VOL. 14 NO. 6 NOVEMBER/DECEMBER 1987

Polarization Measurement: Part 1

WATKINS-JOHNSON COMPANY
Tech-notes

The area of microwave signal analysis has evolved to the point where systems capable of detailed characterizations of signals are in relatively common use. Signal parameters that are routinely measured include frequency, amplitude, phase, direction of propagation, and modulation type and rate. For pulsed signals, properties such as pulse width and PRI are frequently determined. In addition, the time variance of signal parameters is measured because it contains information about scan patterns, frequency agility, Doppler shift and stagger.

Careful analysis of these properties using digital techniques has produced powerful results, but no measurement system can exploit the full vector nature of electromagnetic waves while remaining insensitive to polarization. Unfortunately, information regarding the polarization state of an electromagnetic wave has traditionally been ignored, discarded, or simply unavailable in conventional signal analysis systems, although the utilitarian nature of such information has been discussed at great length.

Applications exist in several fields for precision polarization measurement. The most obvious use for such equipment is in the design and testing of antennas; yet, polarimetry has the potential to enhance system performance in many ways.

As the demand increases for improved, passive-surveillance capabilities, systems will expand to provide previously unavailable measurement data. Since polarization diversity and agility characteristics can be used to distinguish emitters, ELINT collection systems taking advantage of polarization information could add another dimension to the array of parameters used for signal classification. Passive ESM systems could then utilize received polarization data for purposes of emitter identification, or use pulse-by-pulse polarization information as a deinterleaving parameter.

Poelman[1] described a method which allows improvement in target detection performance of active radars in clutter and interference environments through adaptive antenna polarizations, when return polarization information is available. Another polarimetry technique[2] looks promising for improvements in target identification using radar processing that examines the depolarization of back-scattered radiation to obtain geometrical-type information relating to target symmetry and orientation in space.

ECM systems can extend their effective range by adapting the polarization of transmitted jamming signals to match that of the received signal. To optimize such a technique, detailed information about the received polarization is necessary.

Earth-space communication links are susceptible to cross-polarized signal interference resulting from depolarization during propagation, and from small changes in orientation of satellite antennas with respect to ground stations. It has been demonstrated that cancellation of residuals through adaptive antenna polarization is desirable[3], but is dependent on the availability of polarization information about the received signal.

The use of polarization processing techniques is considered the next logical step in the enhancement of many microwave systems. However, taking the step depends on the availability of equipment that is capable of precision polarization measurements.

Many applications require coverage of a broad range of frequencies, rapid processing, and the ability to present measurement results to both human operators and other digital systems. Frequently, the form in which measurement results are presented must be modified to suit a particular application. The need for a powerful, flexible, monopulse polarimeter prompted the development of the Watkins-Johnson Company polarization measurement system.

In the following pages, the basic equations for the vector representation of electromagnetic waves shall be discussed. The concepts of polarization and the Poincaré Sphere shall be presented in order to understand the technique used to measure received polarization. The implementation of the measurement technique used in the Watkins-Johnson Company polarization measurement system and the limitations on the accuracy of such systems will be examined in Part 2.

Coordinate System And Vector Representation

Consider the rectangular Cartesian coordinate system consisting of three mutually orthogonal axes whose variables are denoted x, y, and z. Unit vectors $\vec{a}_x$, $\vec{a}_y$, and $\vec{a}_z$ are oriented as shown in Figure 1. For purposes of analysis we will assume that we are dealing with a plane wave traveling in the z direction. Phase fronts of the wave will be normal to $\vec{a}_z$.

A plane electromagnetic wave traveling in the z direction is composed of electric and magnetic time-varying fields that lie in the xy plane.[4] The fields are perpendicular to each other, and at a specific time can be represented by orthogonal vectors, as shown in Figure 2.

The E and H components of the plane wave vary with time at the same frequency and in the same phase, and the magnitudes are related by a constant. The discussion will, therefore, deal only with the electric field.

The superposition principle states that the total electric-field vector of a wave is the sum of all electric-field vectors composing the wave.[5] This means the total field vector for a wave can be decom-

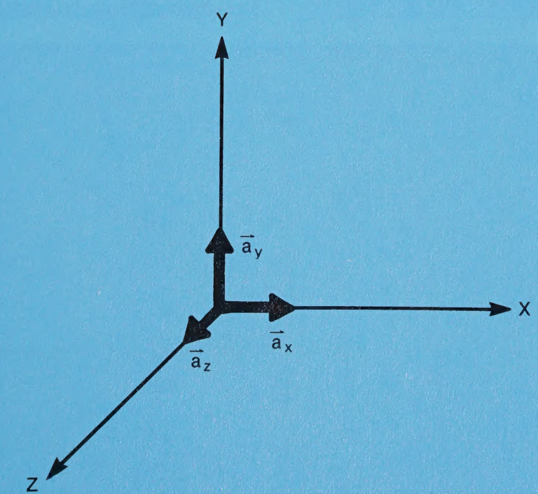

Figure 1. The rectangular coordinate system.

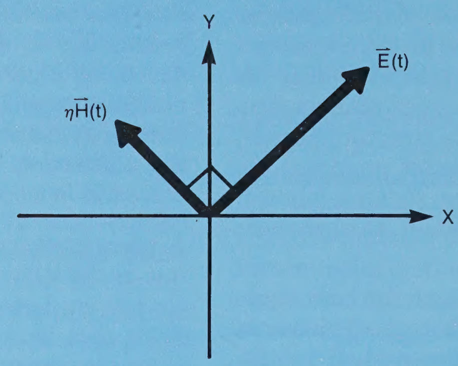

Figure 2. Electric and magnetic fields in a plane wave.

posed into, or constructed from, two orthogonal vector components, as shown in Figure 3.

Component vectors $\vec{E}_x$ and $\vec{E}_y$ are related to the total field vector $\vec{E}$ by the equation,

$$\vec{E}(t) = \vec{E}_x + \vec{E}_y = E_X(t)\vec{a}_x + E_y(t)\vec{a}_y \quad (1)$$

If $\vec{E}(t)$ is a function of a single frequency, the magnitudes of the orthogonal components can be expressed as,

$$E_x(t) = E_x \text{ cox } (\omega t) \quad (2)$$

and

$$E_y(t) = E_y \cos (\omega t + \delta) \quad (3)$$

where, -180 degrees $< \delta <$ 180 degrees.

Polarization

The polarization of a wave is related to the orientation of the electric-field vector with respect to the coordinate system in use.

If an electric-field vector always lies in a given plane, parallel to the direction of propagation, the wave is said to be *linearly polarized.* Since $\vec{E}_x$ and $\vec{E}_y$ fit this description, the orthogonal compo-

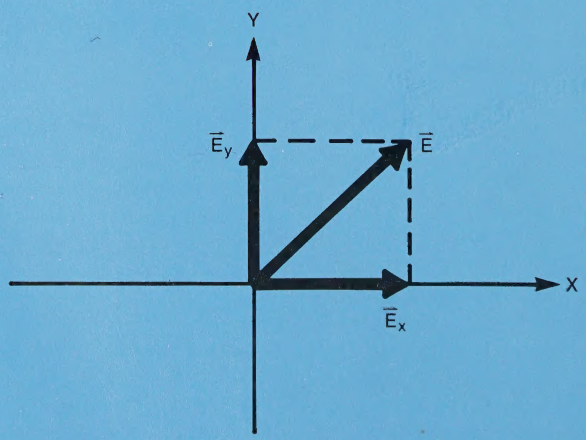

Figure 3. Decomposition of electric-field vector into orthogonal components.

nents we have defined can be thought of as linearly polarized fields which differ in time phase by the angle δ.

In general, any electric field can be resolved into orthogonal linear components of appropriate magnitudes and phase. Combinating $\vec{E}_x$ and $\vec{E}_y$ to form the total field vector, and plotting the locus described by the tip of the resultant vector over time, will generate an ellipse[6] (see Figure 4).

The electric field represented in Figure 4 is said to be *elliptically polarized,* due to the way the field changes with time. The locus is called the *polarization ellipse.* If the tip of the electric field vector had traced out a circle, the polarization would be called *circular*. If the vector remains at a constant angle with respect to the coordinate system, the field is considered linearly polarized. In actuality, all polarizations can be considered elliptical, since the circular and linear polarizations are simply degenerate (special) cases of the polarization ellipse.

Discussions of polarization commonly refer to horizontal, vertical, slant-linear, and circular polarizations.[7] These terms refer to the fact that the coordinate system has been defined in relation to a particular (usually earth-bound) frame of reference. However, polarized waves are rarely of a purely linear or circular polarization, due to reflections, depolarization during propagation, or imperfections in the radiating antenna. Most waves are actually elliptically polarized. However, it is instructive to consider the degenerte cases in order to gain an understanding of the relationships between orthogonal components. The relative amplitudes and phases of $\vec{E}_x$ and $\vec{E}_y$ for these cases are given in Table 1, which assumes that the coordinate system has been aligned such that the x axis is parallel to the horizon.

Polarization	Amplitudes	Phase (δ)
Horizontal	$\lvert\vec{E}_y\rvert = 0$	N/A
Vertial	$\lvert\vec{E}_x\rvert = 0$	N/A
+45 Deg Slant	$\lvert\vec{E}_x\rvert = \lvert\vec{E}_y\rvert$	0 degrees
-45 Deg Slant	$\lvert\vec{E}_x\rvert = \lvert\vec{E}_y\rvert$	180 degrees
Arbitrary Linear Polarization	Don't care	0 or 180 degrees
Right-Hand Circular	$\lvert\vec{E}_x\rvert = \lvert\vec{E}_y\rvert$	-90 degrees
Left-Hand Circular	$\lvert\vec{E}_x\rvert = \lvert\vec{E}_y\rvert$	+90 degrees

Table 1. Amplitude and phase relationships between orthogonal linear components of degenerate polarizations.

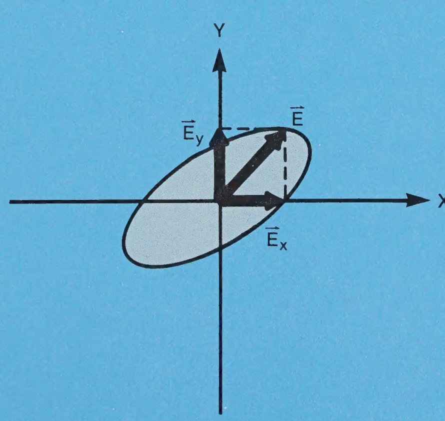

Figure 4. Polarization ellipse generated from the locus of points at the tip of $\vec{E}$ over time.

If the components are in phase, but have unequal amplitudes, the total field is linearly polarized in a direction that makes an angle (τ) with the x axis that is given by the equation,

$$\text{angle } (\tau) = \arctan \left(\frac{|\vec{E}_y|}{|\vec{E}_x|} \right) \tag{4}$$

If δ equals $\pm 90°$, the ellipse is oriented with its major and minor axes aligned with the x and y coordinate axes. Under these conditions, if $E_x > E_y$, the major axis is $2E_x$ and the minor axis is $2E_y$. If $E_x = E_y$, the ellipse degenerates into a circle. If the phase is negative the sense of rotation is clockwise in the z = 0 plane, looking in the direction of propagation, and is called right-handed. If the phase is positive the sense of rotation is counterclockwise.[8]

Axial Ratio, Tilt Angle, Ellipticity Angle

The ratio of the major axis to the minor axis is called the axial ratio (r), which is a frequently used parameter for characterizing polarization. The second parameter used in conjunction with the axial ratio to characterize a polarization is the *sense of rotation.* The final parameter needed to fully define the polarization of a plane wave is the orientation of the major axis with respect to the coordinate system. This parameter is called the *tilt angle* (τ).

Figure 5 shows the major and minor axes of an elliptically polarized field. The tilt angle is also indicated. For consistency, the value of the tilt angle is limited to the range,

$$-90 < \tau < +90 \tag{5}$$

Figure 5 defines an alternate parameter to the axial ratio. The ellipticity angle (ϵ) is simply,

$$\epsilon = \arctan (1/r) \tag{6}$$

Orthogonal Polarizations

Two polarizations are said to be orthogonal if, and only if, their axial ratios are equal and their tilt angles differ by 90 degrees.[9] Any polarization can be thought of as the superposition of two arbitrary, elliptical polarizations.

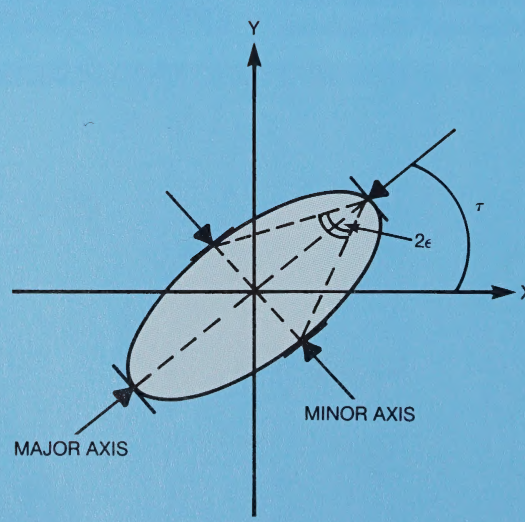

Figure 5. Relationship between major and minor axes, tilt angle, and ellipticity angle.

Equations have been derived to describe polarized fields in terms of the following orthogonal pairs:

- Left-hand/Right-hand Circular
- Horizontal/Vertical
- Left/Right Slant Linear (±45 slant)
- General Case of Orthogonal Elliptical Polarizations

The following discussion will focus on the horizontal and vertical pair.

Power Density and Effective Values

If two fields are orthogonal, the sum of the powers contained in the two fields is equal to the total power in the field.[10] Given orthogonal linear polarizations in the x and y directions, the total power density in the wave (S_w) can be expressed as,

$$S_w = S_x + S_y \tag{7}$$

where,

$$S_x = \frac{E_x^2}{\zeta}, \tag{8}$$

$$S_y = \frac{E_y^2}{\zeta}, \tag{9}$$

and

$$S_w = \frac{E_w^2}{\zeta} \tag{10}$$

E_x, E_y, and E_w are the effective values of the x, y, and total fields. The impedance of space is ζ. For linear polarizations, the effective value of the field is 0.707 times the peak value. For circular polarizations, the effective and peak-field values are equal.

Poincaré Sphere

Poincaré used the relationship in equation (7) to construct the unit sphere shown in Figure 6. The Poincaré Sphere[11] is a very useful graphical aid in visualizing polarization relationships. Poincaré showed that the polarization of a wave can be represented by a unique point on the surface of the sphere. The Poincaré Sphere can be used to express the results of a polarization measurement, rather than

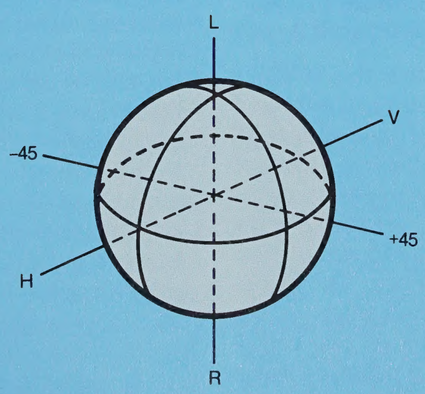

Figure 6. The Poincaré Sphere.

specifying the shape and orientation of the polarization ellipse.

Orthogonal polarizations are located at opposite poles of the sphere. The sphere in Figure 6 is oriented such that the circular polarizations are located at the north and south poles. The family of linear polarizations is located on the equator. The mutually orthogonal axes of the sphere represent the (L, R), (H, V), and (± slant) polarizations.

Defining a point on the surface of the sphere can be done in relation to any of three spherical coordinate systems, although one will be more appropriate for a specific measurement technique than others. The polar angle with respect to each axis is determined by the corresponding polarization ratio. The polarization ratios[12] are defined as,

$$\rho = \tan \gamma = \frac{E_R}{E_L} \quad \text{(circular polarization ratio),} \tag{11}$$

$$\rho_L = \tan \alpha = \frac{E_V}{E_H} \quad \text{(linear polarization ratio),} \tag{12}$$

and

$$\rho_D = \tan \beta = \frac{E_-}{E_+} \quad \text{(diagonal polarization ratio),} \tag{13}$$

where E_R and E_L are the effective values of the right-hand and left-hand circular components of the electric field, E_V and E_H are the effective values of the vertical and horizontal components, and E_- and E_+ are the effective values of the –45 and +45 slant-linear components, respectively. The polarization ratio is an indication of the relative contributions of two orthogonal components of a polarized wave.

The longitude of a point on the sphere about a particular axis is the phase angle by which the component in the numerator of the polarization ratio leads the component in the denominator. The polarization ratio, together with the corresponding phase angle, uniquely specify a polarization.

The Polarization Box

The mutual consistency of the three spherical coordinate systems is confirmed by the polarization box,[13] which is shown in Figure 7. The polarization box is inscribed inside the Poincaré Sphere. It is a graphical aid to understanding the relationships between the three basic sets of polarization components.

The point, W, on the surface of the Poincaré Sphere is located at the corner of the polarization box, opposite the corner located at the origin. The three axes form the edges of the box that intersect at the origin. Specifying the dimensions of the polarization box is equivalent to specifying the polarization.

Polarization Measurement Techniques

The polarization box may be described in several ways. The method of description usually depends on the actual technique used to measure the polarization. The first manner in which the box can be defined is by specifying its three dimensions (i.e., length, width, and height). These dimensions are found directly by measuring the normalized ($\underset{\cdot}{E}$), effective values of the six degenerately polarized components. In other words, the lengths of the three sides are given by,

$$\underset{\cdot}{E}_L^2 - \underset{\cdot}{E}_R^2, \tag{14}$$

$$\underset{\cdot}{E}_V^2 - \underset{\cdot}{E}_H^2, \tag{15}$$

and

$$\underset{\cdot}{E}_-^2 - \underset{\cdot}{E}_+^2 \tag{16}$$

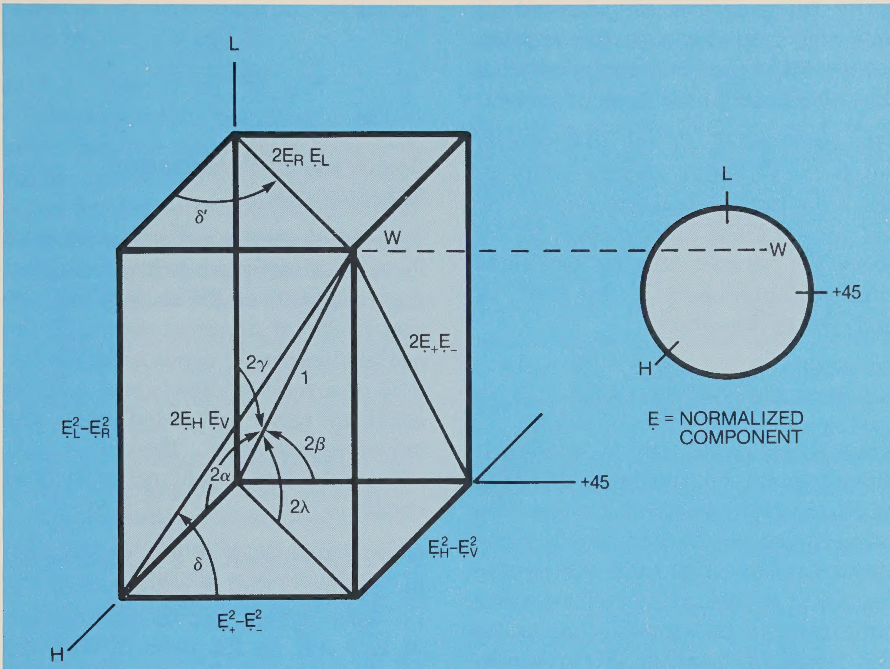

Figure 7. The polarization box.

To measure the contributions of all six cardinal polarizations would require an antenna array with all six receive polarizations available.

Another way to define the polarization box involves specifying the three longitudinal angles about the axes, which are found directly by measuring the phase angles between the same three orthogonal pairs used above. This technique requires the use of the same antenna array with multiple receive polarizations.

Finally, the three polar angles found from the polarization ratios (equations 11, 12, 13) can be specified to define the polarization box, but the direct measurement of the polarization ratios also requires the redundant antenna array described above. Fortunately, it is possible to synthesize the responses of all six cardinal polarizations using only two orthogonal antennas by passing the received signals through the appropriate phase-shifting networks. Alternatively, it is possible to calculate the responses of an orthogonal pair of feeds from the amplitude and phase information extracted from a different orthogonal pair. The inherent relationship between the basic sets of polarization components is represented by the polarization box itself.

To find the lengths of the remaining sides of the polarization box given only the length of one side (which can be measured directly using only two orthogonal antennas) and the longitudinal angle about that side at which the point, W, exists (which is equal to the phase angle between the responses of the two orthogonal antennas used to measure the length of the side), requires simple geometry. In fact, any of the parameters of the polarization box can be calculated from this information. This allows simplification of the antenna array for purposes of polarization analysis, but increases the requirements of the data processing portion of the polarization measurement system.

The relationship of the polarization box to the tilt angle, sense of rotation, and axial ratio is straightforward. The tilt angle[14] (τ) is proportional, to the phase (δ') between the left and right circular components of the field, as given by the equation,

$$\tau = \frac{\delta'}{2} \tag{17}$$

This phase angle can be measured directly or can be calculated using the geometry of the polarization box. The sense of rotation can be found from the circular polarization ratio, which was defined in equation (11). If the circular polarization ratio is greater than unity, the sense of rotation is clockwise, looking in the direction of propagation (right-hand sense). If ρ is less than one, the sense is left-hand. If the orthogonal elements used in the antenna array are not circular, the circular polarization ratio can be calculated from the relationships described above.

The axial ratio is also related to the circular polarization ratio by the equation,

$$r = \frac{\rho + 1}{\rho - 1} \tag{18}$$

Accuracy of Polarization Measurements

Jensen[15] derived a general expression for estimating the errors in polarization measurements on the basis of the relative power of an error signal. If the complex vector $\vec{E}$ denotes the electric field of a received wave, and $\Delta\vec{E}$ the limit of error involved with the measurement of $\vec{E}$, then $\vec{E} + \Delta\vec{E}$ represents the actual measurement result.

If the polarization of $\vec{E}$ is represented by a point D on the surface of the Poincaré Sphere, and the polarization of $\vec{E} + \Delta\vec{E}$ by the point T, then the angular distance between D and T reflects the measurement error. Assuming that the power of the error signal $\Delta\vec{E}$ relative to the power of $\vec{E}$ is known to be less than unity and given by,

$$p = \frac{|\Delta E|^2}{|E|^2} \tag{19}$$

it can be shown that the upper bound

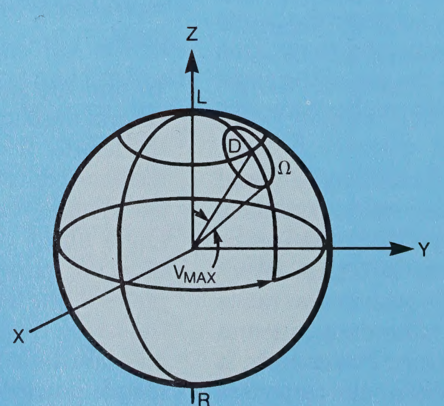

Figure 8. Uncertainty of polarization measurements.

value of V, the angular error, is given by,

$$V_{max} = 2 \arcsin(\sqrt{p}) \qquad (20)$$

If D represents the polarization of the wave, then the measured polarization will be represented by a point T on the sphere located within the circular area, Ω, which is characterized by a maximum angular deviation from D equal to V_{max}, as shown in Figure 8.

The size of the circular area, Ω, on the surface of the Poincaré Sphere can be specified for a particular measurement if the received power level, and the errors in measured parameters, are known. The angular error in a polarization measurement can then be translated into uncertainties in the reported tilt angle, ellipticity and sense, using the geometric relationships of the polarization box.

The Watkins-Johnson Company polarization measurement system discussed in Part 2 allows monopulse measurements of the polarization of received signals in the 2 to 18 GHz region. The system utilizes a spread spectrum multiplexing technique that preserves the amplitude and phase relationships of signals from two orthogonal antennas. The ability to simultaneously measure the relative amplitudes and phase of orthogonal signals without the detrimental effects of multiple receivers greatly improves the accuracy of polarization measurements, while providing the flexibility of broad frequency coverage. In addition, the processing capability incorporated in the system will allow the calculation of polarization parameters as required for presentation to operators or reporting to other systems.

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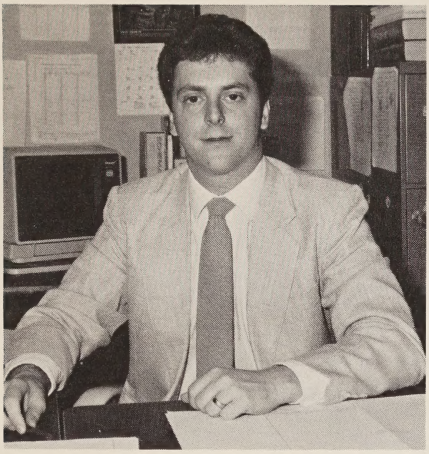

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Printed in U.S.A.