability of intercept.

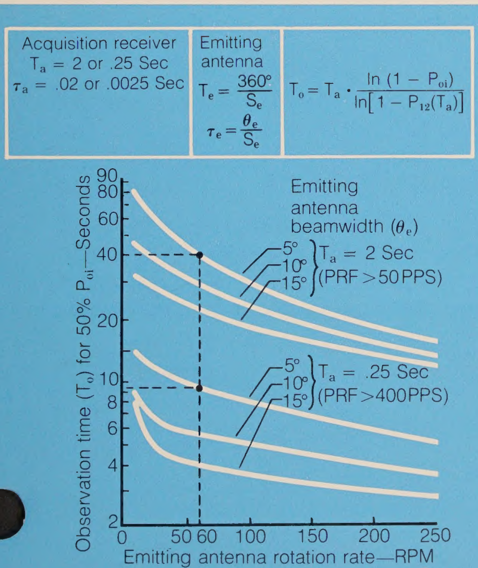


Figure 4. Curves of the observation time (T_o) required to achieve a 50% probability of intercept (P_{oi}) between an emitter and a surveillance receiver which scans a 2000 MHz band in 2 seconds, and again at .25 seconds (T_a).

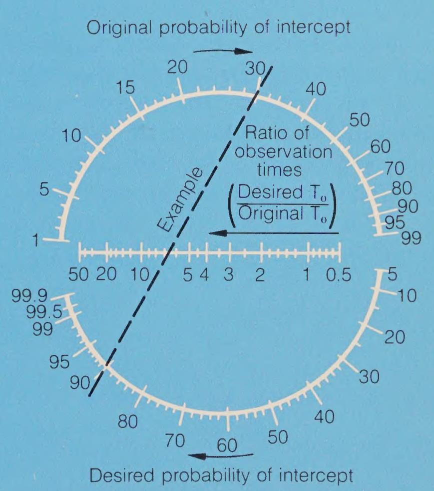


Figure 5. The change in observation time required to achieve a desired probability is simplified by using a probability nomograph. The nomograph is obtained by taking the ratio of the equation for T_o for different values of P_{oi}.

Extending the Probability to Complex Intercepts

Until now, the discussion has focused on simple intercepts: those involving only two window functions. In actual applications, however, an intercept may require the simultaneous coincidence of several window functions. For example, a system may be comprised of a pulsed emitter connected to its rotating antenna, and a scanning receiver connected to its own rotating antenna. In this application, four window functions are involved; that is, the pulsed waveform, the emitting antenna, the receiver frequency scan, and the receiving antenna.

The probability that a simple intercept of two functions will occur within a specified time (T) is now denoted by the double subscript form for probability, $P_{ij}(T)$, where the subscripts (i) and (j) represent the two functions. For any pair of functions (i, j), and for a specified time T, the probability $P_{ij}(T)$ may be computed using the appropriate formula in Table 1.

A complex intercept event requires the mutual coincidence of all the functions involved. If there are N window functions, there are N (N − 1)/2 unique pairs, and the overall probability of intercept is the product of the probability of intercept for each of the pairs. For the case of four window functions (numbered 1-4), the unique pairs are (1, 2), (1, 3), (1, 4), (2, 3), (2, 4) and (3, 4). Thus, the overall probability of intercept for a specified time T is:

$$P_{1234}(T) = P_{12}(T) \cdot P_{13}(T) \cdot P_{14}(T) \cdot P_{23}(T) \cdot P_{24}(T) \cdot P_{34}(T)$$

The observation time necessary to achieve a specific probability of intercept is not calculated as readily as for the simple intercept, since the equation's form does not lend itself conveniently to solve for T_0 in closed form. It is necessary to solve for T_0 iteratively on a computer, or to create a table of $P_{12...N}(T)$ versus T and interpolate the results to obtain the value of T_0 corresponding to a particular value of $P_{12...N}$.

An example which illustrates a complex intercept application is comprised of a rotating emitting antenna, a rotating receiving antenna, and a scanning receiver. Parameters of this complex intercept application are shown in Table 2. If an intercept time of 300 seconds is specified, the probabilities found using the simple probability of intercept formulas in Table 1 are:

$$P_{12}(300) = 0.896934$$
$$P_{13}(300) = 0.637005$$
$$P_{23}(300) = 0.999690$$

where P_{12} is an emitter beam-on-receiver beam intercept, P_{13} is an emitter beam-on-receiver frequency intercept, and P_{23} is a receiver beam-on-receiver frequency intercept. Thus, the probability of intercept for the triple window function with a 300 second intercept time is:

$$P_{123}(300) = 0.5712$$

or 57.12%.

System Trade-Offs

For the system designer, considerations of intercept probability or requi-

System	System parameters	Window function parameters
Emitting Antenna	$S = 24°/\text{Sec}$ $\theta = 2°$	$T_1 = 15$ Sec $\tau_1 = .083$ Sec
Receiving Antenna	$S = 360°/\text{Sec}$ $\theta = 12°$	$T_2 = 1$ Sec $\tau_2 = .033$ Sec
Scanning Receiver	$T = 2$ Sec $\Delta/B = 1/100$	$T_3 = 2$ Sec $\tau_3 = .020$ Sec

Table 2. Operational parameters of a system comprising a complex intercept application.

site observation time are important factors, along with the more conventional—and more familiar—parameters such as sensitivity, dynamic range, weight, cost, power consumption, etc.

One of the first decisions the system engineer may have to make is the type of system he will specify: broad-band or narrow-band. To illustrate how observation time considerations might influence this trade-off, consider the choice between a broad-band receiver and a scanning, narrow-band receiver, where both systems cover the same frequency band and use the same rotating, directive acquisition antenna to maximize sensitivity. The data for this comparison are obtained from the observation times of the beam -on-beam and beam-on-frequency intercept examples shown in Figures 3 and 4. The time required to obtain an acquisition varies radically from system to system and depends on emitter signal strength and receiving system sensitivity.

For an extremely strong emitter, either type of acquisition system can detect the emitting antenna's side and backlobes. As a result, intercept time required for the broad-band system is not longer than one rotation of the receiving antenna. If the frequency scan rate of the scanning, narrow-band receiver is high enough to complete a band scan within each antenna beamwidth dwell time (i.e., if the receiver scan is $\leq \theta/S$), the two systems' intercept time is comparable. Otherwise, the scanning receiver intercept time is that required for a coincidence of the frequency scan window and the antenna scan window.

For weak emitters, a conventional broad-band system cannot detect the emitter at all because of its poorer sensitivity. Clearly, the frequency scanning system's higher sensitivity is the only viable solution.

For moderately strong emitters, a bona fide trade-off exists. In this case, the higher sensitivity of the scanning receiver allows detection of the emitting antenna's side and backlobes. Thus, the intercept is that of the receiving antenna's beam with the receiver's frequency scan. However, the broad-band system's lower sensitivity requires an intercept of the receiving antenna's beam with the emitting antenna's beam to obtain a detection. Since the intercept mechanism is different for the two receivers (viz beam-on-frequency for the scanning receiver versus beam-on-beam for the broad-band receiver) one would expect observation times to be different for the two systems.

For a 50% probability of intercept, observation time for the broad-band system is obtained from Figure 3 for an emitting antenna beamwidth range of 2°–10°, and a rotation rate of .5–4 rpm. Similarly, observation time for the narrow-band system is obtained from Figure 4 for a receiving antenna beamwidth of 5°, and a rotation rate of 60 rpm. Table 3 shows a comparison of the 50% P_{oi} times.

Acquisition system	Observation time
Broad-band receiver (Fig. 3)	30–170 Sec
Scanning, narrow-band receiver (Fig. 4)	
2 Second scan period	40 Sec
0.25 Second scan period	9.2 Sec

Table 3. Comparison of the observation times needed to achieve a 50% probability of intercept for the broad-band and the narrow-band acquisition systems.

Had probability of intercept not been considered, it might have been assumed erroneously that the broadband receiver would react more rapidly to a new emitter. In fact, for moderate power emitters, just the opposite is true: the scanning receiver reacts more rapidly. The ratio of emitter power over which this condition pertains is given approximately by the ratio of the sensitivities of the two systems, or $\sqrt{2\Delta/B}$, where Δ is the surveillance bandwidth, and B is the narrow-band receiver pass-bandwidth. In this example, it is about 12 dB.

Appendix: Derivation of the Probability of Intercept.

A coincidence of the two window functions illustrated in Figure 2 will occur in the interval $0 \leq t \leq$ MINIMUM (T_1, T_2), provided $-\tau_1 \leq (t_1 - t_2) \leq \tau_2$. Since t_1 and t_2 are independent random variables uniformly distributed over their respective periods, the probability density function (PDF) for t_i $(i = 1,2)$ is:

$$p_{t_i}(t) = \frac{1}{T_i}\ (0 \leq t \leq T_i).$$

It is a property of independent random variables that the PDF of their sum is equal to the convolution of their respective PDF's. Defining $z = t_1 - t_2$, where the two random variables are (t_1) and $(-t_2)$, the resulting PDF of z is illustrated in Figure A1 for $T_1 \leq T_2$.

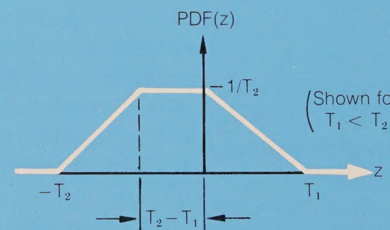


Figure A1. The probability density function (PDF) in terms of the variable z.

The probability of an intercept during the first period is thus:

$$P_{12}(T_1) = \int_{-\tau_1}^{\tau_2} \text{PDF}(z)\, dz.$$

This result is tabulated in Table 1 for the four combinations of conditions which may occur for $T_1 \leq T_2$ and $T_2 \leq T_1$.

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Since joining Watkins-Johnson in 1974, Mr. Hatcher has participated as a staff scientist at Watkins-Johnson's Recon Division, EW Techniques Department. Currently, he is actively engaged in the systems engineering of ECM systems. Mr. Hatcher was previously responsible for technical and administrative direction of major antenna and receiving systems for ground-based and airborne ECM, reconnaissance and surveillance systems, and has substantial experience in the fields of antennas and radar systems. Mr. Hatcher earned his BSEE from the Massachusetts Institute of Technology in 1959, his MSEE in 1964, and has completed course work for his MBA at Northeastern University. He has been an instructor of electromagnetic theory at Lowell Technological Institute. Mr. Hatcher is a senior member of the IEEE, holds a number of patents, and has published numerous articles in the areas of antenna systems, radar systems and transmission lines.

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